CH-3

* Electric Charge: When we rub two bodies of different material against each other, electric charge gets develop on them.
  Nature of charge depends on
  i) Atmospheric condition
  ii) The nature of bodies/material

* Type of Charge:
  i) Positive
  ii) Negative

* 1 Coloumb: The total charge on 6.25×10^{18} electrons is equal to 1 coloumb
  or
  When a current of 1 ampere is flowing ~~sec~~ for one second, the electric charge carried through the conductor is 1 Coloumb.

* Static electricity: It is defined as the force which makes the electrons to move.

* Coloumb's ~~law~~:
  1st law: It states that like charges repel ~~and~~ each other and unlike charges attract each other.

2nd Law: $q_1 \longleftarrow r \longrightarrow q_2$

The force between two charges q_1 & q_2 is directly proportional to product of q_1 & q_2 and inversely proportional to the square of the distance (r) between them.

$$\Rightarrow F \propto \frac{q_1 q_2}{r^2}$$

$$F = \frac{K q_1 q_2}{r^2}$$

$$F = \frac{q_1 \times q_2}{4\pi\varepsilon_0\varepsilon_r r^2}$$

$$\varepsilon_0 = 8.85 \times 10^{-12}$$

eg: Two equal charges are located 100 cm apart from each other in vaccum. The force is 0.05 N. Determine the value of two charges. What would be the force if these charges are placed in a liquid of $\varepsilon_r = 10$.

$$\frac{5}{100} = \frac{q^2 \times 9 \times 10^9}{1 \times 1} \times 1000$$

$$q_1 = q_2 = 2.36\,\mu C$$

$$\Rightarrow F = \frac{2.36 \times 2.36 \times 9 \times 10^{9} \times 10^{2} \times 10^{-12}}{10}$$

* **Electric Field:** It is defined as region around charge body where another charged body experiences an ~~mechanical~~ electrostatic force. of attraction or repulsion

* **Electric Flux lines or electric lines of force:**
It is defined as line along which free positive charge would travel if it is placed in electric field.

* **Properties of electric flux lines:**

i) Electric lines of force originate on positive charge and terminate on negative charge.

ii) They never touch or cross each other.

iii) They always enter or leave the conducting surface at right angle to it.

iv) The electric line of force donot form close loop like magnetic lines of force.

Electric Flux:
It is defined as no of lines of force in particular electric field. It is given by
$$\Psi = Q \text{ Coloumb}$$

* Electric Flux density (D):-

It is defined as flux per unit Area

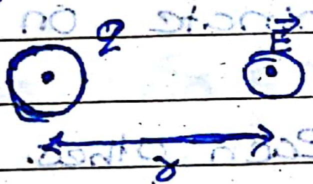
$$D = \frac{\Psi}{A} = \frac{Q}{A}$$

It is also known as displacement density.

* Electric field intensity / strength (E)

It is defined as force experienced by unit positive charge when it is placed in electric field.

* Expression for field intensity:



$$F = E \times q_1$$

$$F = \frac{q_1 q}{4\pi\varepsilon_0\varepsilon_r r^2}$$

$$\Rightarrow \boxed{E = \frac{q}{4\pi\varepsilon_0\varepsilon_r r^2}}$$

Electric field intensity due to multiple Charges:

$$\vec{E} = \vec{E_1} + \vec{E_2} + \cdots + \vec{E_n}$$

The intensity at a point due to multiple charges is vector addition of individual intensity produce by the Charge at that point.

Relation between D & E :

$$D = \frac{\varepsilon}{A} \quad \& \quad E = \frac{\varepsilon}{4\pi\varepsilon_0\varepsilon_r r^2}$$

$$\Rightarrow D \times A = E \times 4\pi\varepsilon_0\varepsilon_r r^2$$

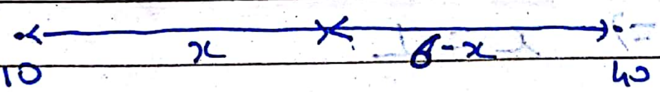
Also,

$$A = 4\pi r^2$$

$$\Rightarrow D \times 4\pi r^2 = E \times 4\pi\varepsilon_0\varepsilon_r r^2$$

$$\Rightarrow \boxed{D = E \cdot \varepsilon}$$

Q: There are two charges of value 10 μC and 40 μC. They are placed at a distance of 6 cm apart in vaccum. What is the location of a point in between them, where the electric intensity is zero.



