Assignment 7

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Outline

Problem Statement

13.4 Q7 [NCERT 12]

A coin is biased so that the head is 3 times as likely to occur as tail. If the coin is tossed twice, find the probability distribution of number of tails.

Random Variable Definition

In this experiment, there are two consecutive Bernoulli trials. Therefore, it is appropriate to define a Binomial Random Variable X as under:

Variable	**Event**
$X = 0$	0 tails are obtained
$X = 1$	1 tails are obtained
$X = 2$	2 tails are obtained

Table 1: Random Variable X

Random Variable Definition

In each Bernoulli trial, let Binomial Random Variable Y be as under:

Variable	**Event**
$Y = 0$	Head obtained
$Y = 1$	Tail obtained

Table 2: Random Variable Y

Probability Mass Function

We know that,

$$
\begin{aligned}
\Pr(Y = 0) &= 3 \times \Pr(Y = 1) && (1) \\
\Pr(Y = 0) + \Pr(Y = 1) &= 1 && (2) \\
\therefore \Pr(Y = 1) &= \frac{1}{4} \text{ and } \Pr(Y = 0) = \frac{3}{4} && (3)
\end{aligned}
$$

Probability Mass Function

The probability of success is $p = \frac{1}{4}$.
Therefore, the probability that X maps to i is given by:

$$\Pr\left(X = i\right) = \binom{2}{i}(1-p)^{2-i}p^{i},\ 0 \leq i \leq 2 \tag{4}$$

The values for i can be substituted in the above formula, and the graph of the PMF can be obtained.

Cumulative Distribution Function

The cumulative probability $\Pr(X \leq i)$ can be defined as under:

$$\Pr(X \leq i) = \sum_{k=0}^{i} \binom{2}{k} (1-p)^{2-k} p^k, \; 0 \leq i \leq 2 \tag{5}$$

The values of i can be substituted in the above equation, and the obtained values can be used to plot the CDF graph.

Solution

We have to find the probability distribution of the number of tails in the trials. So, plugging $i = 0, 1, 2$ in equation (4)

$$\Pr(X = i) = \begin{cases} \frac{9}{16}, i = 0 \\ \frac{3}{8}, i = 1 \\ \frac{1}{16}, i = 2 \end{cases} \tag{6}$$

PMF Graph

The PMF graph is:

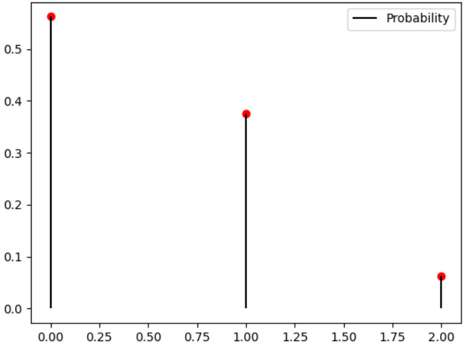

Figure 2: Probability Mass Function

CDF Graph

The CDF graph is:

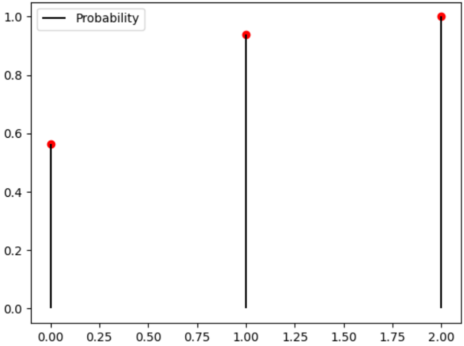

Figure 2: Cumulative Distribution Function