Given data: Catphan phantom CBCT images.

The GOAL: is to compute MTF (Modulation Transfer Function) for the apparatus used to obtain the given CBCT images of Catphan phantom.

Catphan phantom contains several types of gauges – one of them (that is imaged on 66, 67 and 68 CBCT slices) are High Resolution test gauges constructed from equidistantly separated metal plates forming several groups with different "spatial frequency" (67 slice is presented on the figure below – fig.1)

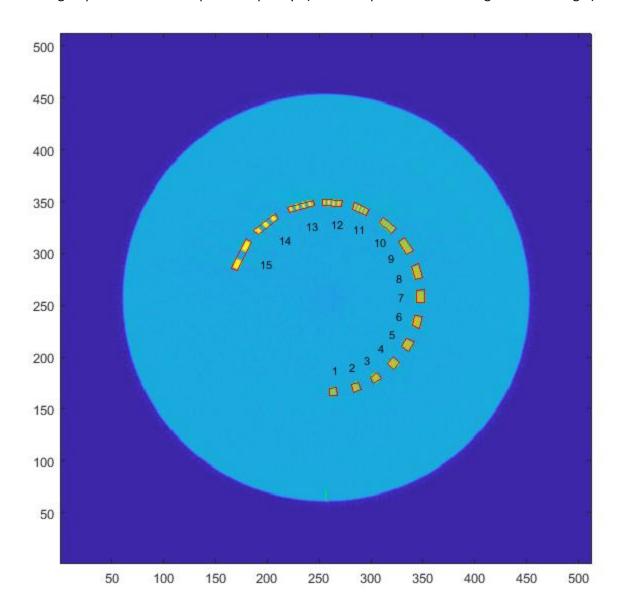


Figure 1 Catphan phantom - CBCT slice #67 (high resolution gauge marked by a red line trapezoid)

Modulation Transfer Function is a measurement of the imaging system's ability to transfer contrast from the specimen to the intermediate image plane at a specific resolution. In the given case, it was decided to use a Droege-Morin method of MTF computing:

$$MTF(f) = \frac{\pi\sqrt{2}}{4} \frac{M(f)}{M_0}, \ M(f) = \sqrt{\left(SD_{gauge}\right)^2 - SD^2}, \ SD^2 = \frac{\left(SD_{metal}\right)^2 + \left(SD_{background}\right)^2}{2}$$
 
$$M_0 = \frac{\left|M_{metal} - M_{background}\right|}{2}$$

Where "SD" means the standard deviation, "M" means the mean value, "gauge" index means the rectangular region on the image that contains the gauge, "metal" index means the region on the image that contains only the metal (here the largest plate on the image is used), "background" index means the region on the image where are no gauge located (here the 20x20 pixels region in the center of the image is taken).

The spatial frequency for each gauge, "f", computed simply as amount of plates in the gauge divided by its length (the length is the distance between the first and the last plate in the gauge).

$$f = \frac{N_{plates}}{L}$$

Assuming that amount of plates in each gauge is a known data the task is transforming into the "gauge localization" task:

To simplify the coding a bit, will assume a "polar rectangular" instead of straight rectangular regions for each gauge – "polar rectangular" is a circular section bounded by two angles and two radius values. In the given case each gauge has a small angular size so this polar rectangular are very close to the straight rectangular (the figure bounded by two x-axis values and two y-axis values), so the using of the "Droege-Morin method" interpretation given above is reasonable.

To localize each gauge region it was decided to move a ray starting from the bottom (near the gauge #1 - -90 degrees) in the counterclockwise direction (from the gauge #1 to gauge #15) making a full circle. Moving the ray with the smallest reasonable angle step (angular size of one pixel), the program analyzes values of the pixels that lies on the ray path (fig.2 – fig.3): if some pixels has a value greater than value of the "free-of-objects" region mean value (20x20 pixels in the center) by 10 standard deviation of the same "free-of-objects" region or more it is classified as a "gauge". The program memorize each such "hit" – the fist point of the ray with the "large" value, the last point of the ray with the "large" value ("radius begin" and "radius end"), the mean value between the first and the last point and the angular position of the ray. Later, analyzing this hits the program will identify "polar rectangles" for each gauge: "radius begin", "radius end", "angle begin", "angle end" (fig.4).

Knowing the boundaries (two angles and two radiuses) for each gauge – it is easy to identify the set of image points (pixels) that lies inside these boundaries and so to identify the set of values for each specific gauge (fig.5). Using these values, it is easy to compute its standard deviation or the mean value that is used for MTF computing ( $SD_{gauge}$ ).

To estimate the  $M_{metal}$  and  $SD_{metal}$  values (that is defined as Mean and Standard Deviation of the pixels values at the largest plate) it is required to identify the set of image pixels that belong only to the largest plate on the image – to do so, the set of pixels that have been found for the largest (#15) gauge is

split to two classes using a k-means clustering analysis, and points with the higher values is assumed to be the "plates" points.

