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Tutorial - 1

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CST - 22

1. What do you understand by Asymptotic notation? Define different asymptotic notation with example.

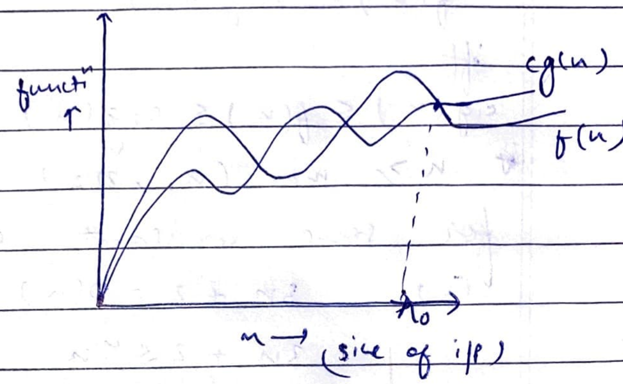
(i) Big O(n)

$f(n) = O(g(n))$

iff

$f(n) \leq g(n)$

$\forall \; n \geq n_0$



for some constant, $c > 0$

$g(n)$ is "tight" upper bound of $f(n)$

Ex → $f(n) = n^2 + n$

$g(n) = n^3$

$n^2 + n \leq c \cdot n^3$

$n^2 + n = O(n^3)$

(ii) Big Omega (Ω)

$f(n) = \Omega(g(n))$

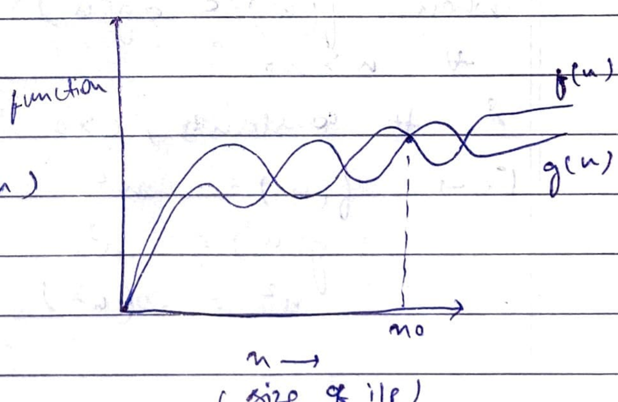
$g(n)$ is "tight" lower bound of function $f(n)$

$f(n) = \Omega(g(n))$

iff

$f(n) \geq c g(n)$

$\forall \; n \geq n_0$



for some constant $c > 0$

Ex - $f(n) = n^3 + 4n^2$

$g(n) = n^2$

$n^3 + 4n^2 = \Omega(n^2)$

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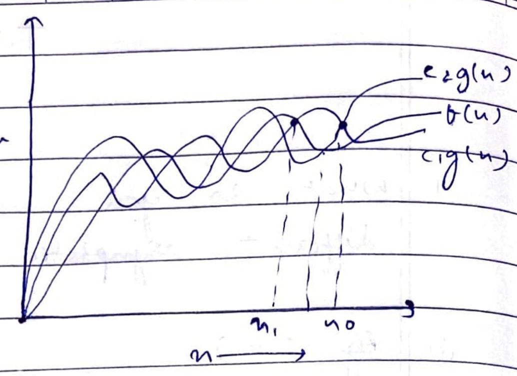
(iii) Big Theta (θ)

$f(n) = \theta(g(n))$

$g(n)$ is both "tight" upper bound & lower bound of function $f(n)$



$f(n) = \theta(g(n))$

iff

$c_1 g(n) \le f(n) \le c_2 g(n)$

$\forall$ $n \ge \max(n_1, n_2)$

for some constant $c_1 > 0$ & $c_2 > 0$

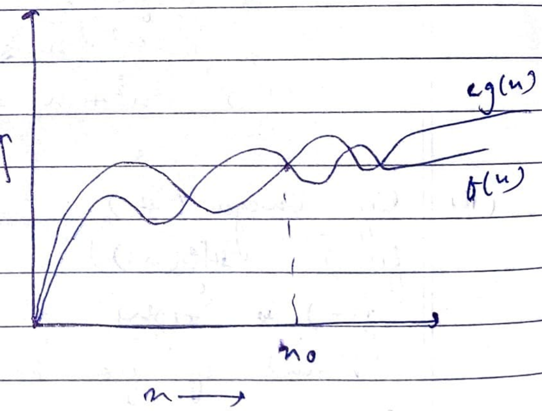
Ex→ $3n + 2 = \theta(n)$ as $3n + 2 \ge 3n$ &

$3n + 2 \le 4n$ for n, $k_1 = 3$, $k_2 = 4$ & $n_0 = 2$

(iv) Small o(o)

