C++ contest library

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Contents

1 Environment and workflow

1.1 Compilation script

```
#!/bin/bash

g++ $1 -o ${1%.*} -std=c++11 -Wall -Wextra -Wshadow -
    ftrapv -Wfloat-equal -Wconversion -Wlogical-op -
    Wshift-overflow=2 -fsanitize=address -fsanitize=
    undefined -fno-sanitize-recover
```

2 General techniques

2.1 Bit tricks

g++ builtin functions:

- `__builtin_clz(x)`: number of zeros in the beginning
- `__builtin_ctz(x)`: number of zeros in the end
- `__builtin_popcount(x)`: number of set bits
- `__builtin_parity(x)`: parity of number of ones

There are separate functions of form `__builtin_clzll(x)` for 64-bit integers.

Iterate subsets of set s:

```
int cs = 0;
do {
        // process subset cs
} while(cs=(cs-s)&s);
```

3 Data structures

3.1 Lazy segment tree

Implements range add and range sum query in $O(log(n))$. 0-indexed.

```
#include <iostream>

using namespace std;
typedef long long ll;

const int N = (1<<18); // segtree max size

ll st[2*N]; // segtree values
ll lz[2*N]; // lazy updates
bool haslz[2*N]; // does a node have a lazy update
    pending

void push(int s, int l, int r) {
        if (haslz[s]) {
                st[s] += (r-l+1)*lz[s]; // change
                    operator+logic

                if (l != r) {
                        lz[2*s] += lz[s]; // change
                            operator
                        lz[2*s+1] += lz[s]; // change
                            operator
                        haslz[2*s] = true;
                        haslz[2*s+1] = true;
                }
```

```
            lz[s] = 0; // set to identity
            haslz[s] = false;
        }
}

ll kysy(int ql, int qr, int s = 1, int l = 0, int r = N
    -1) {
        push(s, l, r);
        if (l > qr || r < ql) {
                return 0; // set to identity
        }
        if (ql <= l && r <= qr) {
                return st[s];
        }

        int mid = (l+r)/2;
        ll res = 0; // set to identity
        res += kysy(ql, qr, 2*s, l, mid); // change
            operator
        res += kysy(ql, qr, 2*s+1, mid+1, r); // change
            operator
        return res;
}

void muuta(int ql, int qr, ll x, int s = 1, int l = 0,
    int r = N-1) {
        push(s, l, r);
        if (l > qr || r < ql) {
                return;
        }
        if (ql <= l && r <= qr) {
                lz[s] += x; // change operator
                haslz[s] = true;
                return;
        }

        int mid = (l+r)/2;
        muuta(ql, qr, x, 2*s, l, mid);
        muuta(ql, qr, x, 2*s+1, mid+1, r);

        st[s] = st[2*s] + st[2*s+1]; // change operator
        if (haslz[2*s]) {
                st[s] += (mid-l+1)*lz[2*s]; // change
                    operator+logic
        }
        if (haslz[2*s+1]) {
                st[s] += (r-(mid+1)+1)*lz[2*s+1]; //
                    change operator+logic
        }
}

void build(int s = 1, int l = 0, int r = N-1) {
        if (r-l > 1) {
                int mid = (l+r)/2;
                build(2*s, l, mid);
                build(2*s+1, mid+1, r);
        }
        st[s] = st[2*s]+st[2*s+1]; // change operator
}

// test code below
int n, q;

/*
        TESTED, correct
        Allowed indices 0..N-1
        2 types of queries: range add and range sum
*/
int main() {
        ios_base::sync_with_stdio(false);
        cin.tie(0);
        cin >> n >> q;
        for (int i = 1; i <= n; ++i) {
                cin >> st[i+N];
        }
        build();
        for (int cq = 0; cq < q; ++cq) {
                int tp;
                cin >> tp;
                if (tp == 1) {
                        int l, r;
                        ll x;
                        cin >> l >> r >> x;
                        muuta(l, r, x);
                }
                else {
                        int l, r;
                        cin >> l >> r;
                        cout << kysy(l, r) << "\n";
                }
```

```
        }
        return 0;
}
```

3.2 Sparse segment tree

Implements point update and range sum query in $O(log(n))$. Memory usage is around 40 MB with a range of $2^{30} = 10^9$ after 10^5 operations. 0-indexed.

```
#include <iostream>

using namespace std;
typedef long long ll;

const int N = 1<<30; // max element index

struct node {
    ll s;
    int x, y;
    node *l, *r;
    node (int cs, int cx, int cy) : s(cs), x(cx), y(cy)
        {
        l = nullptr;
        r = nullptr;
    }
};

node *st = new node(0, 0, N); // segtree root node

void update(int k, ll val, node *nd = st) {
    if (nd->x == nd->y) {
        nd->s += val; // change operator
    }
    else {
        int mid = (nd->x + nd->y)/2;
        if (nd->x <= k && k <= mid) {
            if (nd->l == nullptr) nd->l = new node(0, nd
                ->x, mid);
            update(k, val, nd->l);
        }
        else if (mid < k && k <= nd->y) {
            if (nd->r == nullptr) nd->r = new node(0,
                mid+1, nd->y);
            update(k, val, nd->r);
        }
        ll ns = 0; // set to identity
        if (nd->l != nullptr) ns += (nd->l)->s; //
            change operator
        if (nd->r != nullptr) ns += (nd->r)->s; //
            change operator
        nd->s = ns;
    }
}

ll query(int ql, int qr, node *nd = st) {
    if (ql <= nd->x && nd->y <= qr) return nd->s;
    if (nd->y < ql || nd->x > qr) return 0; // set to
        identity
    ll res = 0; // set to identity
    if (nd->l != nullptr) res += query(ql, qr, nd->l);
        // change operator
    if (nd->r != nullptr) res += query(ql, qr, nd->r);
        // change operator
    return res;
}

int q;

// TESTED, correct
// implements point add and range sum query
int main() {
    ios_base::sync_with_stdio(false);
    cin.tie(0);
    cin >> q;
        for (int i = 0; i < q; ++i) {
                int tp;
                cin >> tp;
                if (tp == 1) {
                        int a, b;
                        cin >> a >> b;
                        cout << query(a, b) << "\n";
                }
                else {
                        int k;
                        ll x;
                        cin >> k >> x;
```

```
                    update(k, x);
                }
            }
    return 0;
}
```

