FEDERAL STATE AUTONOMOUS EDUCATIONAL INSTITUTION OF HIGHER EDUCATION
ITMO UNIVERSITY

Report
On the practical task No. 2
“Algorithms for unconstrained nonlinear optimization. Direct methods”

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St. Petersburg
2022

**Goal:**

Experimental study of the time complexity of different algorithms.

**Formulation of the problem**

The use of direct methods (one-dimensional methods of exhaustive search, dichotomy, golden section search; multidimensional methods of exhaustive search, Gauss (coordinate descent), Nelder-Mead) in the tasks of unconstrained nonlinear optimization.

**Brief theoretical part**

There are lots of ways of solving the minimization problem, in this lab we are going to use several one-dimensional and multidimensional methods, lets describe them below.

- One-dimensional methods:
  - Exhaustive search — also known as generate and test, is a very general problem-solving technique and algorithmic paradigm that consists of systematically enumerating all possible candidates for the solution and checking whether each candidate satisfies the problem's statement;
  - Dichotomy — is a partition of a whole into two parts;
  - Golden section search — is based on evaluating the objective function at different points in the interval. We choose these points in such a way that an approximation to the minimizer may be achieved in as few evaluations as possible.
- Multidimensional methods:
  - Exhaustive search — works the same as one-dimensional,
  - Gauss (coordinate descent) — solves optimization problems by successively performing approximate minimization along coordinate directions or coordinate hyperplanes.,
  - Nelder-Mead — multidimensional unconstrained optimization without derivatives. The method uses the concept of a simplex, which is a special polytope of $n + 1$ vertices in $n$ dimensions. The method approximates a local optimum of a problem with n variables when the objective function varies smoothly and is unimodal. Typical implementations minimize functions, and we maximize $f(x)$ by minimizing $-f(x)$.

**Results**

1) Explain differences (if any) in the results obtained.

```
Min of function f1_x: [0.0, 0.0]
 Number of iterations:1001

Min of function f2_x: [0.0, 0.2]
 Number of iterations:1001

Min of function f3_x: [0.1, -0.05440211108893698]
 Number of iterations:901
```

Pic. 1 — Exhaustive search performance

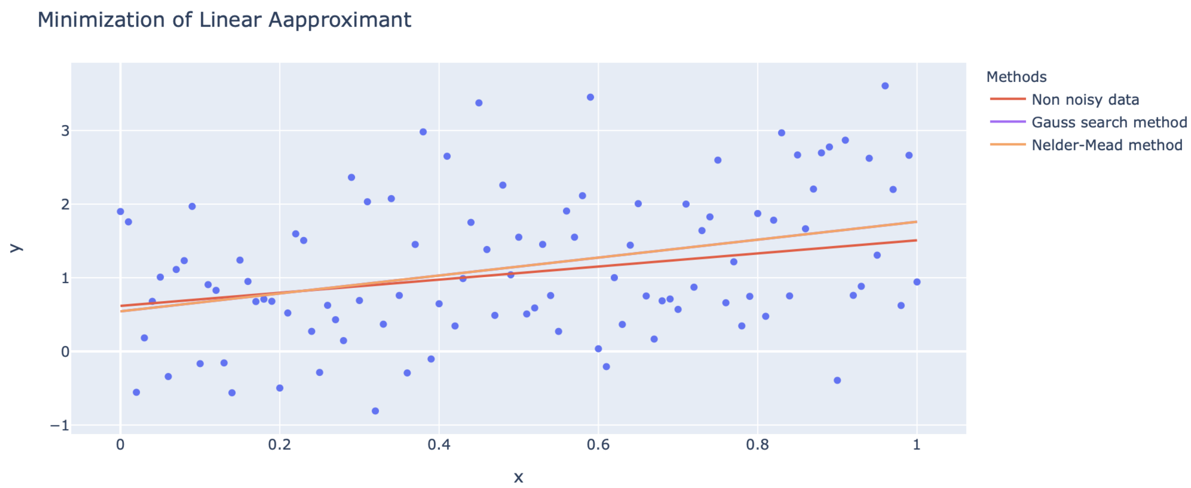
```
1 dichotomy(f1_x, "f1_x", boundaries)

Min of function f1_x: Root is 0 with borders [0.99952294921875, 1.0]
 Number of iterations:10

1 dichotomy(f2_x, "f2_x", boundaries)

Min of function f2_x: Root is 0 with borders [0.99952294921875, 1.0]
 Number of iterations:10

1 dichotomy(f3_x, "f3_x", boundaries[:100])

Min of function f3_x: Root is 0 with borders [0.09866796875, 0.099]
 Number of iterations:7
```

