En utilisant $\Sigma_- = \Sigma_+ = \Sigma$, (1) devient

$$\log\left(\frac{P(Y=+1 \mid X=x)}{P(Y=-1 \mid X=x)}\right) = \log\left(\frac{\pi_+}{\pi_-}\right) + \log\left(\underbrace{\cancel{\frac{\sqrt{\det \Sigma}}{\sqrt{\det \Sigma}}}}_{\to 1}\right) -$$

$$\frac{1}{2}\left[\cancel{x^T \Sigma^{-1} x} - x^T \Sigma^{-1} \mu_+ - \mu_+^T \Sigma^{-1} x + \mu_+^T \Sigma^{-1} \mu_+\right] +$$

$$\frac{1}{2}\left[\cancel{x^T \Sigma^{-1} x} - x^T \Sigma^{-1} \mu_- - \mu_-^T \Sigma^{-1} x + \mu^T \Sigma^{-1} \mu_0\right]$$

$$= \log\left(\frac{\pi_+}{\pi_-}\right) + \log(1) + \frac{1}{2} x^T \Sigma^{-1} \mu_+$$
$$- \frac{1}{2} x^T \Sigma^{-1} \mu_- + \frac{1}{2} \mu_+^T \Sigma^{-1} x - \frac{1}{2} \mu_-^T \Sigma^{-1} x$$
$$+ \frac{1}{2} \mu_-^T \Sigma^{-1} \mu_- - \frac{1}{2} \mu_+^T \Sigma^{-1} \mu_+$$

$$= \log\left(\frac{\pi_+}{\pi_-}\right) + \frac{1}{2} x^T \Sigma^{-1} (\mu_+ - \mu_-)$$
$$+ \frac{1}{2} (\mu_+ - \mu_-)^T \Sigma^{-1} x + \frac{1}{2} \mu_-^T \Sigma^{-1} \mu_-$$
$$- \frac{1}{2} \mu_+^T \Sigma^{-1} \mu_T$$

Finalement,

$$\boxed{\log \frac{P(Y=+1 \mid X=x)}{P(Y=-1 \mid X=x)} = \log\left(\frac{\pi_+}{\pi_-}\right) + x^T \Sigma^{-1} (\mu_+ - \mu_-) - \frac{1}{2} (\mu_+ - \mu_-)^T \Sigma^{-1} (\mu_+ - \mu_-)}$$

ou $$\boxed{\log \frac{P(Y=+1 \mid X=x)}{P(Y=-1 \mid X=x)} = \log\left(\frac{\pi_+}{1-\pi_+}\right) + x^T \Sigma^{-1} (\mu_+ - \mu_-) - \frac{1}{2} (\mu_+ - \mu_-)^T \Sigma^{-1} (\mu_+ - \mu_-)}$$