

Assignment 6

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May 16, 2022

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Question

12th Class NCERT Chapter 13 Example 14:

Prove that if A and B are two independent events, then the probability of occurrence of at least one of A and B is given by $1 - \Pr(A') \Pr(B')$.

Solution

Let the random variable $X \in \{1, 2, \dots, 9\}$ denote the events A and B as follows:

Random Variable	Event
2, 3	A
4, 5	B
1	AB
6, 7, 8, 9	U-A-B

Table

Properties

Given independent events A and B ,

$$\Pr(AB) = \Pr(A) + \Pr(B) - \Pr(A + B) \quad (1)$$

and also,

$$\Pr(AB) = \Pr(A) \Pr(B) \quad (2)$$

$$\Pr(A) = 1 - \Pr(A') \quad (3)$$

Property Derivation

We note that for any events A and B we have the following:

$$A + B = A(B + B') + B(A + A') \quad (4)$$

$$= (AB + BA) + AB' + A'B \quad (5)$$

$$= AB + AB' + A'B \quad (6)$$

Thus, taking probabilities in (6), since all the events are independent,

$$\Pr(A + B) = \Pr(AB') + \Pr(AB) + \Pr(A'B) \quad (7)$$

$$= \Pr(A) + \Pr(A'B) \quad (8)$$

$$= \Pr(A) + \Pr(B) - \Pr(AB) \quad (9)$$

Property Derivation

We know that for any independent events A and B

$$\Pr(A|B) = \Pr(A) \quad (10)$$

$$\frac{\Pr(AB)}{\Pr(B)} = \Pr(A) \quad (11)$$

$$\Pr(AB) = \Pr(A) \Pr(B) \quad (12)$$

Solution contd.

Using (2) and (3) in (1), we get

$$\Pr(A + B) = \Pr(A) + \Pr(B) - \Pr(A) \Pr(B) \quad (13)$$

$$= \Pr(A) + \Pr(B) [1 - \Pr(A)] \quad (14)$$

$$= \Pr(A) + \Pr(B) \Pr(A') \quad (15)$$

$$= 1 - \Pr(A') + \Pr(B) \Pr(A') \quad (16)$$

$$= 1 - \Pr(A') [1 - \Pr(B)] \quad (17)$$

$$= 1 - \Pr(A') \Pr(B') \quad (18)$$