3.3 2D segment tree

Implements point update and subgrid query in $O(log^2(n))$. Grid is 0-indexed.

```
#include <iostream>

using namespace std;
typedef long long ll;

const int N = 1<<11;

int n, q;

ll st[2*N][2*N];

// calculate subgrid sum from {y1, x1} to {y2, x2}
// 0-indexed
ll summa(int y1, int x1, int y2, int x2) {
    y1 += N;
    x1 += N;
    y2 += N;
    x2 += N;

    ll sum = 0;

    while (y1 <= y2) {
        if (y1%2 == 1) {
            int nx1 = x1;
            int nx2 = x2;
            while (nx1 <= nx2) {
                if (nx1%2 == 1) sum += st[y1][nx1++];
                if (nx2%2 == 0) sum += st[y1][nx2--];
                nx1 /= 2;
                nx2 /= 2;
            }
            y1++;
        }
        if (y2%2 == 0) {
            int nx1 = x1;
            int nx2 = x2;
            while (nx1 <= nx2) {
                if (nx1%2 == 1) sum += st[y2][nx1++];
                if (nx2%2 == 0) sum += st[y2][nx2--];
                nx1 /= 2;
                nx2 /= 2;
            }
            y2--;
        }
        y1 /= 2;
        y2 /= 2;
    }
    return sum;
}

// set {y, x} to u
// 0-indexed
void muuta(int y, int x, ll u) {
    y += N;
    x += N;
    st[y][x] = u;
    for (int nx = x/2; nx >= 1; nx /= 2) {
        st[y][nx] = st[y][2*nx]+st[y][2*nx+1];
    }

    for (y /= 2; y >= 1; y /= 2) {
        for (int nx = x; nx >= 1; nx /= 2) {
            st[y][nx] = st[2*y][nx]+st[2*y+1][nx];
        }
    }
}

int main() {
    ios_base::sync_with_stdio(false);
    cin.tie(0);
    cin >> n >> q;
    for (int i = 0; i < n; ++i) {
        for (int j = 0; j < n; ++j) {
            ll a;
            cin >> a;
            muuta(i, j, a);
        }
    }
```

```
    for (int i = 0; i < q; ++i) {
        int tp;
        cin >> tp;
        if (tp == 1) {
            int y, x, u;
            cin >> y >> x >> u;
            muuta(y-1, x-1, u);
        }
        if (tp == 2) {
            int y1, x1, y2, x2;
            cin >> y1 >> x1 >> y2 >> x2;
            cout << summa(y1-1, x1-1, y2-1, x2-1) << "\n
                ";
        }
    }
    return 0;
}
```

3.4 Treap

Implements split, merge, kth element, range update and range reverse in $O(log(n))$. Range update adds a value to every element in a subarray. Treap is 1-indexed.