$$\frac{K \times 10}{x^2} = \frac{K \times 40}{(6-x)^2} \Rightarrow 6 - x = 2x$$

$$x = 2$$

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Q- Calculate the intensity of $\vec{E}$ at a point of 50 cm from a charge of 4.8×10^{-6} C in the medium of dielectric constant = 3.6

$$E = \frac{9 \times 10^9 \times 4.8 \times 10^{-6} \times 4}{3.6 \times}$$

$$= 48 \times 10^3 \text{ N/C}$$

Q- 2 point charges of 15 µC and 60 µC are placed 3 m apart, what is the position of the point in between them where intensity due to both the charges be equal.

$F = \frac{1}{4\pi\varepsilon_0} \times \frac{q^2}{r^2}$

$\frac{N \times m^2}{C^2} = \frac{1}{\varepsilon}$

$$\frac{\cancel{15 \times 10^{-6} \times K}}{x^2} = \frac{\cancel{60 \times 10^{-6} \times K}}{(3-x)^2}$$

$$\Rightarrow x = 1$$

Q- Permittivity : It is the property of medium that allows flux to be established in the material.

Absolute Permittivity (ε): It is defined as ratio of electric flux density (D) to electric field intensity (E)

$$\varepsilon = \frac{D}{E} = \frac{C^2}{Nm^2} = \frac{F}{m}$$

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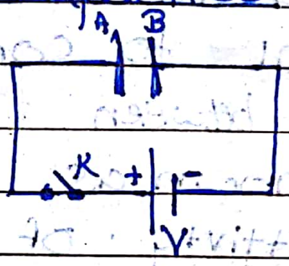
Permittivity of free space (ε_0) :
It is defined as ratio of electric flux density E in free space to corresponding electric field intensity.
Unit is F/m.

Relative permittivity (ε_r)
It is ratio of flux density in a medium to flux density in free space.

$$\varepsilon_r = \frac{\varepsilon}{\varepsilon_0} = \frac{D/E}{D_0/E} = \frac{D}{D_0}$$

Working of Capacitor



$Q \propto V \Rightarrow Q = CV$
$\Rightarrow C = Q/V$

Capacitor having plates A & B is connected across supply voltage V through switch K. When switch K is open, the plates are having no charge. When the switch K is closed, e^- from plate A are attracted by +ve terminal of battery and reach to plate B through battery. As a result plate A becomes more and more positive and plate B becomes more and more -ve.

The flow of e^- from plate A to plate B continues, till the capacitor is charged to supply voltage V. This process is known as charging of capacitor C.

→ Once the Capacitor is charged to Supply voltage V, flow of current stops.

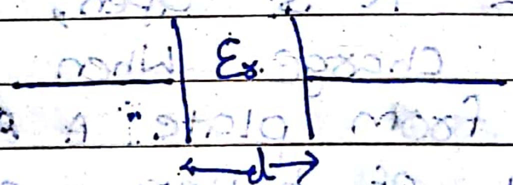
Capacitance :

The property of Capacitor to store the charge is known as ~~Capacitor~~ Capacitance.

Dielectric constant

Capacitor consist of two conducting surfaces seperated by insulating material known as dielectric. The ability of dielectric material to concentrate electric line of force between the plates of capacitor is known as dielectric constant or relative permittivity of the material.

Capacitance of parallel plate Capacitor



Flux density,

$D = \frac{Q}{A}$ $\qquad$ $Q = CV$

Intensity,

$E = \frac{V}{d}$

$$D = \varepsilon_0 \varepsilon_r E$$

$$\Rightarrow \frac{Q}{A} = \varepsilon_0 \varepsilon_r E$$

$$\Rightarrow \frac{Q}{V} = \frac{\varepsilon_0 \varepsilon_r A}{d}$$

$$\Rightarrow \boxed{C = \frac{\varepsilon_0 \varepsilon_r A}{d}}$$

Factors affecting the value of capacitance:

i) ~~Area~~ Cross-sectional area of plates

$C \propto A$

→ Larger the area of plate, greater is the capacitance. This is because larger plates can hold greater value of charge.

ii) Thickness of dielectric (d)

$$C \propto \frac{1}{d}$$

Smaller the thickness, greater is the capacitance. This is because the electrostatic field is intensified when the plates are brought closer to each other, which increases the capacitance.

iii) Relative permittivity (ε_r)

Higher the value of permittivity (ε_r), greater is the capacitance.

$$\varepsilon_r \propto C$$

This is because higher value of dielectric constant allows more electric lines of force

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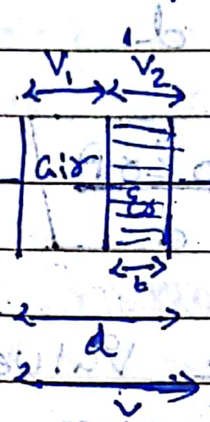
to establish b/w the plates, which increases the Capacitance.

