Assignment 9

1. **2.2 Exercise 3: Prove by induction that 1 + 5 + 9 + ... + (4n-3) = n(2n-1)**
    a. Base Case n = 1: (4(1)-3) = 1(2(1)-1)

       (4-3) = (2-1)

       1=1
    b. So we let k = n +1 for any n integer >=1:

       ... (4n-3) + (4(n+1)-3) = (n+1)(2(n+1)-1)

       ... (4n-3) + (4n+4)-3) = (n+1)((2n+2-1)

       ... (4n-3) + (4n+1) = (n+1)(2n+1)

       ... (4n-3) + 4n+1 = $2n^2$+3n+1
    c. So we can check for any next case of n:

       n=1: 8-2 = 2+3+1 -> 6 = 6

       n=2: 6 ... +8+1 = 8 + 6 +1 -> 15 = 15

       n=3: 15 ... + 12 + 1 = 18+9+1 -> 27 = 27

       So on and so on ...

       This makes the statement true for any n+1 case. Therefore, this statement is true by exhaustive proof and by induction.
2. **Prove that for any positive integer number n, n^3 + 2n is divisible by 3**
    a. Base case n =1: 1^3+2(1) = 3/3 -> true
    b. Case k = n+1 for any n integer >=1:

       = $(n+1)^3$+2(n+1)

       = n^3+3n^2+3n + 1 + 2n +2

       =n^3+3n^2+5n + 3
    c. So the next case n after n = 1, 1^3+3$(1)^2$+5(1) + 3 = 12 / 3 is divisibile by 3.

       So the next case n after n = 2, 2^3+3$(2)^2$+5(2) + 3 = 33 / 3 is divisibile by 3.

       So the next case n after n = 3, 3^3+3$(3)^2$+5(3) + 3 = 72 / 3 is divisibile by 3.

       So on and so on ...

       This makes it true any n+1 case with n greater than 1.

       Therefore, the statement above is true by induction and by exhaustive proof.

3. **Prove that for n >= 1, 9^n − 1 is divisible by 8 for all non-negative integers**

   Hint: 4^(3+1) = 4 * 4^3

   Hint: If 9^n - 1 = 8m, then 9^n = 8m + 1

   a. Base case n = 1: 9^1 - 1 = 9 - 1 = 8/8 -> true
   b. Case k = n + 1 for any n greater than 1:

      $$9^{n+1} - 1 = 9^n * 9 - 1$$

   c. So the next case n after n = 1, 9^{1+1} - 1 = 80 / 8 is divisibile by 8.

      So the next case n after n = 2, 9^{2+1} - 1 = 720 / 8 is divisibile by 8.

      So the next case n after n = 3, 9^{3+1} - 1 == 6560 / 8 is divisibile by 8.

      So on and so on ...

      This makes it true any n+1 case with n greater than 1 that is non-negative.

      Therefore, the statement above is true by induction and by exhaustive proof.