Assignment 2

AI1110: Probability and Random Variables

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ICSE 2018 Grade 12

Q8-a: Find the points on the curve $y = 4x^3 - 3x + 5$ at which the equation of the tangent is parallel to $x - axis$

Solution: $y = f(x)$ is differentiable, and the derivative represents the tangent at a point in f. Since any tangent parallel to the $x - axis$ has slope 0, we equate $f'(x)$ to 0

$$y = f(x) = 4x^3 - 3x + 5 \tag{1}$$
$$\Rightarrow \frac{dy}{dx} = f'(x) = \frac{d}{dx}(4x^3 - 3x + 5) \tag{2}$$
$$\Rightarrow f'(x) = 12x^2 - 3 \tag{3}$$

We want roots for $f'(x) = 0$.

$$\therefore f'(x) = 12x^2 - 3 = 0 \tag{4}$$
$$\Rightarrow x^2 = \frac{1}{4} \tag{5}$$
$$\Rightarrow x = \pm\sqrt{\frac{1}{4}} \tag{6}$$
$$\Rightarrow x = \pm\frac{1}{2} \tag{7}$$

Now at $x = \pm\frac{1}{2}$ we get from equation (1),

$$y = 4 \times \left(\pm\frac{1}{2}\right)^3 - 3 \times \left(\pm\frac{1}{2}\right) + 5 \tag{8}$$
$$\Rightarrow y = \pm\frac{1}{2} \mp \frac{3}{2} + 5 \tag{9}$$
$$\Rightarrow y = 4, 6 \tag{10}$$

$\therefore$ At $(x, y) = \left(\frac{1}{2}, 4\right), \left(\frac{-1}{2}, 6\right)$ the tangents are parallel to x axis.

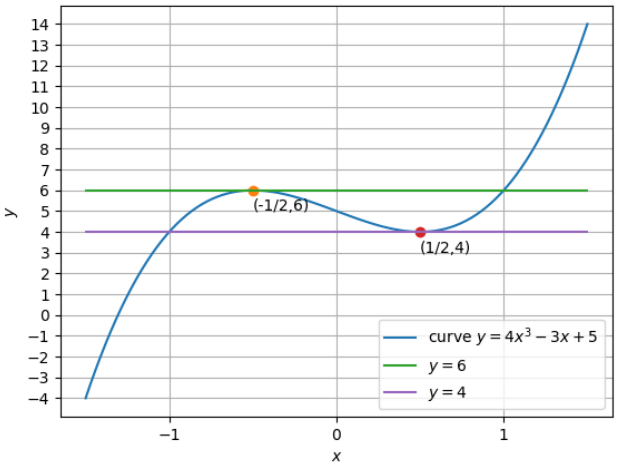

Fig. 1. Plot showing curve and appropriate tangents