$f(n) = o(g(n))$

$g(n)$ is upper bound of funct^n $f(n)$



$f(n) = o(g(n))$

when $f(n) < cg(n)$

$\forall$ $n > n_0$

& $\forall$ constants, $c > 0$

Ex → $f(n) = n^2$

$g(n) = n^3$

$n^2 = o(n^3)$

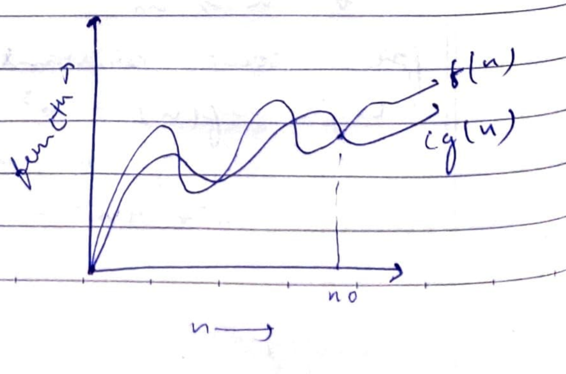
(v) Small omega (ω)

$f(n) = \omega(g(n))$

$g(n)$ is lower bound of functn $f(n)$

$f(n) = \omega(g(n))$ when

$f(n) > c \cdot g(n)$



$\forall \quad n > n_0$

& $\forall$ constants, $c > 0$

2. What should be time complexity of -

for (i=1 to n) { i = i * 2i }

for (i = 1 to n)
{

$i = i * 2i \quad \rightarrow \quad O(1)$

}

$i = \overbrace{1, 2, 4 \ldots, n}^{k}$

$a = 1, \quad r = \frac{b_2}{b_1} = 2$

$t_k = ar^{k-1}$ (k^{th} value of GP)

$t_k = 2^{k-1}$

$t_k = \frac{2^k}{2}$ Let $t_k = n$

$2n = 2^k$

$\log_2(2n) = k \log_2 2$

$\log_2 2 + \log_2 n = k$

$\log_2 n + 1 = k$

$O(\log_2 n + \cancel{1}) = O(\log_2 n)$.

3. $T(n) = \{ 3T(n-1)$ if $n > 0$, otherwise 1 }

$T(n) = 3T(n-1)$ — ①

put $n = n-1$ in ①

$T(n-1) = 3T(n-2)$ — ②

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put ② in ①

$T(n) = 9T(n-2)$ — ③

put $n = n-2$ in ①

$T(n-2) = 3T(n-3)$ — ④

Put ④ in ③

$$T(n) = 27T(n-3) \quad — ④$$

generalising,

$$T(n) = 3^K\, T(n-K) \quad — ⑤$$

Let $n - K = 1$

$$K = n-1 \quad — ⑥$$

Put ⑥ in ⑤

$$T(n) = 3^{n-1}\, T(1)$$

$$T(n) = 3^{n-1} \cdot 1$$

$$T(n) = \frac{3^n}{3}$$

$$\boxed{= O(3^n)}$$

4. $T(n) = \{ 2T(n-1) - 1$ if $n > 0$, otherwise $1 \}$

$$T(n) = 2T(n-1) - 1 \quad — ①$$

Put $n = n-1$

$$T(n-1) = 2T(n-2) - 1 \quad — ②$$

Put ② in ①

$$T(n) = 4T(n-2) - 2 - 1 \quad — ③$$

Put $n = n-2$ in ①

$$T(n-2) = 2T(n-3) - 1 \quad — ④$$

Put ④ in ③

$$T(n) = 8T(n-3) - 4 - 2 - 1$$

generalising,

$$T(n) = 2^K\, T(n-K) - 2^{K-1} - 2^{K-2} \cdots 2^0$$

Let $n - K = 1$

$$K = n-1$$

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$$T(n) = 2^{n-1}\, T(1) - 2^K \left(\frac{1}{2} + \frac{1}{2^2} \cdots \frac{1}{2^K} \right)$$

$$= 2^{n-1} - 2^{n-1} \left(\frac{1}{2} + \frac{1}{2^2} \cdots \frac{1}{2^{n-1}} \right)$$

$a = \frac{1}{2}$, $r = \frac{1}{2}$

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$$= 2^{n-1}\left(1 - \left(\cancel{\tfrac{1}{2}}\left(\frac{1-(1/2)^{n-1}}{\cancel{1-1/2}}\right)\right)\right)$$

$$= 2^{n-1}\left(\cancel{1} - \cancel{1} + (1/2)^{n-1}\right)$$

$$= \frac{\cancel{2^{n-1}}}{\cancel{2^{n-1}}} = \boxed{O(1)}$$

5. What should be time complexity of

```
int i = 1 , s = 1;
while ( s <= n )
{      i++ ;    s = s + i ;
   printf(" # ");
}
```