Note: Memory management tools warn of about 30 MB memory leak for 500 000 elements. This is because nodes are not deleted when exiting program and is irrelevant in a competition. Deleting nodes would slow treap down by a factor of 3.

```
#include <iostream>
#include <cstdlib>
#include <algorithm>

using namespace std;
typedef long long ll;

struct node {
        ll val; // change data type (char, integer...)
        int prio, size;
        bool lzinv;
        ll lzupd;
        bool haslz;
        node *left, *right;

        node(ll v) {
                val = v;
                prio = rand();
                size = 1;
                lzinv = false;
                lzupd = 0;
                haslz = false;
                left = nullptr;
                right = nullptr;
        }
};

int gsize(node *s) {
        if (s == nullptr) return 0;
        return s->size;
}

void upd(node *s) {
        if (s == nullptr) return;
        s->size = gsize(s->left) + 1 + gsize(s->right);
}

void push(node *s) {
        if (s == nullptr) return;

        if (s->haslz) {
                s->val += s->lzupd; // operator
        }
        if (s->lzinv) {
                swap(s->left, s->right);
        }

        if (s->left != nullptr) {
                if (s->haslz) {
                        s->left->lzupd += s->lzupd; //
                            operator
                        s->left->haslz = true;
                }
                if (s->lzinv) {
                        s->left->lzinv = !s->left->lzinv
                            ;
                }
        }
```

```
	if (s->right != nullptr) {
		if (s->haslz) {
			s->right->lzupd += s->lzupd; //
			    operator
			s->right->haslz = true;
		}
		if (s->lzinv)  {
			s->right->lzinv = !s->right->
			    lzinv;
		}
	}

	s->lzupd = 0; // operator identity value
	s->lzinv = false;
	s->haslz = false;
}

// split a treap into two treaps, size of left treap = k
void split(node *t, node *&l, node *&r, int k) {
	push(t);
	if (t == nullptr) {
		l = nullptr;
		r = nullptr;
		return;
	}
	if (k >= gsize(t->left)+1) {
		split(t->right, t->right, r, k-(gsize(t
		    ->left)+1));
		l = t;
	}
	else {
		split(t->left, l, t->left, k);
		r = t;
	}
	upd(t);
}

// merge two treaps
void merge(node *&t, node *l, node *r) {
	push(l);
	push(r);
	if (l == nullptr) t = r;
	else if (r == nullptr) t = l;
	else {
		if (l->prio >= r->prio) {
			merge(l->right, l->right, r);
			t = l;
		}
		else {
			merge(r->left, l, r->left);
			t = r;
		}
	}
	upd(t);
}

// get k:th element in array (1-indexed)
ll kthElem(node *t, int k) {
	push(t);
	int cval = gsize(t->left)+1;
	if (k == cval) return t->val;
	if (k < cval) return kthElem(t->left, k);
	return kthElem(t->right, k-cval);
}

// do a lazy update on subarray [a..b]
void rangeUpd(node *&t, int a, int b, ll x) {
	node *cl, *cur, *cr;
	int tsz = gsize(t);
	bool lsplit = false;
	bool rsplit = false;
	cur = t;
	if (a > 1) {
		split(cur, cl, cur, a-1);
		lsplit = true;
	}
	if (b < tsz) {
		split(cur, cur, cr, b-a+1);
		rsplit = true;
	}
	cur->lzupd += x; // operator
	cur->haslz = true;
	if (lsplit) {
		merge(cur, cl, cur);
	}
	if (rsplit) {
		merge(cur, cur, cr);
	}
	t = cur;
}
```

```cpp
// reverse subarray [a..b]
void rangeInv(node *&t, int a, int b) {
	node *cl, *cur, *cr;
	int tsz = gsize(t);
	bool lsplit = false;
	bool rsplit = false;
	cur = t;
	if (a > 1) {
		split(cur, cl, cur, a-1);
		lsplit = true;
	}
	if (b < tsz) {
		split(cur, cur, cr, b-a+1);
		rsplit = true;
	}
	cur->lzinv = !cur->lzinv;
	if (lsplit) {
		merge(cur, cl, cur);
	}
	if (rsplit) {
		merge(cur, cur, cr);
	}
	t = cur;
}


// test code below

int n, q;

/*
	TESTED, correct.

	Treap, allows split, merge, kth element, range
	    update and range reverse in O(log n)
	It's also possible to implement range sum query
	    (ioi16-treap IV)

	Implemented range update adds a value to every
	    element in a subarray.

	NOTE: Memory management tools warn of a ~ 30MB
	    memory leak for 500 000 nodes. This is
	    because nodes are not deleted on program
	    exit. Deleting would severely harm
	    performance (over 3 times slower) and is
	    unnecessary in a contest setting since the
	    program is terminated anyway. Leak can be
	    fixed by deleting nodes recursively on exit
	    starting from leaf nodes and progressing
	    towards root (post-order dfs).
*/
int main() {
	ios_base::sync_with_stdio(false);
	cin.tie(0);

	cin >> n >> q;
	node *tree = nullptr;
	for (int i = 1; i <= n; ++i) {
		node *nw = new node(0);
		merge(tree, tree, nw); // treap
		    construction
	}

	for (int cq = 0; cq < q; ++cq) {
		char tp;
		cin >> tp;
		if (tp == 'G') {
			int cind;
			cin >> cind;
			cout << kthElem(tree, cind) << "
			    \n";
		}
		else if (tp == 'R') {
			int a, b;
			cin >> a >> b;
			rangeInv(tree, a, b);
		}
		else {
			int a, b;
			ll d;
			cin >> a >> b >> d;
			rangeUpd(tree, a, b, d);
		}
	}
	return 0;
}
```

3.5 Indexed set (policy-based data structures)

Works like `std::set` but adds support for indices. Set is 0-indexed. Requires g++. Has two additional functions:

1. `find_by_order(x)`: return an iterator to element at index x
2. `order_of_key(x)`: return the index that element x has or would have in the set, depending on if it exists

Both functions work in $O(log(n))$.

Changing `less` to `less_equal` makes the set work like multiset. However, elements can't be removed.

```
#include <iostream>
#include <ext/pb_ds/assoc_container.hpp>

using namespace std;
using namespace __gnu_pbds;
typedef tree<int, null_type, less<int>, rb_tree_tag,
    tree_order_statistics_node_update> indexed_set;

indexed_set s;

int main() {
        s.insert(2);
        s.insert(4);
        s.insert(5);

        auto x = s.find_by_order(1);
        cout << *x << "\n"; // prints 4

        cout << s.order_of_key(5) << "\n"; // prints 2
        cout << s.order_of_key(3) << "\n"; // prints 1
        return 0;
}
```

3.6 Union-find

Uses path compression, `id(x)` has amortized time complexity $O(a^{-1}(n))$ where a^{-1} is inverse Ackermann function.

```
#include <iostream>
#include <algorithm>

using namespace std;

int k[100005];
int s[100005];

int id(int x) {
        int tx = x;
        while (k[x] != x) x = k[x];
        return k[tx] = x;
}

bool equal(int a, int b) {
        return id(a) == id(b);
}

void join(int a, int b) {
        a = id(a);
        b = id(b);
        if (s[b] > s[a]) swap(a, b);
        s[a] += s[b];
        k[b] = a;
}

int n;

int main() {
        for (int i = 0; i < n; ++i) {
                k[i] = i;
                s[i] = 1;
        }
        return 0;
}
```

4 Mathematics

4.1 Number theory

- Prime factorization of n: $p_1^{\alpha_1} p_2^{\alpha_2} \dots p_k^{\alpha_k}$

- Number of factors: $\tau(n) = \prod_{i=1}^{k} (\alpha_i + 1)$
- Sum of factors: $\sigma(n) = \prod_{i=1}^{k} \frac{p_i^{\alpha_i+1}-1}{p_i-1}$
- Product of factors: $\mu(n) = n^{\tau(n)/2}$

Euler's totient function $\varphi(n)$ $(1, 1, 2, 2, 4, 2, 6, 4, 6, 4, \dots)$: counts numbers coprime with n in range $1 \dots n$

$$\varphi(n) = \begin{cases} n-1 & \text{if } n \text{ is prime} \\ \prod_{i=1}^{k} p_i^{a_i-1}(p_i-1) & \text{otherwise} \end{cases}$$

Fermat's theorem: $x^{m-1} \mod m = 1$ when m is prime and x and m are coprime. It follows that $x^k \mod m = x^{k \mod (m-1)} \mod m$.

