Bachelor of Science (Major) Examination, 2024

(First Year, First Semester)

Mathematics

Paper : CORE – 02

(Geometry and Linear Algebra)

Time : 2 *Hours* *Full Marks* : 40

Use separate answer scripts for each Part.

PART—I (20 Marks)

Answer ***any five*** questions from the following : (4×5)

1. A sphere of radius $2k$ passes through the origin and meets the axes in A, B and C respectively. Show that the locus of the centroid of the tetrahedron $OABC$ is the sphere $(x^2 + y^2 + z^2) = k^2$.

2. Find the equation of the cone with vertex at origin, which passes through the curve of intersection of plane $lx + my + nz = p$ and $ax^2 + by^2 + cz^2 = 0$.

3. PSP' is a focal chord of the conic. Prove that the angle between tangents at P and P' is $\tan^{-1}\left(\dfrac{2e\sin\alpha}{1-e^2}\right)$, where α is the angle between the chord and the major axis.

4. Obtain the equation of the cylinder, whose generators intersect the plane curve $ax^2 + by^2 = 1$, $z = 0$ and are parallel to the straight line $\frac{x}{l} = \frac{y}{m} = \frac{z}{n}$.

5. Prove that the length of the common chord of circles $(x-a)^2 + (y-b)^2 = c^2$ and $(x-b)^2 + (y-a)^2 = c^2$ is $\sqrt{4c^2 - 2(a-b)^2}$.

6. Show that the area enclosed by the curve in which the plane $z = h$ cuts the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ is $\pi ab\left(1 - \frac{h^2}{c^2}\right)$.

PART—II (20 Marks)

Let $\mathbb{R}$ denote the field of all real numbers.

Answer ***any four*** questions : 4×5

1. Define a *subspace* of a vector space. Determine whether S is a subspace of $\mathbb{R}^5$, where

$$S = \left\{(a_1, a_2, a_3, a_4, a_5) \in \mathbb{R}^5 \mid a_1^2 + a_3^2 = 0, 2a_2 + 3a_5 = 5a_4\right\}$$

Find a basis and the dimension of S over $\mathbb{R}$ if S is a subspace of $\mathbb{R}^5$. 1+4

2. Solve the following system of linear equations by Gaussian elimination process :

$$\begin{array}{rcrcrcrcrcr} x_1 & - & 2x_2 & & & + & 2x_4 & - & 6x_5 & = & 4 \\ 2x_1 & - & 4x_2 & + & 2x_3 & & & + & 4x_5 & = & 6 \\ x_1 & - & 2x_2 & + & 3x_3 & - & 3x_4 & + & 10x_5 & = & 16 \end{array}$$ 5

3. Define a *basis* of a vector space. Find a basis of $\mathbb{R}^5$ that contains $\{(1, 0, -4, 3, 5), (-2, 1, 2, 2, -3)\}$. 1+4

4. Define V be a finite dimensional vector space over $\mathbb{R}$. Let $T : V \to V$ be a linear transformation. Prove that T is one-to-one if and only if T is onto. 5

5. Let $T : \mathbb{R}^4 \to \mathbb{R}^2$ be the linear transformation defined by

$T(x, y, z, t) = (x + 5y - 3z + t, 4z - 5t)$.

Find the matrix representation of T with respect to the ordered bases

$\{(1, -1, 1, 0), (0, 2, -2, 1), (1, 1, 1, 1), (3, 2, 1, 0)\}$ and $\{(2, 3), (5, 7)\}$ of $\mathbb{R}^4$ and $\mathbb{R}^2$ respectively. 5

6. Find eigenvalues and corresponding eigen-spaces of the matrix and determine whether it is diagonalizable.

$$\begin{pmatrix} 2 & 0 & -1 \\ 4 & 1 & -4 \\ 2 & 0 & -1 \end{pmatrix}$$

1+2+2

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