2021 IEEE International Conference on Robotics and Automation (ICRA 2021)
May 31 - June 4, 2021, Xi'an, China

Robust Motion Averaging under Maximum Correntropy Criterion

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Abstract— **Recently, the motion averaging method has been introduced as an effective means to solve the multi-view registration problem. This method aims to recover global motions from a set of relative motions, where the original method is sensitive to outliers due to using the Frobenius norm error in the optimization. Accordingly, this paper proposes a novel robust motion averaging method based on the maximum correntropy criterion (MCC). Specifically, the correntropy measure is used instead of utilizing Frobenius norm error to improve the robustness of motion averaging against outliers. According to the half-quadratic technique, the correntropy measure based optimization problem can be solved by the alternating minimization procedure, which includes operations of weight assignment and weighted motion averaging. Further, we design a selection strategy of adaptive kernel width to take advantage of correntropy. Experimental results on benchmark data sets illustrate that our method has superior performance on accuracy and robustness for multi-view registration. What's more, it can be applied to robot mapping.**

I. INTRODUCTION

Point set registration is a fundamental and important technique in many domains, such as compute vision, robotics, and computer graphics, etc. For each range scan given in a set-centered frame, the registration goal is to find an optimal rigid transformation (global motion) and transform it into the reference frame. Depend on the number of involved point sets, point set registration can be divided into pair-wise registration and multi-view registration problems. In the past few decades, lots of effective methods have been proposed to solve the pair-wise registration problem. Among these methods, the iterative closest point (ICP) algorithm [1] is one of the most popular methods. Based on this basic algorithm, many variants [2] have been proposed to improve the performance of pair-wise registration from different perspectives. For convenience, we will use the term rigid transformation and motion interchangeably throughout this paper.

Different from pair-wise registration, the multi-view registration problem is more complex and has attracted less attention. However, some methods have been proposed to solve this difficult problem. Chen et al. [3] proposed the alignment-and-merging method, which repeatedly aligns and merges two scans until all scans are merged into the whole model. This method is straightforward but suffers from the error accumulation problem. To address this issue, Evangelidis et al. [4] proposed the JRMPC method, which assumes that all points are realizations of a unique Gaussian mixture model (GMM) and therefore casts the registration into a clustering problem. Subsequently, the expectation-maximization (EM) algorithm is utilized to estimate GMM parameters as well as all global motions for multi-view registration. This method is time-consuming due to the large number of parameters required to be estimated. Therefore, Zhu et al. [5] introduced the k-means algorithm to solve the multi-view registration problem. Compared with the JRMPC, this method is more efficient and likely to obtain better registration results.

For multi-view registration, another feasible solution is to recover global motions from a set of relative motions. To this end, Govindu [6] proposed the motion averaging (MA) algorithm, which avoids averaging of motion in Lie groups but performs average in the Lie-algebra of the underlying motion representation. With an initial guess, global motions can be simultaneously recovered from a set of relative motions by MA algorithm, which was further extended to solve multi-view registration problem [7]. Although these two algorithm is effective, it is sensitive to outliers due to utilizing Frobenius norm error in optimization. Govindu [8] combined graph-based sampling scheme and Random sample consensus (RANSAC) method to remove motion outliers. This method is more robust, but the efficiency is seriously reduced with the increase of scan number. For robot mapping, Grisetti et al. [9] proposed the general framework for graph optimization called G2O, which takes the same inputs as that of MA algorithm. Similar to MA algorithm, it is effective but sensitive to outliers.

Besides, Bourmaud et al. [10] proposed Bayesian MA algorithm for robot mapping. It is more complex than the original MA and its performance is greatly affected by the assignment of a reasonable covariance to each relative motion, which is very difficult in real applications. Meanwhile, Arrigoni et al. [11] introduced the low-rank and sparse (LRS) matrix decomposition to solve multi-view registration, which concatenates all available relative motions into a large matrix and then decomposes it into one sparse matrix and one low-rank matrix. This method can be viewed as another MA method and that is robust to outliers, but it requires more relative motions to achieve good registration. What's more, these methods treat each relative motion equally, which will reduce the performance of registration. Accordingly, Guo et al. [12] proposed weighted MA algorithm and Jin et al. [13] proposed weighted LRS algorithm, which can really improve the performance of multi-view registration with each relative motion assigned by a suitable weight, e.g. reliable motions assigned with high weights. However, it is difficult to manually assign a suitable weight to each relative motion.