Modular inverse $x^{-1} = x^{\varphi(m)-1}$. If m is prime, $x^{-1} = x^{m-2}$. Inverse exists if and only if x and m are coprime.

4.2 Combinatorics

Binomial coefficients:

$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$$

$$\binom{n}{0} = \binom{n}{n} = 1$$

Catalan numbers $(1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796 \dots)$:

$$C_n = \frac{1}{n+1}\binom{2n}{n}$$

Classic examples of Catalan numbers: number of balanced pairs of parentheses, number of mountain ranges (n upstrokes and n downstrokes all staying above the original line), number of paths from upper left corner to lower right corner staying above the main diagonal in a $n \times n$ square, ways to trianguate a $n+2$ sided regular polygon, ways to shake hands between $2n$ people in a circle such that no arms cross, number of rooted binary trees with n nodes that have 2 children, number of rooted trees with n edges, number of permutations of $1 \dots n$ that don't have an increasing subsequence of length 3.

Number of derangements (no element stays in original place) of $1, 2, \dots, n$ $(1, 0, 1, 2, 9, 44, 265, 1854, 14833, 133496, 1334961, \dots)$:

$$f(n) = \begin{cases} 0 & n = 1 \\ 1 & n = 2 \\ (n-1)(f(n-2) + f(n-1)) & n > 2 \end{cases}$$

4.3 Matrices

Matrix $A = a \times n$, matrix $B = n \times b$. Matrix multiplication:

$$AB[i, j] = \sum_{k=1}^{n} A[i, k] \cdot B[k, j]$$

Let linear recurrence $f(n) = c_1 f(n-1) + c_2 f(n-2) + \cdots + c_k f(n-k)$ with initial values $f(0), f(1), \dots, f(k-1)$. $c_1, c_2, \dots, c_n$ are constants.

Transition matrix X:

$$X = \begin{pmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 1 \\ c_k & c_{k-1} & c_{k-2} & \dots & c_1 \end{pmatrix}$$

Now $f(n)$ can be calculated in $O(k^3 log(n))$:

$$\begin{pmatrix} f(n) \\ f(n+1) \\ \vdots \\ f(n+k-1) \end{pmatrix} = X^n \cdot \begin{pmatrix} f(0) \\ f(1) \\ \vdots \\ f(k-1) \end{pmatrix}$$

```cpp
#include <iostream>
#include <cstring>

using namespace std;
typedef long long ll;

const int N = 2; // matrix size
const ll M = 1000000007; // modulo

struct matrix {
    ll m[N][N];
    matrix() {
        memset(m, 0, sizeof m);
    }
    matrix operator * (matrix b) {
        matrix c = matrix();
        for (int i = 0; i < N; ++i)
            for (int j = 0; j < N; ++j)
                for (int k = 0; k < N; ++k) {
                    c.m[i][j] = (c.m[i][j] + m[i][k] * b
                        .m[k][j])%M;
                }
        return c;
    }
    matrix unit() {
        matrix a = matrix();
        for (int i = 0; i < N; ++i) a.m[i][i] = 1;
        return a;
    }
};

matrix p(matrix a, ll e) {
    if (e == 0) return a.unit();
    if (e%2 == 0) {
        matrix h = p(a, e/2);
        return h*h;
    }
    return (p(a, e-1)*a);
}

ll n;

// prints nth Fibonacci number mod M
int main() {
    cin >> n;
    matrix x = matrix();
    x.m[0][1] = 1;
    x.m[1][0] = 1;
    x.m[1][1] = 1;
    x = p(x, n);
    cout << x.m[0][1] << "\n";
    return 0;
}
```

4.4 Miller-Rabin

Deterministic primality test for all 64-bit integers. Requires `__int128` support to test over 32-bit integers.

```cpp
#include <iostream>

using namespace std;
typedef long long ll;
typedef __int128 lll;

// required bases to make test deterministic for 64-bit
    integers
ll mrb[12] = {2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31,
    37};

lll modpow(lll k, lll e, lll m) {
        if (e == 0) return 1;
        if (e == 1) return k;
        if (e%2 == 0) {
                lll h = modpow(k, e/2, m)%m;
                return (h*h)%m;
        }
        return (k*modpow(k, e-1, m))%m;
}

bool witness(ll a, ll x, ll u, ll t) {
        lll cx = modpow(a, u, x);
        for (int i = 1; i <= t; ++i) {
                lll nx = (cx*cx)%x;
                if (nx == 1 && cx != 1 && cx != (x-1))
                    return true;
                cx = nx;
        }
        return (cx != 1);
```

```
}

// TESTED, correct
// determines if x is prime
// deterministic for all 64-bit integers
bool miller_rabin(ll x) {
        if (x == 2) return true;
        if (x < 2 || x%2 == 0) return false;

        ll u = x-1;
        ll t = 0;
        while (u%2 == 0) {
                u /= 2;
                t++;
        }

        for (int i = 0; i < 12; ++i) {
                if (mrb[i] >= x-1) break;
                if (witness(mrb[i], x, u, t)) return
                    false;
        }
        return true;
}

int t;

int main() {
        ios_base::sync_with_stdio(false);
        cin.tie(0);
        cin >> t;
        for (int i = 0; i < t; ++i) {
                ll n;
                cin >> n;
                if (miller_rabin(n)) cout << "YES\n";
                else cout << "NO\n";
        }
        return 0;
}
```

4.5 Pollard-Rho

Finds a prime factor of x in $O(\sqrt[4]{x})$. Requires `__int128` support to factor over 32-bit integers.

