Bachelor of Science Examination, 2024

(First Year, First Semester)

Mathematics

Paper : MDC – 01

(Discrete Mathematics)

Time : *Two Hours* *Full Marks* : 40

Unexplained Symbols & Notations have the usual meaning.
Use separate Answer scripts for each Part.

PART—I (24 Marks)

Answer ***any four*** questions : 6×4=24

1. *(a)* Find the minimum number of students needed to guarantee that five of them belong to the same batch among the batches UG I, UG II, UG III, PG I and PG II. 3

   (b) There are 20 students in a class. In how many ways can the 20 students take 4 different tests if each test is taken by 5 students? 3

2. *(a)* Solve the difference equation $2a_n = 7a_{n-1} - 3a_{n-2}$, $n \geq 2$ for the given initial conditions $a_0 = 1, a_1 = 1$. 3

   (b) Using generating function solve the recurrence relation $a_n = 3a_{n-1} + 2$, where $n \geq 1$ and $a_0 = 2$. 3

3. (a) Define a *graph*. Let G be a simple graph with at least two vertices. Then show that G has at least two vertices of the same degree. 1+2

(b) Prove that for any simple graph G with 6 vertices, either G or its complement $\overline{G}$ contains a triangle as a subgraph. 3

4. (a) Define a *bipartite graph*. Let $G = (V, E)$ be a regular bipartite graph with bipartition $V = X \cup Y$. Show that $X = Y$. 1+3

(b) Let G be a graph with 6 components and 20 edges. Find the maximum possible number of vertices in G. 2

5. (a) Let T be an acyclic graph with n vertices and k components. Determine the number of edges of T. 2

(b) Show that a connected even graph is Eulerian. 4

6. (a) Define a *planar graph*. Prove that the complete bipartite graph $K_{3,3}$ is not planar. 1+2

(b) Find all Hamiltonian cycles of $K_{3,3}$. 3

PART—II (16 Marks)

Answer **Q. No. 1** and ***any one*** from the rest : 8×2=16

1. Draw the Hasse diagram of the poset $\{\{a,b,c,d\}, \{a,b\}, \{a,c\}, \{a,d\}, \{a\}\}$ with respect to set inclusion as partial order.

Determine if this poset is

(i) a lattice;

(ii) a distributive lattice;

(iii) a modular lattice. 8

2. *(a)* Give an example, with proper explanation, of a Boolean algebra with 16 elements.

   (b) Let P be a totally ordered set. Then P is a lattice. — True or False! Give reason in support of your answer.

   (c) Give an example of a poset which is not a lattice. Explain with reason. 4+2+2

3. *(a)* Let L be a distributive lattice. Prove that for all $a, b, c \in L$,

   $a \vee b = a \vee c$ and $a \wedge b = a \wedge c$ implies that $b = c$.

   (b) Prove that in a distributive lattice, complement of an element (if exists) is unique. 6+2

4. *(a)* Prove that the lattice of subspaces of a vector space is a modular lattice with respect to set inclusion as partial order.

   (b) Suppose V is a vector space over a field F. Prove that lattice of subspaces of V is not a sub lattice of the power set lattice $P(V)$, both with respect to set inclusion as partial order. 6+2

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