Previous MA methods use Frobenius norm error in op-

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timization, and they perform well under the assumption of Gaussian noises. However, in practice, a relative motion set often includes outliers. In this case, the Frobenius norm error can not properly capture error statistics, which may seriously degrade the performance. Recently, correntropy [14] has been proposed as an information theoretic learning measure to solve robust learning problems [15], [16]. Compared with Frobenius norm, correntropy includes all even moments of the error. Therefore, the correntropy measure is robust against outliers and can achieve better learning performance especially when data contain outliers.

Accordingly, this paper introduces the correntropy measure to reformulate the MA problem, which is difficult to be solved directly. To this end, the half-quadratic (HQ) [17] technique is utilized to transform the problem into a half-quadratic optimization problem, which can be solved by the traditional optimization method. Further, we design an adaptive selection strategy for kernel width to take advantage of correntropy properties. Compared with Frobenius norm error, the negative effects of outliers are therefore alleviated by the correntropy measure. In summary, the main contributions of this paper are delivered as 1) It proposes a novel cost function for robust motion averaging. 2) It develops an effective MA algorithm by the HQ technique. 3) Experiments carried out on benchmark data sets confirm its superior performance over other related algorithms.

The remainder of this paper is organized as follows. Section 2 briefly reviews the concepts of MCC and HQ optimization theory. Section 3 formulates the correntropy based objective function for motion averaging and proposes the HQ based algorithm. Following that is section 4, in which the proposed method is tested and evaluated on benchmark data sets. Finally, conclusions are drawn in Section 5.

II. Preliminaries

This section briefly reviews MCC and HQ optimization theory, which are bases of the proposed method.

A. Maximum correntropy criterion

Given two random variables X and Y, the correntropy is defined by:

$$V(X,Y) = E(\kappa(X,Y)) = \int \kappa(x,y) dF_{XY}(x,y) \tag{1}$$

where $\kappa(\cdot,\cdot)$ denotes a shift-invariant Mercer kernel and $F_{XY}(x,y)$ is the joint probability distribution function (PDF) of (X,Y). In practice, the joint PDF is unknown and only a finite number of data points are available. With finite samples $\{x_i, y_i\}_{i=1}^{N}$, the correntropy can be approximated as:

$$\hat{V} = \frac{1}{N}\sum_{i=1}^{N} \kappa(x_i, y_i). \tag{2}$$

Usually, the correntropy kernel utilizes Gaussian Kernel:

$$\kappa(x,y) = G_\sigma(e) = \exp(-\frac{e^2}{2\sigma^2}), \tag{3}$$

where σ is the kernel width and $e = (x-y)$ is the error.

Obviously, the correntropy is a local and nonlinear similarity measure between two random variables within a "window" in the joint space defined by the kernel width. Compared with traditional measures, the correntropy contains all the even moments of the difference between X and Y, and it is robust to outliers. In supervised learning, the correntropy measure based loss function is usually given by:

$$J_{G_\sigma} = \frac{1}{N}\sum_{i=1}^{N} \sigma^2(1 - G_\sigma(e(i))) \tag{4}$$

which is referred to as the MCC.

B. Half-quadratic optimization theory

Usually, it is difficult to directly optimize the correntropy based objective function, which is non-quadratic. Therefore, the HQ technique has been introduced to solve this problem.

According to the HQ theory [17], there is a convex conjugated function φ corresponding to $G_\sigma(e)$ and they have the following relationship:

$$G_\sigma(e) = \max_t(\frac{e^2 t}{\sigma^2} - \varphi(t)), \tag{5}$$

where $t \in \mathbb{R}$ and the maximum is achieved at $t = -G_\sigma(e)$. Equivalently, Eq. (5) can also be transformed into:

$$\sigma^2(1 - G_\sigma(e)) = \min_t(-e^2 t + \sigma^2\varphi(t) + \sigma^2). \tag{6}$$

By defining $w = -t$ and $\phi(w) = \sigma^2\varphi(-w) + \sigma^2$, Eq. (6) can be further derived as:

$$\min_e \sigma^2(1 - G_\sigma(e)) = \min_{e,w}(e^2 w + \phi(w)). \tag{7}$$

Based on the HQ technique, the non-quadratic cost function is reformulated as the augmented objective function in enlarged parameter space $\{e, w\}$ by introducing auxiliary variable w.

