Assignment 5

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***Abstract*—This document contains the solution for Assignment 5 (NCERT Class 12 Ex13.1 Q9)**

Ex13.1 Q9 [NCERT 12] : Mother, Father and son line up at random for a family picture. Events E and F are defined as below:

E : son on one end F : father in middle Find the probability of E given that F has already occurred $(E|F)$

Solution:

Let the random variables X_i map to the set $\{0, 1\}$ as described in Table I

Variable	Event
$X_1 = 1$	E
$X_2 = 1$	F
x	son
y	mother
z	father

TABLE I

(i) The sample space for the arrangement is given by $\mathcal{S} = \{(x, y, z) : x, y, z \in \{left, middle, right\} \wedge x \neq y \neq z\}$.

$$\therefore \left|\mathcal{S}\right| = 3! = 6 \tag{1}$$

(ii) The sample space for event E is given by $\mathcal{S}_E = \{(x, y, z) : x, y, z \in \{left, middle, right\} \wedge x \in \{left, right\} \wedge y \neq z\}$.

$$\therefore \left|\mathcal{S}_E\right| = \binom{2}{1} \times 2! = 4 \tag{2}$$

(iii) The sample space for event F is given by $\mathcal{S}_F = \{(x, y, z) : x, y, z \in \{left, middle, right\} \wedge z = middle \wedge x \neq y\}$.

$$\therefore \left|\mathcal{S}_F\right| = 2! = 2 \tag{3}$$

(iv) The sample space for both event E and event F simultaneously occurring is given by $\mathcal{S}_{E\cap F} = \{(left, right, middle), (right, left, middle)\}$.

$$\left|\mathcal{S}_{E\cap F}\right| = 2 \tag{4}$$

Probability	Value
$\Pr(X_1 = 1)$	$\frac{4}{6} = \frac{2}{3}$
$\Pr(X_2 = 1)$	$\frac{2}{6} = \frac{1}{3}$
$\Pr(X_1 = 1, X_2 = 1)$	$\frac{2}{6} = \frac{1}{3}$
$\Pr(X_1 = 1 \mid X_2 = 1)$	?

TABLE II

The probabilities for different values of X_i can therefore be found.

The values are given in Table II

Now,

$$\Pr(X_1 = 1|X_2 = 1) = \frac{\Pr(X_1 = 1, X_2 = 1)}{\Pr(X_2 = 1)} \tag{5}$$

Therefore, substituting the values from Table II, we have:

$$\Pr(X_1 = 1|X_2 = 1) = \frac{\frac{1}{3}}{\frac{1}{3}} = 1 \tag{6}$$