Assignment 9

1. **2.2 Exercise 3: Prove by induction that $1 + 5 + 9 + ... + (4n-3) = n(2n-1)$**
   a. Base Case $n = 1$: $(4(1)-3) = 1(2(1)-1)$

   $$(4-3) = (2-1)$$
   $$1=1$$

   b. So we let $k = n + 1$ for any n integer >=1:

   $$n(2n-1) + (4(n+1)-3) = (n+1)(2(n+1)-1)$$
   $$2n^2-n + (4n+4)-3) = (n+1)((2n+2-1)$$
   $$2n^2-n + + (4n+1) = (n+1)(2n+1)$$
   $$2n^2+3n+1 = 2n^2+3n+1$$

   c. So we can check for any next case of n:

   n=1: 2 +3 +1= 2+3+1 -> 6 = 6

   n=2: 8 +6 +1 = 8 + 6 +1 -> 15 = 15

   n=3: 18 + 9 + 1 = 18+9+1 -> 27 = 27

   So on and so on ...

   This makes the statement true for any n+1 case. Therefore, this statement is true by exhaustive proof and by induction.

2. **Prove that for any positive integer number n, n^3 + 2n is divisible by 3**
   a. Base case $n =1$: $1^3+2(1) = 3/3$ -> true

   b. Case $k = n+1$ for any n integer >=1:

   $$= (n+1)^3+2(n+1)$$
   $$= n^3+3n^2+3n + 1 + 2n +2$$
   $$=n^3+3n^2+5n + 3$$

   c. So the next case n after n = 1,

   $1^3+3(1)^2+5(1) + 3 = 12 / 3 = 4$ is divisibile by 3.

   So the next case n after n = 2,

   $2^3+3(2)^2+5(2) + 3 = 33 / 3 = 11$is divisibile by 3.

   So the next case n after n = 3,

   $3^3+3(3)^2+5(3) + 3 = 72 / 3 = 24$ is divisibile by 3.

   So on and so on ...

This makes it true any n+1 case with n greater than 1.

Therefore, the statement above is true by induction and by exhaustive proof.

3. **Prove that for n >= 1, 9^n − 1 is divisible by 8 for all non-negative integers**

   Hint: 4^(3+1) = 4 * 4^3

   Hint: If 9^n - 1 = 8m, then 9^n = 8m + 1

   a. Base case n = 1: $9^1 - 1 = 9 - 1 = 8/8$ -> true
   b. Case k = n + 1 for any n greater than 1:

      $$9^{n+1} - 1 = 9^n * 9 - 1$$

   c. So the next case n after n = 1, $9^{1+1} - 1 = 80 / 8$ is divisibile by 8.

      So the next case n after n = 2, $9^{2+1} - 1 = 720 / 8$ is divisibile by 8.

      So the next case n after n = 3, $9^{3+1} - 1 == 6560 / 8$ is divisibile by 8.

      So on and so on ...

      This makes it true any n+1 case with n greater than 1 that is non-negative.

      Therefore, the statement above is true by induction and by exhaustive proof.