III. Robust Motion Averaging under MCC

This section states the MA problem in multi-view registration and then proposes a robust solution under MCC.

A. Problem statement

Given multiple range scans, the goal of multi-view registration is to estimate the rigid transformation for each scan to the reference coordinate frame. For simplicity, the rigid transformation can be defined in the form of motion $\mathbf{M}_i$ as: $\mathbf{M}_i = \begin{bmatrix} \mathbf{R}_i & \vec{t}_i \\ 0 & 1 \end{bmatrix}$. where $\mathbf{R}_i$ and $\vec{t}_i$ denote the rotation matrix and translate vector, respectively. Compared with the multi-view registration problem, the pair-wise registration problem is much easier. Therefore, it is reasonable to achieve multi-view registration based on pair-wise registration, which arises the MA problem. Given a set of estimated relative motions $\hat{\mathbf{M}}_{ij} \in \Omega$, it requires to recover the global motion $\{\mathbf{M}_i\}_{i=1}^{N}$ for multi-view registration. Accordingly, the multi-view registration can be formulated the following optimization problem:

$$\arg\min_{\mathbf{M}_i,\mathbf{M}_j} \sum_{\hat{\mathbf{M}}_{ij}\in\Omega} \left\| \hat{\mathbf{M}}_{ij} - {\mathbf{M}_i}^{-1}\mathbf{M}_j \right\|_F^2. \tag{8}$$

As either $\mathbf{M}_i$ or $\mathbf{M}_j$ denotes the variable of global motion, we only preserve $\mathbf{M}_i$ as the variable for simplicity. This problem has been solved by the original MA algorithm [6], which is sensitive to outliers due to the application of Frobenius norm error in the optimization.

To improve the robustness, we introduce correntropy as the error measure and reformulate the multi-view registration problem as the following optimization problem:

$$\min_{\mathbf{M}_i} J_{G_\sigma}(\mathbf{M}_i) = \sum_{\hat{\mathbf{M}}_{ij}\in\Omega} \sigma^2(1 - G_\sigma(\left\|\hat{\mathbf{M}}_{ij} - \mathbf{M}_i{}^{-1}\mathbf{M}_j\right\|_F)),$$

which denotes a non-convex and non-quadratic cost function, which is difficult to be directly minimized by traditional methods. To this end, the HQ technique should be utilized to minimize this function.

B. Optimization by the HQ theory

As shown in Eq. (7), minimizing the correntropy measure based loss function in terms of e equals to minimizing an augmented cost function in an enlarged parameter space $\{e, w\}$. Accordingly, the correntropy measure based objective function can be further formulated as:

$$J_{G_\sigma}(\mathbf{M}_i) = \min_{w_{ij}} \sum_{\hat{\mathbf{M}}_{ij}\in\Omega} \left[\left\|\hat{\mathbf{M}}_{ij} - \mathbf{M}_i{}^{-1}\mathbf{M}_j\right\|_F^2 w_{ij} + \phi(w_{ij})\right]. \quad (9)$$

Further, we can define the augmented cost function:

$$J_{HQ}(\mathbf{M}_i, w_{ij}) = \sum_{\hat{\mathbf{M}}_{ij}\in\Omega} \left[\left\|\hat{\mathbf{M}}_{ij} - \mathbf{M}_i{}^{-1}\mathbf{M}_j\right\|_F^2 w_{ij} + \phi(w_{ij})\right] \quad (10)$$

According to the HQ optimization theory, we obtain the equivalent relation as follows:

$$\min_{\mathbf{M}_i} J_{G_\sigma}(\mathbf{M}_i) = \min_{\mathbf{M}_i, \mathbf{W}_{ij}} J_{HQ}(\mathbf{M}_i, w_{ij}). \quad (11)$$

This optimization problem can then be solved by the alternating minimization procedure as follows:

(1) Optimization of w_{ij}: According to Eq. (5) and Eq. (7), the minimum of the objective function J_{HQ} is achieved by $w = G_\sigma(e)$ for given a certain e. Therefore, the optimal solution of w_{ij} can be estimated for the fixed $\mathbf{M}_i$ as:

$$w_{ij} = G_\sigma(\left\|\hat{\mathbf{M}}_{ij} - \mathbf{M}_i{}^{-1}\mathbf{M}_j\right\|_F). \quad (12)$$

This procedure can be viewed as the weight assignment operation, which assigns different weights to each relative motion based on the residual motion error. According to the property of Gaussian function, a relative motion with small error will be assigned with a large weight, and vice versa. Different from previous methods, we do not manually estimate a weight for each relative motion, but automatically calculate them by the residual motion error. Therefore, suitable weight can be assigned to each relative motion due to properties of the correntropy measure.

(2) Optimization of $\mathbf{M}_i$: For the fixed w_{ij}, Eq. (10) is simplified into the following optimization problem:

$$\mathbf{M}_i = \arg\min_{\mathbf{M}_i} \sum_{\hat{\mathbf{M}}_{ij}\in\Omega} w_{ij} \left\|\hat{\mathbf{M}}_{ij} - \mathbf{M}_i{}^{-1}\mathbf{M}_j\right\|_F^2. \quad (13)$$

Eq. (13) denotes the weighted MA problem, where the negative impact of outliers can be seriously reduced due to the small weight assigned by the first procedure. Since this problem can be solved by the variant of original MA algorithm, we present the solution without any proof.

C. Weighted motion averaging

Given the relative motion set $\{\hat{\mathbf{M}}_{ij,h}\}_{h=1}^H$, the motion averaging algorithm requires initial global motions $\{\mathbf{M}_i^0\}_{i=1}^N$ to achieve multi-view registration by iterations. For one relative motion $\hat{\mathbf{M}}_{ij,h}$ and previous global motion $\{\mathbf{M}_i^{k-1}\}_{i=1}^N$, the residual relative motion is defined as:

$$\begin{aligned} \Delta\mathbf{M}_{ij} &= \mathbf{M}_i^{k-1}\hat{\mathbf{M}}_{ij}(\mathbf{M}_j^{k-1})^{-1} \\ &= (\Delta\mathbf{M}_i)^{-1}\Delta\mathbf{M}_j. \end{aligned} \quad (14)$$

Eq. (14) can be converted into the equivalent formulation:

$$\Delta\mathbf{m}_{ij,h} = (\Delta\mathbf{m}_{j,h} - \Delta\mathbf{m}_{i,h}), \quad (15)$$

where $\Delta\mathbf{m} = \log(\Delta\mathbf{M})$. Subsequently, the function $vec()$ is utilized to extract parameters from $\Delta\mathbf{m}$ to form a column wise vector Δv and then Eq. (15) is transformed into the following form:

$$\Delta v_{ij,h} = (\Delta v_{j,h} - \Delta v_{i,h}), \quad (16)$$

where $\Delta v = vec(\Delta\mathbf{m})$.

As each relative motion is assigned with a weight, the $6 \times 6N$ block-matrix $\mathbf{D}_{ij,h}$ is constructed with the ith and jth block-elements filling with $-w_{ij,h}\mathbf{I}_6$ and $w_{ij,h}\mathbf{I}_6$:

$$\mathbf{D}_{ij,h} = \begin{bmatrix} \cdots & -w_{ij,h}^k\mathbf{I}_6 & \cdots & w_{ij,h}^k\mathbf{I}_6 & \cdots \end{bmatrix}, \quad (17)$$

where $\mathbf{I}_6$ denotes the 6×6 identity matrix. According to Eq. (16), there exists the following relationship:

$$\mathbf{D}_{ij,h}\mathfrak{A} = w_{ij,h}^k \Delta v_{ij,h}, \quad (18)$$

where $\mathfrak{A} = \begin{bmatrix} \Delta v_1; & \Delta v_2; & \cdots & \Delta v_N \end{bmatrix}$. To refine global motions, Eq. (18) can be extended to the situation of many relative motions $\{\hat{\mathbf{M}}_{ij,h}\}_{h=1}^H$ as follows:

$$\mathbf{D}\mathfrak{A} = \Delta\mathrm{V}_{ij}, \quad (19)$$

where $\Delta\mathbf{V} = \begin{bmatrix} w_{ij,1}^k\Delta v_{ij,1}; & \cdots & w_{ij,H}^k\Delta v_{ij,H} \end{bmatrix}$ and $\mathbf{D} = \begin{bmatrix} \mathbf{D}_{ij,1}; & \cdots; & \mathbf{D}_{ij,H} \end{bmatrix}$. This formulation leads to vector $\mathfrak{A}$ including parameters of all residual global motions:

$$\mathfrak{A} = \mathbf{D}^\dagger\Delta\mathrm{V}_{ij}, \quad (20)$$

where $\mathbf{D}^\dagger$ is the pseudo-inverse matrix of $\mathbf{D}$. Finally, elements of $\mathfrak{A}$ is utilized to update each global motion as:

$$\mathbf{M}_i^k = \mathbf{M}_i^{k-1}\exp(\Delta\mathbf{m}_i), \quad (21)$$

where $\Delta\mathbf{m}_i = rvec(\Delta v_i)$ is residual global motions and $rvec()$ denotes the inverse function of $vec()$.

Algorithm 1 Robust motion averaging under MCC

Input: Initial guess $\{\mathbf{M}_i^0\}_{i=1}^N$ and relative motions $\{\hat{\mathbf{M}}_{ij,h}\}_{h=1}^H$
Output: Global motions $\{\mathbf{M}_i\}_{i=1}^N$

$k = 0$;
repeat
  $k = k + 1$;
  Compute the residual error $e_{\mathbf{M},k}$ by Eq. (22);
  Obtain the kernel width $\sigma_k = \alpha e_{\mathbf{M},k}$;
  Calculate the weight $w_{ij,h}^k$ for $\mathbf{M}_{ij,h}^k$ by Eq. (12);
  for $(h = 1 : H)$ **do**
    Generate $\mathbf{D}_{ij,h}$ by Eq. (17);
    Concatenate $\mathbf{D}_{ij,h}$ into the matrix $\mathbf{D}$;
    Stack $w_{ij,h}^k \Delta v_{ij,h}$ into the vector $\Delta \mathrm{V}_{ij}$;
  end for
  $\mathfrak{A} = \mathbf{D}^{\dagger} \Delta \mathrm{V}_{ij}$;
  $\forall i \in [2, N],\ \mathbf{M}_i^k = \mathbf{M}_i^{k-1} \exp(\Delta \mathbf{m}_i)$
until ($\mathbf{M}'$s change is negligible) and $(k \geq K)$

D. Implementation

Obviously, our method is local convergent. To obtain desired results, initial guess should be provided for global motions in advance. Besides, its performance is affected by the kernel width σ in correntropy measure. In the literature, lots of works have illustrated that relatively large kernel width can offer high convergence speed but suffer from less accuracy, and vice versa. As our method achieves multi-view registration by iterations, it is better to use an adaptive kernel width. Specifically, the kernel width is set to be large at the beginning of the iteration and it should decrease with the increase of the iteration number. As residual motion error decreases with the increase of iteration number, it is reasonable to set the kernel width to be proportional with the residual error of all global motions, e.g. $\sigma_k = e_{\mathbf{M},k}$, where $e_{\mathbf{M},k}$ denotes the residual motion error defined as:

$$e_{\mathbf{M},k} = \sum\nolimits_{h=1}^{H} \left\| \hat{\mathbf{M}}_{ij} - \left(\mathbf{M}_i^k\right)^{-1} \mathbf{M}_j^k \right\|_F \Big/ H. \tag{22}$$

This setting can well balance the convergence speed and accuracy of the proposed method. Accordingly, the proposed method is summarized in **Algorithm 1**