If x is prime, algorithm might not terminate or it might return 1. Primality must be checked separately.

```
#include <iostream>
#include <cstdlib>
#include <algorithm>

using namespace std;

typedef long long ll;
typedef __int128 lll;

ll n;

ll f(lll x) {
    return (x*x+1)%n;
}

ll gcd(ll a, ll b) {
    if (b == 0) return a;
    return gcd(b, a%b);
}

// return a prime factor of a
// st is a starting seed for pseudorandom numbers, start
     with 2, if algorithm fails (returns -1), increment
    seed
ll pollardrho(ll a, ll st) {
    if (n%2 == 0) return 2;

    ll x = st, y = st, d = 1;
    while (d == 1) {
        x = f(x);
        y = f(f(y));
        d = gcd(abs(x-y), a);
        if (d == a) return -1;
    }
    return d;
}

/*
        TESTED, correct.
    Finds a prime factor of n in O(root_4(n))
    If n is prime, alg might not terminate or it might
        return 1. Check for primality.
```

```
*/
int main() {
    cin >> n;
    ll fa = -1;
    ll st = 2;
    while (fa == -1) {
        fa = pollardrho(n, st++);
    }
    cout << min(fa, n/fa) << "␣" << max(fa, n/fa) << "\n
        ";
    return 0;
}
```