* Capacitance of parallel plate Capacitor (Medium Partly Area)



Now,

$$v = v_1 + v_2$$

$$v_1 = E_1(d - t)$$

$$v_2 = E_2(t)$$

$$\Rightarrow v = E_1(d - t) + E_2(t)$$

$$D = \varepsilon_0 \varepsilon_r E = \frac{Q}{A}$$

$$\Rightarrow E_1 = \frac{Q}{A\varepsilon_0} \qquad E_2 = \frac{Q}{A\varepsilon_0\varepsilon_r}$$

$$\Rightarrow \frac{v}{Q} = \frac{d - t}{\varepsilon_0 A} + \frac{t}{\varepsilon_0 A \varepsilon_r}$$

$$\Rightarrow \frac{Q}{V} = \frac{\varepsilon_0 A}{\left(d - t + \frac{t}{\varepsilon_r}\right)}$$

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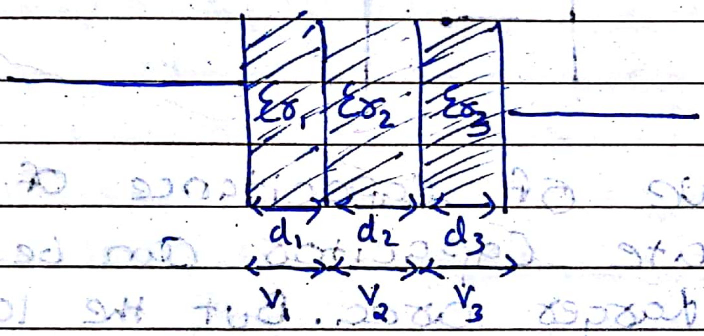
$$\Rightarrow \boxed{C = \frac{\varepsilon_0 A}{\left(d - t + \frac{t}{\varepsilon_r}\right)}}$$

Note:- From this eqn it can be seen that distance between the plate decreases which will increase the value of capacitance. To bring the capacitance back to its orignal value, the capacitor plates will have to be further seperated by that much distance in air.

* Capacitance of parallel plate Capacitor (With composite medium)



$$V = V_1 + V_2 + V_3$$

$$V_1 = E_1 d_1$$

$$V_2 = E_2 d_2$$

$$V_3 = E_3 d_3$$

$$V = E_1 d_1 + E_2 d_2 + E_3 d_3$$

Now,

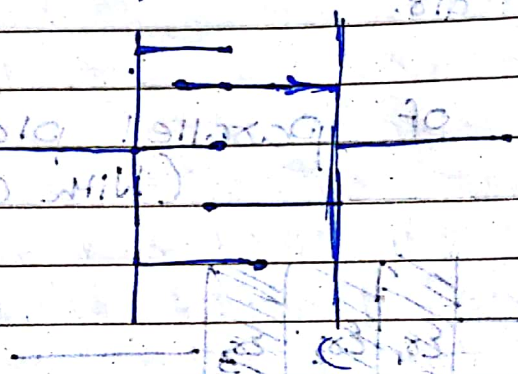
$$D = \frac{Q}{A} = \varepsilon_0 \varepsilon_r E \Rightarrow E_1 = \frac{Q}{A \varepsilon_0 \varepsilon_{r_1}}$$

When n plates are meet, Connected in parallel

$$V = \frac{Q}{A\varepsilon_0}\left(\frac{d_1}{\varepsilon_{r_1}} + \frac{d_2}{\varepsilon_{r_2}} + \frac{d_3}{\varepsilon_{r_3}}\right)$$

$$\Rightarrow \quad C = \frac{A\varepsilon_0}{\left(\frac{d_1}{\varepsilon_{r_1}} + \frac{d_2}{\varepsilon_{r_2}} + \frac{d_3}{\varepsilon_{r_3}}\right)}$$