IV. Experiments

This section tests our method on five benchmark data sets, where four data sets are taken from the Stanford 3D Scanning Repository [18] and the other data set is the Gazebo data set [19]. The first four data sets are acquired from one object model in multiple views and the last data set was acquired in the outdoor environment for SLAM (Simultaneous Localization and Mapping). In all these data sets, ground truth of rigid transformations was provided with multiple scans for the evaluation of registration results. But they are only utilized to assist for the final assessment. As the proposed method takes relative motions as its input to recover global motions, we estimate relative motions for each scan pair by utilizing the pair-wise registration method proposed in [20], which can obtain reliable results for these scan pairs with non-low overlap percentage. For accuracy comparison, the registration error of rotation matrix and translation vector are defined as $e_{\mathbf{R}} = \frac{1}{M}\sum_{i=1}^{M} \arccos(\frac{\mathrm{tr}(\mathbf{R}_{m.i}(\mathbf{R}_{g,i})^T)-1}{2})$ and $e_t = \frac{1}{M}\sum_{i=1}^{M} \|t_{m,i} - t_{g,i}\|_F$, respectively. Here, $(\mathbf{R}_{g,i}, t_{g,i})$ indicates the ground truth of the i th rigid transformation and $(\mathbf{R}_{m,i}, t_{m,i})$ denotes the one estimated by multi-view registration method. Table I demonstrates the scan number, relative motion number, and unit for each data set. Besides, Table II lists statistics information of all relative motions in each data set, including the mean, median, RMSE of rotation and translation errors. As shown in Table II, four object data sets contain many unreliable relative motions or outliers. While the environment data set includes less unreliable relative motions or outliers.

TABLE I
Details of benchmark datasets.

Dataset	Armadillo	Buddha	Bunny	Dragon	Gazebo
Scans	12	15	10	15	32
Motions	68	79	46	103	465
Unit	mm	mm	mm	mm	m

TABLE II
Statistics information of relative motions for each data set.

	Rotation error (rad.)			Translation error (mm|m)		
	Mean	Median	RMSE	Mean	Median	RMSE
Armadillo	0.2871	0.0045	0.6101	9.857	0.6271	21.5347
Buddha	0.1655	0.0088	0.5280	1.8631	0.7141	4.4277
Bunny	0.0253	0.0054	0.0624	2.4320	0.0906	6.1294
Dragon	0.1157	0.0043	0.4459	4.5063	0.6799	12.9122
Gazebo	0.0112	0.0103	0.0060	0.0603	0.0426	0.0526

To demonstrate the performance, the proposed method is tested on five data sets and compared with some related methods, including the multi-view registration method based on general framework for graph optimization [9], the original motion averaging algorithm [7], and weighted motion averaging algorithm [12], which are abbreviated as G2O, WA, and wWA, respectively. For object data sets, initial global motions are obtained by the LRS [11] method. While, for the environment data set, initial global motions are estimated from sequentially scan matching. All experiments are performed on a four-core 3.6 GHz computer with 8 GB of memory. Experimental results are reported in the form of run time, rotation error, and translation error. These registration results are all recorded in Table III. To compare these methods in a more intuitive manner, Fig. 1 displays multi-view registration results in the form of cross-section for four object data sets and Fig. 2 illustrates location results of the environment data set as well as our SLAM result.

As the multi-view registration problem is similar to the SLAM problem, both of them can be solved by G2O, which takes initial global motions and relative motions its inputs.

TABLE III

REGISTRATION RESULTS OF DIFFERENT METHODS TESTED ON FIVE DATA SETS, WHERE THE NUMBERS IN BOLD DENOTE THE BEST PERFORMANCE.

	Armadillo			Buddha			Bunny			Dragon			Gazebo		
	$e_{\mathbf{R}}$(rad.)	$e_{\vec{t}}$(mm)	T(s)	$e_{\mathbf{R}}$(rad.)	$e_{\vec{t}}$(mm)	T(s)	$e_{\mathbf{R}}$(rad.)	$e_{\vec{t}}$(mm)	T(s)	$e_{\mathbf{R}}$(rad.)	$e_{\vec{t}}$(mm)	T(s)	$e_{\mathbf{R}}$(rad.)	$e_{\vec{t}}$(m)	T(s)
Initial	0.2291	8.6053	/	0.1386	1.9209	/	0.0459	3.5322	/	0.1962	11.7375	/	0.0241	0.1069	/
G2O	0.2679	8.0389	0.0270	0.0471	1.2784	0.0364	0.0377	2.6475	0.0372	0.5644	25.5545	0.0339	0.0131	0.0510	0.2688
MA	0.4677	10.0962	0.6842	0.3239	3.0887	0.9874	0.0522	4.5743	0.1221	0.5977	20.5170	0.9533	0.0095	0.0525	10.1827
wMA	0.0861	2.9548	0.5562	**0.0090**	1.0112	0.7099	0.0269	2.2822	0.1320	0.1841	13.7669	0.4624	0.0094	0.0519	10.1036
Ours	**0.0044**	**1.2808**	0.5868	0.0111	**0.9871**	1.1065	**0.0086**	**0.6749**	0.2563	**0.0163**	**1.3837**	0.8753	**0.0088**	**0.0337**	21.7301