5 Geometry

```
#include <iostream>
#include <complex>
#include <vector>
#include <algorithm>
#include <iomanip>

using namespace std;
typedef long double ct; // coordinate type
typedef complex<ct> point;
#define X real()
#define Y imag()
#define F first
#define S second

const ct EPS = 0.000000001; // 1e-9
const ct PI = 3.14159265359;

// floating-point equality comparison
bool equal(ct a, ct b) {
        return abs(a-b) < EPS;
}

// point equality comparison
bool equal(point a, point b) {
        return (equal(a.X, b.X) && equal(a.Y, b.Y));
}

// comparer for sorting points
// check if a < b
bool point_comp(point a, point b) {
        if (equal(a.X, b.X)) {
                return a.Y < b.Y;
        }
        return a.X < b.X;
}

struct line {
        point first, second;

        line(point a, point b) {
                if (point_comp(b, a)) swap(a, b);
                first = a;
                second = b;
        }

        // construct line from point and angle of
            elevation
        line(point a, ct ang) : line(a, a+polar((ct)1.0,
             ang)) {}

        // construct line from standard equation
            coefficients
        // assume that a != 0 or b != 0
        // TESTED
        line(ct a, ct b, ct c) {
                if (equal(b, 0.0)) {
                        // vertical line
                        ct cx = c/(-a);
                        first = {cx, 0};
                        second = {cx, 1};
                }
                else {
                        first = {0, c/(-b)};
                        second = {1, (a+c)/(-b)};
                }
                if (point_comp(second, first)) swap(
                    first, second);
        }
};

struct line_segment {
        point first, second;
```

```
        // implicit conversion
        operator line() {
                return line(first, second);
        }

        line_segment(point a, point b) {
                if (point_comp(b, a)) swap(a, b);
                first = a;
                second = b;
        }

        line_segment(point a, ct ang, ct len) :
            line_segment(a, a+polar(len, ang)) {};
};

// assume that the first and last vertices are the same
typedef vector<point> polygon;

// radians to degrees
ct rad_to_deg(ct arad) {
        return (arad*((ct)180.0/PI));
}

// degrees to radians
ct deg_to_rad(ct adeg) {
        return (adeg*(PI/(ct)180.0));
}

// dot product, > 0 if a, b point to same direction, 0
    if perpendicular, < 0 if pointing to opposite
    directions
ct dot(point a, point b) {
        return (conj(a)*b).X;
}

// 2D cross product, > 0 if a+b turns left, 0 if
    collinear, < 0 if turns right
ct cross(point a, point b) {
        return (conj(a)*b).Y;
}

// euclidean distance
// TESTED
ct dist(point a, point b) {
        return abs(a-b);
}

// squared distance
ct sq_dist(point a, point b) {
        return norm(a-b);
}

// angle from a to b
// [0, 2*pi[
// TESTED
ct angle(point a, point b) {
        ct cres = arg(b-a);
        if (cres < 0) cres = 2*PI+cres;
        return cres;
}

// angle of elevation
// [-pi/2, pi/2]
ct elev_ang(point a, point b) {
        if (point_comp(b, a)) swap(a, b);
        return arg(b-a);
}

// angle of elevation
ct elev_ang(line l) {
        return elev_ang(l.F, l.S);
}

// slope of line
ct slope(point a, point b) {
        return tan(elev_ang(a, b));
}

// slope of line
ct slope(line l) {
        return tan(elev_ang(l));
}

// length of line segment
ct segment_len(line_segment ls) {
        return dist(ls.F, ls.S);
}

// rotate a around origin by ang
```

```
point rot_origin(point a, ct ang) {
        return (a*polar((ct)1.0, ang));
}

// rotate a around ps by ang
point rot_pivot(point a, point ps, ct ang) {
        return ((a-ps)*polar((ct)1.0, ang)+ps);
}

// check if a -> b -> c turns counterclockwise
bool ccw(point a, point b, point c) {
        return cross({b.X-a.X, b.Y-a.Y}, {c.X-a.X, c.Y-a
            .Y}) > 0;
}

// < 0 if point is left, ~0 if on line, > 0 if right
// TESTED
ct point_line_side(point a, line l) {
        return cross(a-l.F, a-l.S);
}

// check if point is on line
// TESTED
bool point_on_line(point a, line l) {
        return equal(point_line_side(a, l), (ct)0.0);
}

// check if point is on line segment
// TESTED
bool point_on_seg(point a, line_segment ls) {
        if (!point_on_line(a, ls)) return false;
        if (equal(a, ls.F) || equal(a, ls.S)) return
            true;
        return (point_comp(ls.F, a) && point_comp(a, ls.
            S));
}

// get projection of a on l
// TESTED
point point_line_proj(point a, line l) {
        return (l.F+(l.S-l.F)*dot(a-l.F, l.S-l.F)/norm(l
            .S-l.F));
}

// reflect a across l
point point_line_refl(point a, line l) {
        return (l.F+conj((a-l.F)/(l.S-l.F))*(l.S-l.F));
}

// angle a-b-c
// [0, PI]
// TESTED
ct ang_abc(point a, point b, point c) {
        return abs(remainder(arg(a-b)-arg(c-b), (ct)2.0*
            PI));
}

// shortest distance between point a and line l
// TESTED
ct point_line_dist(point a, line l) {
        point proj = point_line_proj(a, l);
        return dist(a, proj);
}

// shortest distance between point a and line segment ls
// TESTED
ct point_segment_dist(point a, line_segment ls) {
        point proj = point_line_proj(a, ls);
        if (point_on_seg(proj, ls)) {
                return dist(a, proj);
        }
        return min(dist(a, ls.F), dist(a, ls.S));
}

// get intersection point of two lines
// first return val 0 = no intersection, 1 = single
    point, 2 = infinitely many
// second return val = intersection point if first
    return val = 1, otherwise undefined
// TESTED (only non-degenerate cases, single
    intersection point)
pair<int, point> intersect(line a, line b) {
        ct c1 = cross(b.F-a.F, a.S-a.F);
        ct c2 = cross(b.S-a.F, a.S-a.F);
        if (equal(c1, c2)) {
                if (point_on_line(b.F, a)) {
                        return {2, a.F};
                }
                return {0, a.F};
        }
```

```
        return {1, (c1*b.S-c2*b.F)/(c1-c2)};
}

// sort comparer for seg_intersect
bool pi_comp(pair<point, int> p1, pair<point, int> p2) {
        if (equal(p1.F, p2.F)) return p1.S < p2.S;
        return point_comp(p1.F, p2.F);
}

// get intersection point of two line segments
// first return val 0 = no intersection, 1 = single
    point, 2 = infinitely many
// second return val = intersection point if first
    return val = 1, otherwise undefined
pair<int, point> seg_intersect(line_segment a,
    line_segment b) {
        ct alen = segment_len(a);
        ct blen = segment_len(b);

        if (equal(alen, (ct)0) && equal(blen, (ct)0)) {
                return (equal(a.F, b.F) ? make_pair(1, a
                    .F) : make_pair(0, a.F));
        }
        else if (equal(alen, (ct)0)) {
                return (point_on_seg(a.F, b) ? make_pair
                    (1, a.F) : make_pair(0, a.F));
        }
        else if (equal(blen, (ct)0)) {
                return (point_on_seg(b.F, a) ? make_pair
                    (1, b.F) : make_pair(0, b.F));
        }

        auto tres = intersect(a, b);
        if (tres.F == 0) {
                return tres;
        }
        else if (tres.F == 2) {
                vector<pair<point, int>> v = {{a.F, 1},
                    {a.S, 1}, {b.F, 2}, {b.S, 2}};
                sort(v.begin(), v.end(), pi_comp);
                if (v[0].S != v[1].S) return {2, a.F};
                    // overlapping segments

                // common vertex
                if (equal(a.S, b.F)) return {1, a.S};
                if (equal(a.F, b.S)) return {1, a.F};

                // not intersecting but on the same line
                return {0, a.F};
        }
        if (point_on_seg(tres.S, a) && point_on_seg(tres
            .S, b)) {
                return tres;
        }
        return {0, a.F};
}

// get polygon area
// O(n)
// TESTED
ct pgon_area(polygon pg) {
        ct cres = 0;
        for (int i = 0; i < pg.size()-1; ++i) {
                cres += cross(pg[i], pg[i+1]);
        }
        return (abs(cres)/(ct)2.0);
}

// check if point is inside polygon
// 0 = outside, 1 = inside, 2 = on polygon edge
// O(n)
int point_in_pgon(point a, polygon pg) {
        for (int i = 0; i < pg.size()-1; ++i) {
                if (point_on_seg(a, line_segment(pg[i],
                    pg[i+1]))) {
                        return 2;
                }
        }
        // arbitrary angle, try to avoid polygon
            vertices (likely lattice points)
        line_segment tl = line_segment(a, (ct)
            1.03077640640, (ct)10000000000.0); // length
             10^10
        int icnt = 0;
        for (int i = 0; i < pg.size()-1; ++i) {
                auto cur = seg_intersect(tl,
                    line_segment(pg[i], pg[i+1]));
                if (cur.F > 0) {
                        icnt++;
                }
```

```
        }
        return (icnt%2 == 1);
}

// return the points that form given point set's convex
    hull
// O(n log n)
vector<point> convex_hull(vector<point> ps) {
        vector<point> ch;
        sort(ps.begin(), ps.end(), point_comp);
    for (int cv = 0; cv < 2; ++cv) {
        for (int i = 0; i < ps.size(); ++i) {
            int cs = ch.size();
            while (cs >= 2 && ccw(ch[cs-2], ch[cs-1], ps
                [i])) {
                ch.pop_back();
                --cs;
            }
            ch.push_back(ps[i]);
        }
        ch.pop_back();
        reverse(ps.begin(), ps.end());
    }
    return ch;
}


int main() {
        ios_base::sync_with_stdio(false);
        cin.tie(0);
        int t;
        cin >> t;
        for (int cte = 0; cte < t; ++cte) {
                long long ax, ay, bx, by, cx, cy, dx, dy
                    ;
                cin >> ax >> ay >> bx >> by >> cx >> cy
                    >> dx >> dy;
                line_segment a = line_segment({ax, ay},
                    {bx, by});
                line_segment b = line_segment({cx, cy},
                    {dx, dy});
                auto res = seg_intersect(a, b);
                if (res.F == 0) {
                        cout << "NO\n";
                }
                else if (res.F == 1) {
                        cout << "POINT\n";
                }
                else {
                        cout << "SEGMENT\n";
                }
        }
        return 0;
}
```

6 Graph algorithms

6.1 Kosaraju's algorithm

Finds strongly connected components in a directed graph in $O(n+m)$.