Pic. 2 — Dichotomy method performance

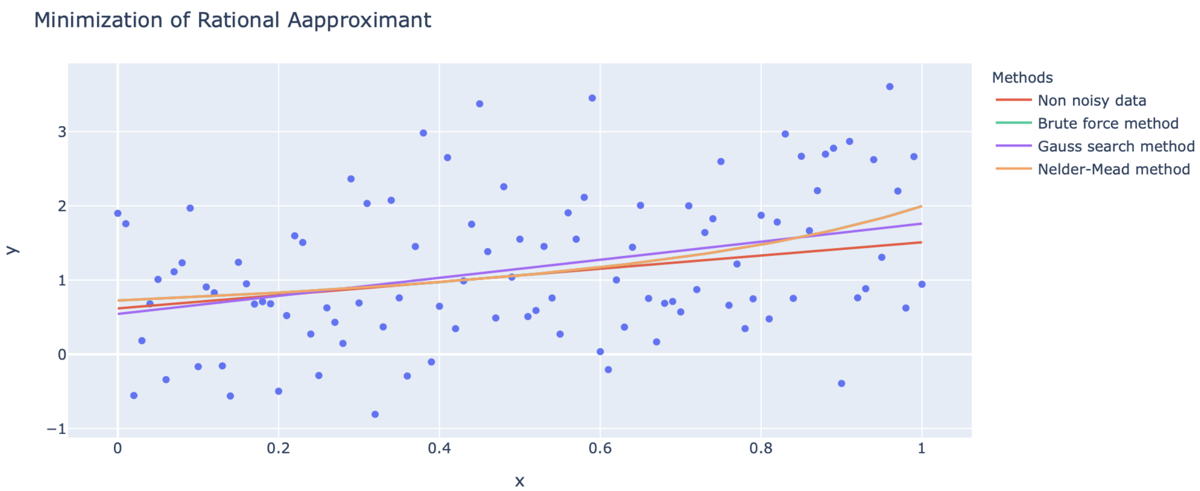
```
1 golden_section_search(f1_x, "f1_x", boundaries)

Min of function f1_x: Root is 0.00036668424059301056
 Number of iterations:14

1 golden_section_search(f2_x, "f2_x", boundaries)

Min of function f2_x: Root is 0.20007975243568882
 Number of iterations:14

1 golden_section_search(f3_x, "f3_x", boundaries)

Min of function f3_x: Root is 0.22255262862968567
 Number of iterations:14
```

Pic. 3 — Golden section search performance

2) Analyze the results obtained (in terms of number of iterations, precision, number of function evaluations, etc.)




Pic. 4 — Linear approximation results




Pic. 5 — Rational approximation results

## Conclusions

As can be seen from the graphs and console output above, all methods have good results in minimizing the function. This happened because the value of the function and the parameters are close for both linear and rational functions. It is worth noting the difference in execution time and resource usage.

### Questions for self-monitoring from Russian educational materials

1. Provide a definition of direct optimization methods.

By definition, they use when searching for $x^*$ only the values of the function $f$ itself, but not its derivatives.

2. For a convex function $\boldsymbol{f : [0, 1] \rightarrow R}$ which of the methods — brute force, dichotomy or golden ratio — will for a fixed $\boldsymbol{\varepsilon > 0}$ use more values of ***f*** on the interval **[0, 1]**? What is the fundamental difference between the dichotomy and the golden ratio methods?

   Golden section search method will be using more values of ***f***. *x1, x2, f(x1)* and *f(x2)* and $(\frac{x_1+x_2}{2} = \frac{a_0+b_0}{2})$, The first iteration requires to calculate two points and two values of ***f*** *and* on the forthcoming iterations, calculate one point and one corresponding value of ***f***.

   However when we are using the dichotomy method we have to calculate ***x1, x2, f(x1)*** and **f(x2)** and repeat the reducing procedure by calculating both.

**Appendix**

DataLore : site. – URL: https://datalore.jetbrains.com/notebook/RemqSkuJwmr1PM4Gc3cBqB/sMEjCnY4wbBlCSCZL0IjJq (circulation date: 13.09.2022)