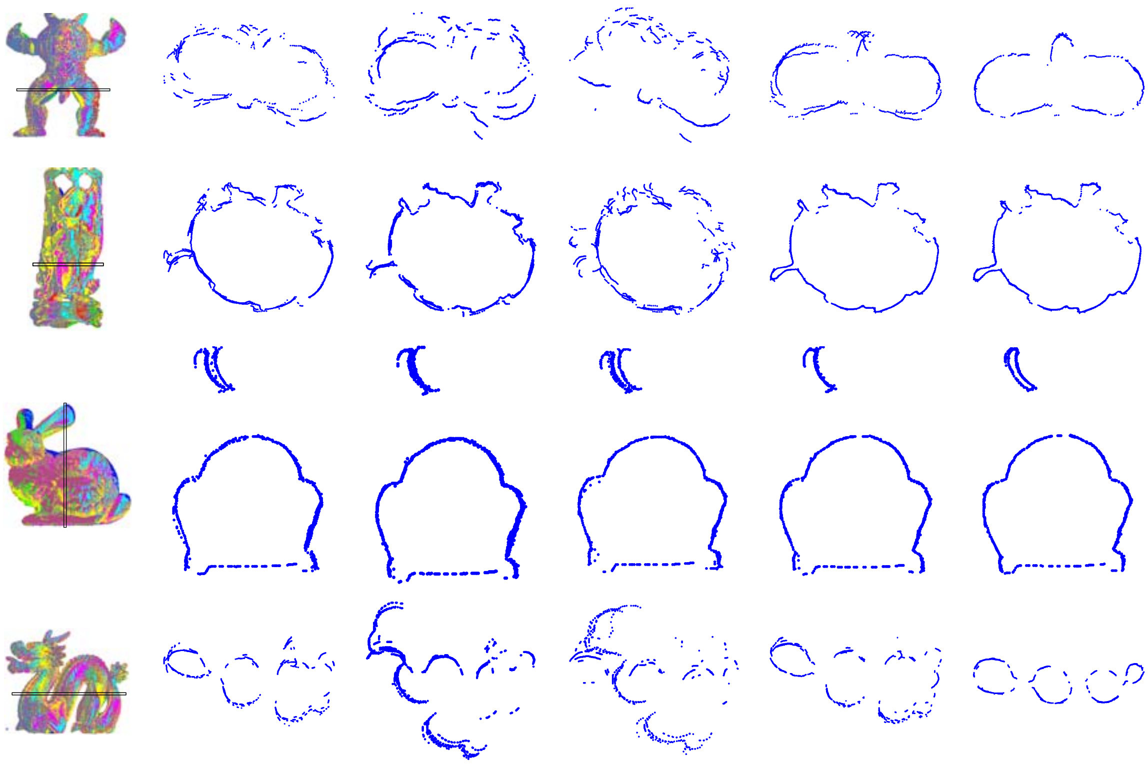

Fig. 1. Multi-view registration results in the form of cross-section for object data sets. (a) Aligned 3D models. (b) Initial results. (c) G2O results. (d) MA results. (e) wMA results. (f) Our results.

In this method, each relative motion should be assigned with one covariance matrix to denotes its uncertainty or reliability. Here, we assign the identity matrix to each relative motion due to the lack of prior information. As Table III and Fig. 1 illustrate, G2O is efficient but is unable to obtain promising registration results for object data sets. To obtain the desired results, good initial motions are required and each relative motion requires one appropriate covariance matrix, which is very difficult to be estimated in most practical applications. Since good initial motions are provided for the environment data set, G2O can obtain promising SLAM result.

