Tutorial 6 answers

Jun 2022

Problem 1

Probability densities in region $x<0$:

(a) $$|\Psi(x,t)|^2 = 2|A|^2\left(\frac{k_1^2+k_2^2}{(k_1+k_2)^2} + \frac{k_1-k_2}{k_1+k_2}\cos 2k_1x\right)$$

(b) $$|\Psi(x,t)|^2 = 2|A|^2\left(1+\frac{k_1^2-k_2'^2}{k_1^2+k_2'^2}\cos 2k_1x - \frac{2k_1k_2'}{k_1^2+k_2'^2}\sin 2k_1x\right)$$

Problem 2

$$R = \left|\frac{B}{A}\right|^2 = \left(\frac{\sqrt{E+V_0}-\sqrt{E}}{\sqrt{E+V_0}+\sqrt{E}}\right)^2 = \frac{1}{9}$$

Answers to prob 3, tut 6

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(a) time-independent schrödinger equation

$$-\frac{\hbar^2}{2m}\nabla^2\psi + V\psi = E\psi \quad \text{(three dimension)}$$

$$\psi = \psi(x,y,z) = X(x)\,Y(y)\,Z(z)$$

$$-\frac{\hbar^2}{2m}\frac{d^2X}{dx^2} = E_x X \quad ; \; k_x = \frac{\sqrt{2mE_x}}{\hbar}$$

$$X(x) = A_x \sin(k_x x) + B_x \cos(k_x x)$$

similarly $Y(y) = \ldots$

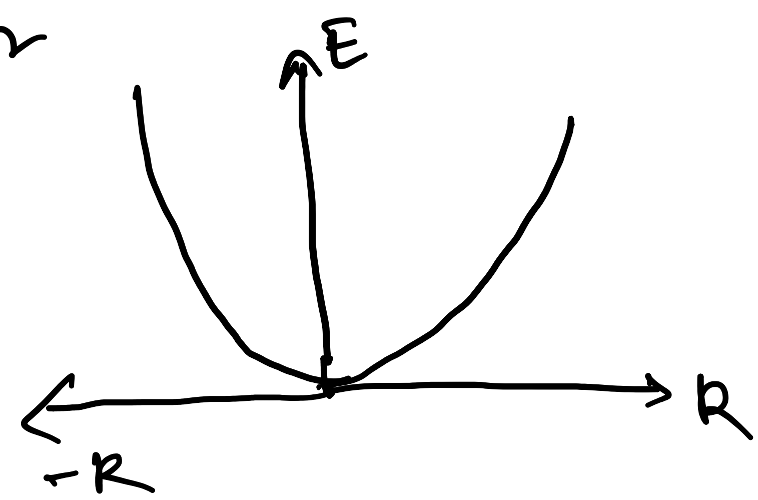
$Z(z) = \ldots$

$$E = \frac{\hbar^2}{2m}(k_x^2 + k_y^2 + k_z^2)$$

$$= \frac{\hbar^2}{2m}k^2$$

Since $L_x, L_y, L_z = \infty$; $k_x, k_y, k_z \to$ Continuous.

(b) $E = \hbar^2/2m \; k^2$



(c) $L_x, L_y, L_z \to$ finite

Since, free electrons can not leave the metal block

$$X(x=0) = 0 \;;\; X(x=L_x) = 0$$

$$Y(y=0) = 0 \;;\; Y(y=L_y) = 0$$

$\ldots$

Now, k_x, k_y and k_z will be quantized.

Answers to prob 4, tut 6

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$$\cos(qa) = \cos(ka) + \frac{m\alpha a}{\hbar^2}\frac{\sin(ka)}{ka} \quad \text{—(1)}$$

$$\frac{m\alpha a}{\hbar^2} \ll 1$$

$q=0$ is the zone-center.

Energy of electron is related to k

$$E = \frac{\hbar^2 k^2}{2m}. \quad \text{—(2)}$$

At $q=0$, we get from (1)

$$\cos(ka) + \frac{m\alpha a}{\hbar^2}\frac{\sin(ka)}{ka} = 1$$

We are trying to find out lowest energy at $q=0$. So, from eq. (2) we get,

$$k \to 0$$

$$\sin(ka) = ka - \frac{(ka)^3}{3!} + \dots$$

Since $\frac{m\alpha a}{\hbar^2}$ is $\ll 1$, so first term is significant in the following equation.

$$\cos(ka) + \frac{m\alpha a}{\hbar^2}\frac{\cancel{ka}}{\cancel{ka}} = 1$$

$$\Rightarrow \cancel{1} - \frac{(ka)^2}{2} + \frac{m\alpha a}{\hbar^2} = \cancel{1} \qquad [ka \to 0$$

$$\Rightarrow \frac{k^2a^2}{2} = \frac{m\alpha a}{\hbar^2}$$

$$\Rightarrow k^2 = \frac{2m\alpha a}{a^2\hbar^2}$$

$$\Rightarrow \frac{\hbar^2 k^2}{2m} = m\alpha/a$$

$$\boxed{E = \frac{m\alpha}{a}} \quad \text{at } q \to 0 \text{ (for lowest energy band)}$$