1. Create an inverse graph where all edges are reversed.
2. Do a DFS traversal on original graph and add all nodes in post-order to a vector.
3. Reverse the previous vector.
4. Iterate the vector. If a node doesn't belong to a component, create new component and assign current node to it, and do a DFS search **in inverse graph** from current node and add all reachable nodes to the component that was just created.

6.2 Bridges

An edge $u - v$ is a bridge if there is no edge from the subtree of v to any node with lower depth than u in DFS tree. $O(n+m)$.

```
#include <iostream>
#include <vector>
#include <algorithm>

using namespace std;

int n, m;
```

```
vector<int> g[200010];

int v[200010];
int d[200010];

// found bridges
vector<pair<int, int>> res;

// find bridges
int bdfs(int s, int cd, int p) {
    if (v[s]) return d[s];
    v[s] = 1;
    d[s] = cd;

    int minh = cd;

    for (int a : g[s]) {
        if (a == p) continue;
        minh = min(minh, bdfs(a, cd+1, s));
    }

    if (p != -1) {
        if (minh == cd) {
            res.push_back({s, p});
        }
    }
    return minh;
}

int main() {
    ios_base::sync_with_stdio(false);
    cin.tie(0);
    cin >> n >> m;
    for (int i = 0; i < m; ++i) {
        int a, b;
        cin >> a >> b;
        g[a].push_back(b);
        g[b].push_back(a);
    }
    for (int i = 1; i <= n; ++i) {
        if (!v[i]) bdfs(i, 1, -1);
    }
    cout << res.size() << "\n";
    for (auto a : res) {
        cout << a.first << "␣" << a.second << "\n";
    }

    return 0;
}
```

6.3 Articulation points

A vertex u is an articulation point if there is no edge from the subtree of u to any parent of u in DFS tree, of if u is the root of DFS tree and has at least 2 children. $O(n+m)$ if removing duplicates doesn't count.

Set `res` can be replaced with a vector if duplicates are removed afterwards.

```
#include <iostream>
#include <vector>
#include <algorithm>
#include <set>

using namespace std;

int n, m;
vector<int> g[200010];
int v[200010];
int dt[200010];
int low[200010];

// found articulation points
// can be replaced with vector, but duplicates must be
    removed
set<int> res;

int curt = 1;

void adfs(int s, int p) {
    if (v[s]) return;
    v[s] = 1;
    dt[s] = curt++;
    low[s] = dt[s];

    int ccount = 0;
```

```
    for (int a : g[s]) {
        if (!v[a]) {
            ++ccount;
            adfs(a, s);
            low[s] = min(low[s], low[a]);

            if (low[a] >= dt[s] && p != -1) res.insert(s
                );
        }
        else if (a != p) {
            low[s] = min(low[s], dt[a]);
        }

        if (p == -1 && ccount > 1) {
            res.insert(s);
        }
    }
}


int main() {
    ios_base::sync_with_stdio(false);
    cin.tie(0);
    cin >> n >> m;
    for (int i = 0; i < m; ++i) {
        int a, b;
        cin >> a >> b;
        g[a].push_back(b);
        g[b].push_back(a);
    }

    for (int i = 1; i <= n; ++i) {
        if (!v[i]) adfs(i, -1);
    }
    cout << res.size() << "\n";
    for (int a : res) cout << a << "\n";
    return 0;
}
```

6.4 Maximum flow (scaling algorithm)

Scaling algorithm, uses DFS to find an augmenting path where each edge weight is larger than or equal to a certain threshold. Time complexity $O(m^2 log(c))$, where c is the starting threshold (sum of all edge weights on the graph).

```
#include <iostream>
#include <vector>
#include <algorithm>

using namespace std;
typedef long long ll;

const int N = 105; // vertex count
const ll LINF = 1000000000000000005;

int n, m;
vector<int> g[N];
ll d[N][N]; // edge weights

int v[N];
vector<int> cp; // current augmenting path

ll res = 0;

// find augmenting path using scaling
// prerequisities: clear current path, divide threshold
    by 2, increment cvis
ll dfs(int s, int t, ll thresh, int cvis, ll cmin) {
    if (v[s] == cvis) return -1;
    v[s] = cvis;
    cp.push_back(s);
    if (s == t) return cmin;

    for (int a : g[s]) {
        if (d[s][a] < thresh) continue; // scaling
        ll cres = dfs(a, t, thresh, cvis, min(cmin, d[s
            ][a]));
        if (cres != -1) return cres;
    }

    cp.pop_back();
    return -1;
}

int main() {
    ios_base::sync_with_stdio(false);
    cin.tie(0);
```

```
    cin >> n >> m;
    ll cthresh = 0;
    for (int i = 0; i < m; ++i) {
        int a, b;
        ll c;
        cin >> a >> b >> c;
        g[a].push_back(b);
        g[b].push_back(a);
        d[a][b] += c;
        d[b][a] = 0;
        cthresh += c;
    }
    int cvis = 0;
    while (true) {
        cvis++;
        cp.clear();
        ll minw = dfs(1, n, cthresh, cvis, LINF);
        if (minw != -1) {
            res += minw;
            for (int i = 0; i < cp.size()-1; ++i) {
                d[cp[i]][cp[i+1]] -= minw;
                d[cp[i+1]][cp[i]] += minw;
            }
        }
        else {
            if (cthresh == 1) break;
            cthresh /= 2;
        }
    }
    cout << res << "\n";
    return 0;
}
```

6.5 Theorems on flows and cuts

Maximum flow is always equal to minimum cut. Minimum cut can be found by running a maximum flow algorithm and dividing the resulting flow graph into two sets of vertices. Set A contains all vertices that can be reached from source using positive-weight edges. Set B contains all other vertices. Minimum cut consists of the edges between these two sets.