* Multiplate Capacitor

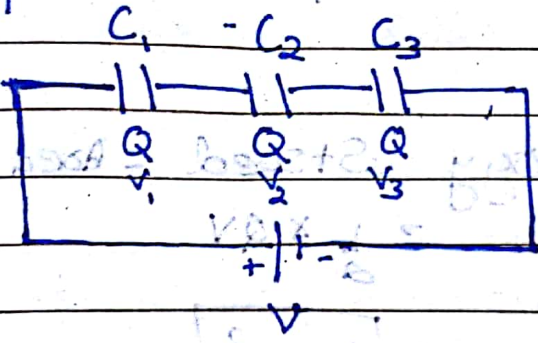


Large value of capacitance of a parallel plate capacitor can be achieved by using larger area. But the larger area increases the size of capacitance. So to obtain higher value of capacitance, without increasing the size of capacitor, multiplate capacitor is used.

When n plates are connected in parallel, it is equivalent to n-1 capacitors in parallel. So the total capacitance is given by

$$\boxed{C = \frac{(n-1)\, A\,\varepsilon_0\, \varepsilon_r}{d}}$$

* Capacitor in Series



$$V = V_1 + V_2 + V_3$$

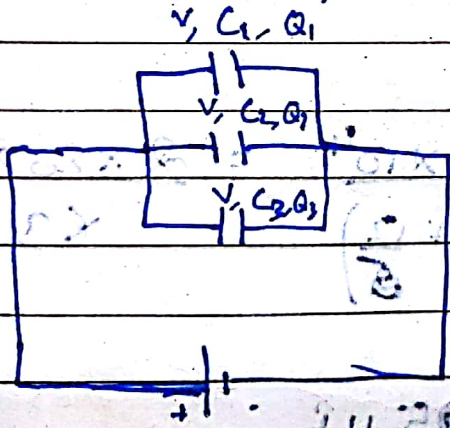
Also,

$$Q = C_1V_1 = C_2V_2 = C_3V_3$$

$$\Rightarrow \frac{Q}{C} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$

$$\Rightarrow \boxed{\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$$

* Capacitor in parallel:



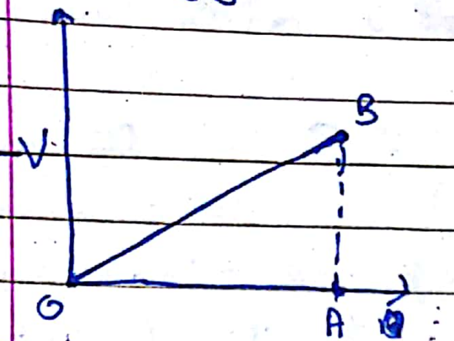
$$Q = Q_1 + Q_2 + Q_3$$

Also,

$$Q_1 = C_1V, \quad Q_2 = C_2V, \quad Q_3 = C_3V$$

$$\Rightarrow \boxed{C_{eq} = C_1 + C_2 + C_3}$$

* Energy Stored in a Capacitor



Energy Stored = Area of ΔOAB

$= \frac{1}{2} \times QV$

$= \boxed{\frac{1}{2} CV^2}$

1. Parallel plate capacitor has plates of area $2m^2$, seperated by 3 slabs of different dielectric. The relative permittivites are 2, 3 and 6 and the thickness are 0.4mm, 0.6mm and 1.2 mm resp. Calculate the total capacitance and electric stress in each material, when the applied voltage is 1000V.

Ans

$$C = \frac{2 \times \varepsilon_0 \times 10^4}{\left(\frac{4}{2} + \frac{6}{3} + \frac{12}{6}\right)} = \frac{2 \times \varepsilon_0 \times 10^4}{63}$$

$= 29.05 \mu f$

$E_1 = \frac{V_1}{d_1}$

$Q = 2.95 \times 10^{-9}$

$$E_1 = \frac{D}{\varepsilon_0 \varepsilon_{r_1}} = \frac{Q}{A\varepsilon_0 \varepsilon_{r_1}} = \frac{2.95 \times 10^{-8}}{}$$

$$= \frac{\varepsilon_0 \times 10^4 \times 10^3}{3 \times \varepsilon_0 \times 2} = 832.956 \text{ K V/m}$$

