

1. Prove: If  $n = 25, 100$ , or  $169$ , then  $n$  is a perfect square and is the sum of two perfect squares. Indicate which method of proof you used.
  - a. Suppose  $n^2 = 25$ .  $n$  is equal to  $5$ .  $5^2 = 25$ . Then,  $3^2 + 4^2$  which the sum of two perfect squares  $9 + 16 = 25$ .
  - b. Suppose  $n^2 = 100$ .  $n$  is equal to  $10$ .  $10^2 = 100$ . Then,  $6^2 + 8^2$  which the sum of two perfect squares  $36 + 64 = 100$ .
  - c. Suppose  $n^2 = 169$ .  $n$  is equal to  $13$ .  $13^2 = 169$ . Then,  $5^2 + 12^2$  which the sum of two perfect squares  $25 + 144 = 169$ .

For a-c, I've used the exhaustive proof in which I tried all possible combinations of  $n^2$  sums from  $n = 1$  to  $n = 20$ . At the end, I realized that I could have used pathgorean therom.

2. Prove: The sum of two odd integers is even. Hint: By definition, even integers can be expressed as  $2n$ , thus odd integers can be expressed as  $2n + 1$ 
  - a. We let  $x$  and  $y$  be odd integers.  
 By definition, even integers can be expressed as  $2n$ , thus odd integers can be expressed as  $2n + 1$   
 $\exists x(2n_x + 1 \text{ is odd})$  and  $\exists y(2n_y + 1 \text{ is odd})$  So we claim:  $x + y$  is even.  
 $x + y = 2n_x + 1 + 2n_y + 1 = 2(n_x + n_y) + 2 = 2(n) + 2$   
 We simplify the two equations to  $2n$  which by definition is even and plus 2 which is also even. Therefore  $x + y$  is even thus the sum of two odd integers are also even.
3. Prove: The sum of an even integer and it's square is even
  - a. We let the sum of an even integer be  $n$ . So the squared of  $n$  is  $(n)^2 = n^2$  which is also even. For a base case  $n = 2$ ,  $1(2) = 2$  and  $(1*2)^2 = 4$  are both even. For a case  $n = 4$ ,  $1(4) = 4$  and  $(1*4)^2 = 16$  are both even. For any other  $n$  case greater than 2 that is even, let  $k = n + 2$  is even then  $(n + 2)^2 = n^2 + 4n + 4$  is also even. Therefore, the sum of an even integer and it's square is even by induction and exhuastion proof.
4. Prove by Contradiction: If  $n$  squared is odd, then  $n$  is odd
  - a.  $P$  is  $n$  squared is odd  
 $Q$  is  $n$  is odd

So lets assume  $P$  is  $n$  squared is not odd with  $n = 2n + 1$  and  $Q$  is  $n^2 = (2n + 1)^2 = 4n^2 + 4n + 1$  is odd. By definition,  $2n$  is even (divisible by two) and  $2n + 1$  is odd by definition. So,  $(2n + 1)$  is odd by definition which contradicts with our earlier

statement of P. Even  $\rightarrow$  Odd    Contradiction.

So the statement of  $n$  squared is odd, then  $n$  is odd is true by contradiction.