$i = 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad \dots$

$S = 1 + 3 + 6 + 10 + 15 + \dots$

sum of $S = 1 + 3 + 6 + 10 + \dots + n$ —— ①

Also $S = 1 + 3 + 6 + 10 + \dots T_{n-1} + T_n$ —— ②

$0 = 1 + 2 + 3 + 4 + \dots n - T_n$

$T_k = 1 + 2 + 3 + 4 + \dots + k$

$T_k = \frac{1}{2} k (k+1)$

for k iterations

$1 + 2 + 3 + \dots k <= n$

$\frac{k(k+1)}{2} <= n$

$\frac{k^2 + k}{2} <= n$

$O(k^2) <= n$

$k = O(\sqrt{n}) \Rightarrow \boxed{T(n) = O(\sqrt{n})}$

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6\. Time complexity of

```
void fn(int n)
{
    int i, count = 0;
    for (i=1; i*i <=n; ++i)
}
```

As $i^2 <= n$

$i <= \sqrt{n}$

$i = 1, 2, 3, 4, \ldots \sqrt{n}$

$\sum_{i=1} 1 + 2 + 3 + 4 + \cdots + \sqrt{n}$

$T(n) = \frac{\sqrt{n} \times (\sqrt{n} + 1)}{2}$

$T(n) = \frac{n \times \sqrt{n}}{2}$

$T(n) = O(n)$

7\. Time complexity of

```
void fn(int n)
{
    int i, j, k , count = 0;
    for (i=n/2; i<=n; ++i)
        for (i=1; j<=n; j=j*2)
            for(K=1; K<=n; K=K*2)
                count++;
}
```

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for $K = K^2$

$K = 1, 2, 4, 8 \ldots, n$

GP. ⇒ $a = 1$, $r = 2$

$= \frac{a(r^n - 1)}{r - 1}$

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$= \frac{1(2^{k} - 1)}{1}$

$n \Rightarrow 2^{k} - 1$

$\log(n) \neq k$

i	j	k
1	log(n)	log n * log(n)
2	log(n)	log n * log n
:	:	:
n	log(n)	log(n) * log(n)

$\Rightarrow O(n * \log n * \log n)$

$\Rightarrow O(n \log^2 n)$

8. Time complexity of

```
function (int n )
{       if (n==1 ) return;
        for (i=1  to n )
            for ( j=1  to  n)
                printf ("*");
            }
        }
        function (n-3 );
}
```

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for :- for (i=1 to n)
we get j = n times every turn

$\because i \times j = n^2$

Now, $T(n) = n^2 + T(n-3);$

$T(n-3) = (n^2 3)^2 + T(n-6);$ } k Time

$T(n-6) = (n3\ 6)^2 + T(n-5);$

$T(1) = 1;$

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Now subs each value in T(n)

$T(n) = n^2 + (n-3)^2 + (n-6)^2 + \dots + 1$

Let

$n - 3k = 1$

$k = (n-1)/3$

Total terms = $k + 1$

$T(n) = n^2 + (n-3)^2 + (n-6)^2 + \dots + 1$

$T(n) \simeq k n^2$

$T(n) \simeq \frac{(n-1)}{3} \times n^2$

$\therefore T(n) = O(n^3)$

9. Time complexity of -

```
void function (int n){
    for (i = 1 to n) {
        for (j = 1 ; j <= n ; j = j + i)
            printf ("*");
    }
}
```

for : $i = 1$ $j = 1 + 2 + \dots$ $(n \geq j + i)$

$i = 2$ $j = 1 + 3 + 5 + \dots$ ''

$i = 3$ $j = 1 + 4 + 7 \dots$ ''

⋮

m^{th} terms of AP is

$T(M) = a + d \times m$

$T(m) = 1 + d \times m$

$(n-1)/d = m$

for $i = 1$ $(n-1)/1$ times

$i = 2$ $(n-1)/2$ times

⋮

$i = n-1$ 1

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we get,

$$T(n) = i_1 j_1 + i_2 j_2 \dots i_{n-1} j_{n-1}$$

$$= \left(\frac{n-1}{1}\right) + \left(\frac{n-2}{2}\right) + \left(\frac{n-3}{3}\right) + \dots 1$$

$$= n + \frac{n}{2} + \frac{n}{3} + \dots \frac{n}{n-1} - n \times 1$$

$$= n\left[1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{n-1}\right] - n + 1$$

$$= n \times \log n - n + 1$$

Since $\int \frac{1}{x} = \log x$

$$T(n) = O(n \log n)$$

10. For the funct^ns n^k & c^n, what is the asymptotic relationship b/w these functions? Assume that $k >= 1$ & $c > 1$ are constants. Find out the value of c & n_0 for which relationship holds.

As given n^k & c^n

relationship b/w n^k & c^n is

$$n^k = O(c^n)$$

$$n^k \leq a(c^n)$$

$\forall$ $n \geq n_0$ &

constant, $a > 0$

for $n_0 = 1$

$c = 2$

$\Rightarrow 1^k \leq a^{2 \cdot 1}$

$\therefore n_0 = 1$ & $c = 2$.

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