As MA utilizes Frobenius norm error for the estimation of global motions, it is sensitive to outliers and difficult to obtain promising registration results due to the exiting of outliers. Therefore, this method is difficult to obtain good muti-view registration results for all object data sets. Different from MA, wMA pays more attention to reliable relative motions by assigning high weights. When each relative motion is assigned with one appropriate weight, e.g. outliers assigned with very low weight, wMA can obtain promising registration results, such as Stanford Buddha. However, the weight of each relative motion is estimated and assigned by some manual methods in wMA, it may assign a non-low weight to outliers, which can lead to the failure of multi-view registration. For the environment data set, good initial global motions are provided with reliable relative motions. Subsequently, both MA and wMA are able to obtain more promising SLAM results than G2O.

Different from other competed methods, the proposed method utilizes the correntropy measure to achieve MA for muti-view registration. Compared with the Frobenius norm error, the correntropy measure can effectively alleviate the impact of large errors caused by outliers. For the balance of registration accuracy and convergence speed, adaptive kernel width has been selected by the well-designed strategy. Therefore, the proposed method can achieve the most promising

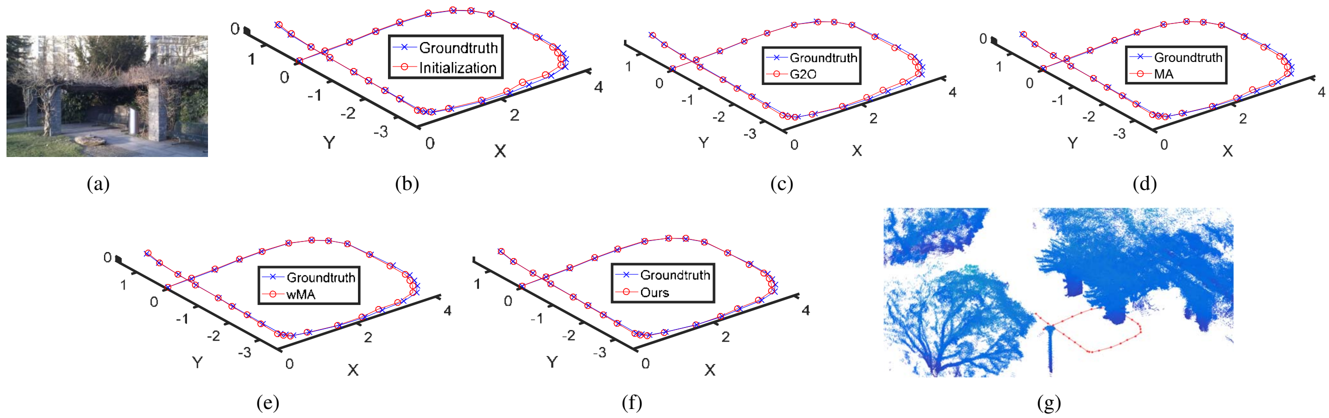

Fig. 2. Comparison of localization results for different methods tested on Gazebo data set. (a) Scene image (b) Initialization. (c) G2O result. (d) MA result. (e) wMA result. (f) Our localization result. (g) Our SLAM result.

results for both multi-view registration and robot mapping, even the input of relative motion set contains unreliable motions or outliers. The weakness is that our method is less efficient than other competed methods due to the weight update and the calculation of $\mathbf{D}^{\dagger}$ in Eq. (20) in each iteration. Since accuracy is one of the most important performances for the graph-based SLAM algorithm, the proposed method can be applied to robot mapping.

V. CONCLUSIONS

In this paper, we proposed a novel and robust MA method for multi-view registration. To improve the robustness against outliers, it first utilizes the correntropy measure to design the objective function of MA, which arises a non-quadratic optimization problem. By the HQ theory, the correntropy based optimization problem can be solved by an alternating minimization procedure, which includes the operation of weight assignment and weighted MA derived from original MA algorithm. Further, the selection strategy of adaptive kernel width is proposed to balance the accuracy and convergent speed of our algorithm. Experiments tested on benchmark data sets illustrate that the proposed method can achieve multi-view registration and SLAM with better performance than related methods on accuracy and robustness.

ACKNOWLEDGEMENTS

This work is supported in part by the National Natural Science Foundation of China under Grants 61573273 and 61976175; by the Key Research and Development Program of Shaanxi under Grant 2021GY-025; by the Fundamental Research Funds for Central Universities under Grant xzy012019045.

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