Number of edge-disjoint (= each edge can be used at most once) paths in a graph is equal to maximum flow on graph where capacity of each edge is 1.

Number of vertex-disjoint paths can be found the same way as edge-disjoint paths, but each vertex is duplicated and an edge is added between the two vertices. All incoming edges go to the first vertex and all outgoing edges start from the second vertex.

Maximum matching of a bipartite graph can be found by adding a source and a sink to the graph and connecting source to all left vertices and sink to all right vertices. Maximum matching equals maximum flow on this graph.

König's theorem: sizes of a minimum vertex cover (= minimum set of vertices such that each edge has at least one endpoint in the set) and a maximum matching are always equal in a bipartite graph. Maximum independent set (= maximum set of vertices such that no two vertices in the set are connected with an edge) consists of the vertices not in a minimum vertex cover.

6.6 Heavy-light decomposition

Supports updates and queries on path between two vertices a and b in $O(log^2(n))$.

Doesn't explicitly look for LCA, instead climbs upwards from the lower chain until both vertices are in the same chain.

Requires a segment tree implementation that corresponds to the queries. Lazy segtree, for example, can be pasted directly in.

```
#include <iostream>
#include <vector>
#include <algorithm>

using namespace std;
typedef long long ll;

const int S = 100005; // vertex count
const int N = (1<<18); // segtree size, must be >= S

vector<int> g[S];

int sz[S], de[S], pa[S];
```

```
int cind[S], chead[S], cpos[S];
int cchain, cstind, stind[S];

// IMPLEMENT SEGMENT TREE HERE
// st_update() and st_query() should call segtree
    functions
ll st[2*N];

void hdfs(int s, int p, int cd) {
    de[s] = cd;
    pa[s] = p;
    sz[s] = 1;
    for (int a : g[s]) {
        if (a == p) continue;
        hdfs(a, s, cd+1);
        sz[s] += sz[a];
    }
}

void hld(int s) {
    if (chead[cchain] == 0) {
        chead[cchain] = s;
        cpos[s] = 0;
    }
    else {
        cpos[s] = cpos[pa[s]]+1;
    }
    cind[s] = cchain;

    stind[s] = cstind;
    cstind++;

    int cmx = 0, cmi = -1;
    for (int i = 0; i < g[s].size(); ++i) {
        if (g[s][i] == pa[s]) continue;
        if (sz[g[s][i]] > cmx) {
            sz[g[s][i]] = cmx;
            cmi = i;
        }
    }

    if (cmi != -1) {
        hld(g[s][cmi]);
    }
    for (int i = 0; i < g[s].size(); ++i) {
        if (i == cmi) continue;
        if (g[s][i] == pa[s]) continue;
        cchain++;
        cstind++;
        hld(g[s][i]);
    }
}

// do a range update on underlying segtree
// sa and sb are segtree indices
void st_update(int sa, int sb, ll x) {

}

// do a range query on underlying segtree
// sa and sb are segtree indices
ll st_query(int sa, int sb) {

}

// update all vertices on path from vertex a to b
// a and b are vertex numbers
void path_update(int a, int b, ll x) {
    while (cind[a] != cind[b]) {
        if (de[chead[cind[b]]] > de[chead[cind[a]]])
            swap(a, b);
        st_update(stind[chead[cind[a]]], stind[a], x);
        a = pa[chead[cind[a]]];
    }
    if (stind[b] < stind[a]) swap(a, b);
    st_update(stind[a], stind[b], x);
}

// query all vertices on path from vertex a to b
// a and b are vertex numbers
ll path_query(int a, int b) {
        ll cres = 0; // set to identity
        while (cind[a] != cind[b]) {
        if (de[chead[cind[b]]] > de[chead[cind[a]]])
            swap(a, b);
        cres += st_query(stind[chead[cind[a]]], stind[a
            ]); // change operator
        a = pa[chead[cind[a]]];
    }
```

```
    if (stind[b] < stind[a]) swap(a, b);
    cres += st_query(stind[a], stind[b]); // change
        operator
    return cres;
}


int n, m;

// TESTED, correct
// do updates and queries on paths between two nodes in
    a tree
// interface: path_update() and path_query()
int main() {
    ios_base::sync_with_stdio(false);
    cin.tie(0);
    cin >> n >> m;
    for (int i = 0; i < n-1; ++i) {
        int a, b;
        cin >> a >> b;
        g[a].push_back(b);
        g[b].push_back(a);
    }

    // init hld
    hdfs(1, -1, 0);
    hld(1);

    // handle queries
    return 0;
}
```

7 String algorithms

7.1 Polynomial hashing

If hash collisions are likely, compute two hashes with two distinct pairs of constants of magnitude 10^9 and use their product as the actual hash.

```
#include <iostream>
using namespace std;

const ll A = 957262683;
const ll B = 998735246;

string s;
ll h[1000005];
ll p[1000005];

ll ghash(int a, int b) {
        if (a == 0) return h[b];
        ll cres = (h[b]-h[a-1]*p[b-a+1])%B;
        if (cres < 0) cres += B;
        return cres;
}

int main() {
        cin >> s;

        h[0] = s[0];
        p[0] = 1;

        for (int i = 1; i < s.length(); ++i) {
                h[i] = (h[i-1]*A+s[i])%B;
                p[i] = (p[i-1]*A)%B;
        }
        return 0;
}
```