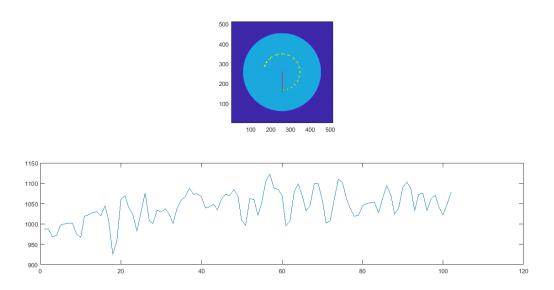


Figure 2 Gauge localization: Ray out of the gauge

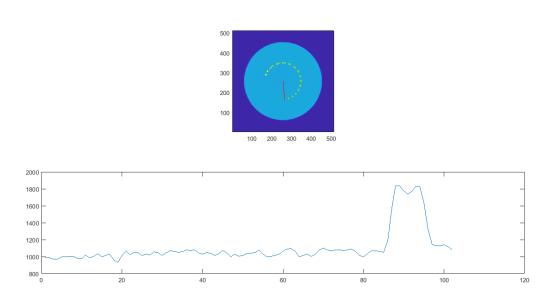


Figure 3 Gauge localization: Ray hit the gauge

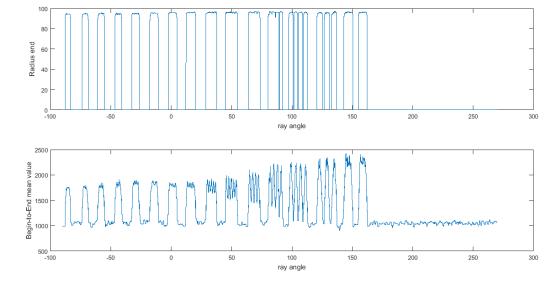


Figure 4 Ray hits: "radius end" AND the "mean value"

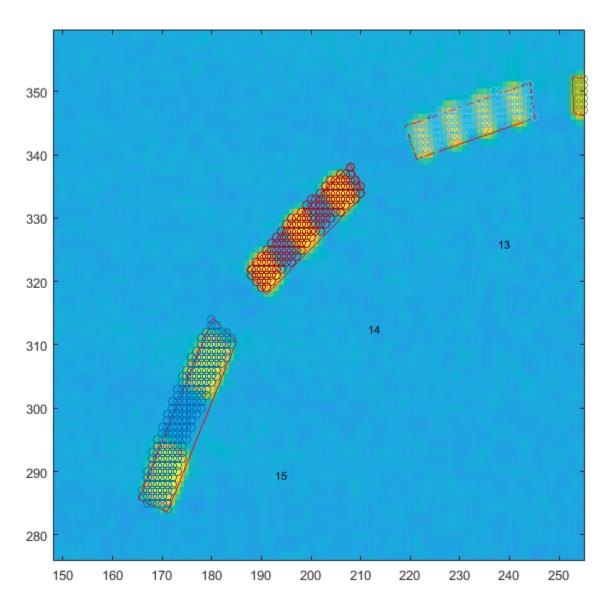


Figure 5 Gauge localization: find the gauge pixels

Resulting MTF is given on the fig.6:

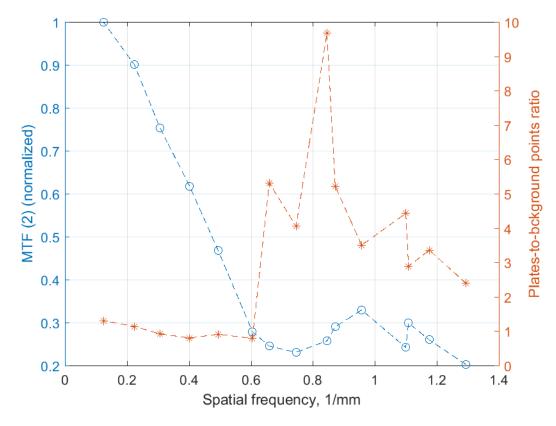


Figure 6 MTF (along with the "metal"-to-"water" amount of points ratio in the gauge region)

Each gauge points were additionally split to two classes: with low and high values, and the ratio between these two sets were computed. It was assumed that each gauge region should contain nearly equal or comparable amount of "plate" and "background" points in case if plates are separable for the specific gauge. The fig.6 contains the ratio between "plate" and "water" points for each gauge (for each spatial frequency) and It is evident that for large gauge where plates can be easily detected by eye – the ratio is near the [0.5-1], but for gauges where it is hard or impossible to detect a separate plate this ratio has a very high value. In the case of automotization this parameter could be used to identify if the plates are really separable for a specific gauge, and, for example set all MTF values for "non-separable" gauges to zero or just assume the last separable gauge as a limit for resolution instead of using a trusted (10% or 20%) level on the MTF curve.