FEDERAL STATE AUTONOMOUS EDUCATIONAL INSTITUTION OF HIGHER EDUCATION
ITMO UNIVERSITY

Report
On the practical task No. 4
“Algorithms for unconstrained nonlinear optimization. Stochastic and metaheuristic algorithms”

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St. Petersburg
2022

Goal

The use of stochastic and metaheuristic algorithms (Simulated Annealing, Differential Evolution, Particle Swarm Optimization) in the tasks of unconstrained nonlinear optimization and the experimental comparison of them with Nelder-Mead and Levenberg-Marquardt algorithms

Formulation of the problem

The use of the methods of stochastic and metaheuristic algorithms in the tasks of unconstrained nonlinear optimization.

Brief theoretical part

There are lots of ways of solving the minimization problem, in this lab we are going to use methods of stochastic and metaheuristic algorithms, lets describe them below.

- **Simulated Annealing** — is a metaheuristic algorithm that solves the optimization problem similar to the process of annealing in metallurgy. This method can be applied to Traveling Salesman Optimization (TSP-OPT) which is a NP-hard problem and Traveling Salesman Search(TSP) is NP-complete.
    - Theoretical Time Complexity — $O(\sqrt{m})$, optimal cooling schd. of length m
- **Differential Evolution** — is an metaheuristic algorithm that solves the optimization problem by maintaining a population of agents, i.e. candidate solutions, creating new agents by combining existing ones and further keeping the best one.
    - Theoretical Time Complexity — $O(MN^2)$, where *M − number of objectives, N − population size*
- **Particle Swarm Optimization** — is a metaheuristic algorithm that solves the optimization problem by iterative moving candidate solutions (particles) with certain velocity. Each particle's movement is influenced by its local best known position and is guided by the swarm's best known position that is updated as better positions are found by other particles.

However, until we have a new theory, every metaheuristic has a **complexity of infinity** — since there is no theory developed that establishes that any metaheuristic finds and identifies an optimal solution for a large enough class of problems.

Results

1) Explain differences (if any) in the results obtained.

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Nelder-Mead method: [-0.99697241  0.99745732 -2.00094551  1.00096141] function evaluations: 777 iterations: 463
```

Pic. 1 — Nelder-Mead method

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Levenberg-Marquardt method: [-0.99989498  1.00035571 -2.00084629  1.00086232] function evaluations: 131
```

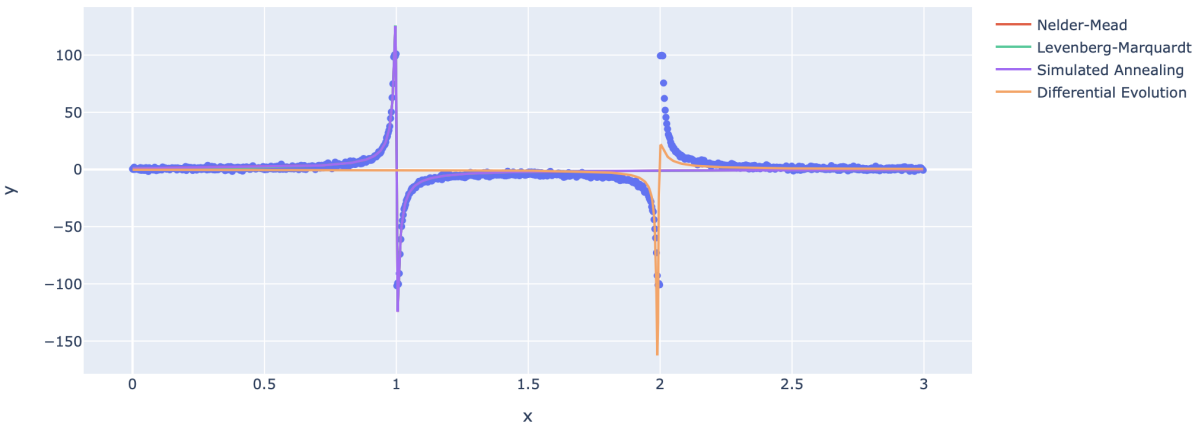
Pic. 2 — Levenberg-Marquardt method

```
Simulated Annealing: [-0.99673356  0.99721816 -2.00094519  1.00096108] function evaluations: 9006 iterations: 1000
```

Pic. 3 — Simulated Annealing

```
Differential Evolution: [ 0.4799358  -0.95813982 -3.98301231  3.9661123 ] function evaluations: 1765 iterations: 5
```

Pic. 4 — Differential Evolution



Pic. 5 — The results of all methods

Pic. 6 — First (random) iteration route

Pic. 7 — Best route

Conclusions

As can be seen from the graphs and console output above, all methods have good results in minimizing the function. This happened because the value of the function and the parameters are close for both linear and rational functions. It is worth noting the difference in execution time and resource usage.

Questions for self-monitoring from Russian educational materials

1. What stochastic and metaheuristic algorithms for nonlinear are you aware of? Name at least three.

Honestly, we took the answer from the 4th lecture, but to be more interested in topic let's translate terms, that gets artificial complication:

- Stochastic = random;
- Metaheuristic = do not fully provide a proper formalization of the intuitive concept of algorithm.

Monte Carlo (stochastic) methods — are a wide class of algorithms that rely on repeated random sampling to obtain solutions to an optimization problem. The methods are most useful when it is impossible or difficult to apply other approaches.

Genetic algorithm — inspired by the process of natural selection that belongs to the larger class of evolutionary algorithms (EA). Genetic algorithms are commonly used to generate high-quality solutions to optimization and search problems by relying on biologically inspired operators such as mutation, crossover and selection.

DE and **PSO** algorithms were discussed in the theory part.

2. Do such algorithms always find the global extremum of the target of a target function in a nonlinear optimization problem?

Not always, because the discontinuities of the objective function may be due to the nature of the optimized object in the mathematical model (for example, shock effects, resonance phenomena, jumps in geometric dimensions or material properties, etc.). In some cases, the discontinuity points are known; but at the same time, there are problems with no a priori estimates of discontinuity points. It is known, however, that such points do exist. To be more exact, we could compare GD and SA.

Appendix

DataLore : site. – URL: https://datalore.jetbrains.com/notebook/RemqSkuJwmr1PM4Gc3cBqB/CZyH5NsMrCbi54ZMd6g0E3 (circulation date: 28.09.2022)