Assignment 6

Kushagra Gupta (cs21btech11033)

***Abstract*—This document contains the solution for Assignment 7 (NCERT Class 12 Chapter 13 Example 14)**

13 E14 [NCERT 12] : If A and B are two independent events, then the probability of occurrence of at least one of A and B is given by $1-\Pr(A')\Pr(B')$.
Solution: We will map the numbers $\{1, 2, \dots 9\}$ to events A and B to simulate a favourable sample space We know that, given independent events A and B,

Sample Space	Event
$1, 2, 3$	A
$1, 4, 5$	B
1	AB
$6, 7, 8, 9$	U-A-B

TABLE I

$$\Pr(AB) = \Pr(A) + \Pr(B) - \Pr(A+B) \quad (1)$$

and also,

$$\Pr(AB) = \Pr(A)\Pr(B) \quad (2)$$
$$\Pr(A) = 1 - \Pr(A') \quad (3)$$

Using (2) and (3) in (1), we get

$$\Pr(A+B) = \Pr(A) + \Pr(B) - \Pr(A)\Pr(B) \quad (4)$$
$$= \Pr(A) + \Pr(B)[1 - \Pr(A)] \quad (5)$$
$$= \Pr(A) + \Pr(B)\Pr(A') \quad (6)$$
$$= 1 - \Pr(A') + \Pr(B)\Pr(A') \quad (7)$$
$$= 1 - \Pr(A')[1 - \Pr(B)] \quad (8)$$
$$= 1 - \Pr(A')\Pr(B') \quad (9)$$

Note: Derivation of (1) using Boolean Algebra: We note that for any events A and B we have the following:

$$A + B = A(B + B') + B(A + A') \quad (10)$$
$$= (AB + BA) + AB' + A'B \quad (11)$$
$$= AB + AB' + A'B \quad (12)$$

Thus, taking probabilities in (12), since all the events are independent,

$$\Pr(A+B) = \Pr(AB') + \Pr(AB) + \Pr(A'B) \quad (13)$$
$$= \Pr(A) + \Pr(A'B) \quad (14)$$
$$= \Pr(A) + \Pr(B) - \Pr(AB) \quad (15)$$