2. Capacitor consist of 2 parallel square plates, each of = 120 mm side, seperated by 1 mm in the air. When a voltage of 1000 V is applied between the plates an average current of 12 mA flows for 5 second. Calculate:

i) The charge on the capacitor
ii) Electric Flux
iii) " " " density
iv) " Field intensity

i) $C = \frac{\varepsilon_0 A}{d} = \varepsilon_0 \times 120$

$\Rightarrow Q = \varepsilon_0 \times 120 \times 10^3$

$\phi = \frac{Q}{\varepsilon_0}$

ii) $\phi = \frac{12 \times 10^{4}}{}$ $\frac{Q}{\varepsilon_0}$

iii) $= \frac{12 \times 10^4}{12} \times 10^2 = 10^6$

iv)

~~Q- Capacitor consist of two parallel Square plates, each of 120 mm side, Seperated~~

Q- Capacitor is constructed from two Square metal plates of side 120 mm. The plates are seperated by dielectric of thickness 2 mm and relative permitivity = 5. Calculate the capacitance. If the electric field strength in the dielectric is 12.5 KV/mm Calculate the total charge on each plate.

$$C = \frac{\varepsilon_0 \times 5 \times \overset{60}{\cancel{120}} \times 120 \times 10^{-3}}{\cancel{2}}$$

$$= 36\,\varepsilon_0 = 36 \times 8.85 \times 10^{-12}$$

$$C = 3.18 \times 10^{-10}$$

$$\vec{E} = 12.5 \text{ KV/mm}$$

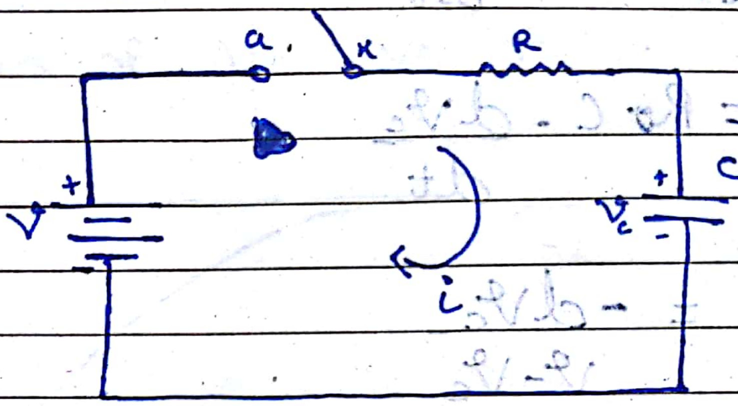
$$\cancel{12.5 = \frac{9 \times 10^9 \times q}{5 \times 2 \times 2}}$$

$$\cancel{q = 27.7 \times 10^{?}}$$

$$V = Ed \quad \text{and} \quad Q = CV$$

$$\frac{Q}{C} = E \cdot d$$

Charging of Capacitor



V_c = Voltage across capacitor
i = Charging current

At $t = 0$
$V_c = 0$

$$i = \frac{V - V_c}{R} = \frac{V - 0}{R}$$

$$i = \frac{V}{R} \text{ (Max Im)}$$

As, $t \uparrow$, $V_c \uparrow$ and $i \downarrow$

Using KCL,

$$V - iR - V_c = 0$$

$$\Rightarrow V - V_c = iR$$

$$\Rightarrow \frac{V - V_c}{R} = i$$

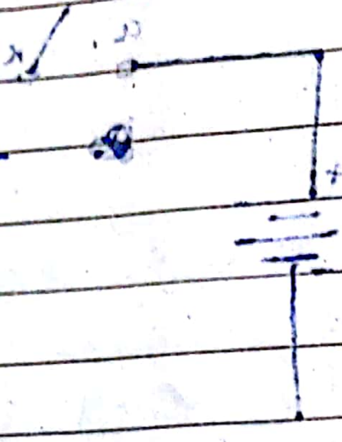
Here i is continously changing

$$\Rightarrow i = \frac{dq}{dt}$$

And

$$q = C \cdot V_c$$

$$i = C \cdot \frac{dv_c}{dt} = \frac{d(C \cdot v_c)}{dt} = \frac{dq}{dt}$$



$$\Rightarrow v - v_c = R \cdot C \cdot \frac{dv_c}{dt}$$

$$\Rightarrow \frac{-dt}{RC} = \frac{-dv_c}{v - v_c}$$

On integrating both the sides, we get

$$\log_e(v - v_c) = \frac{-t}{RC} + K$$

where K = integration constant.

At $t = 0$

$v_c = 0$

$$\Rightarrow K = \log_e v$$

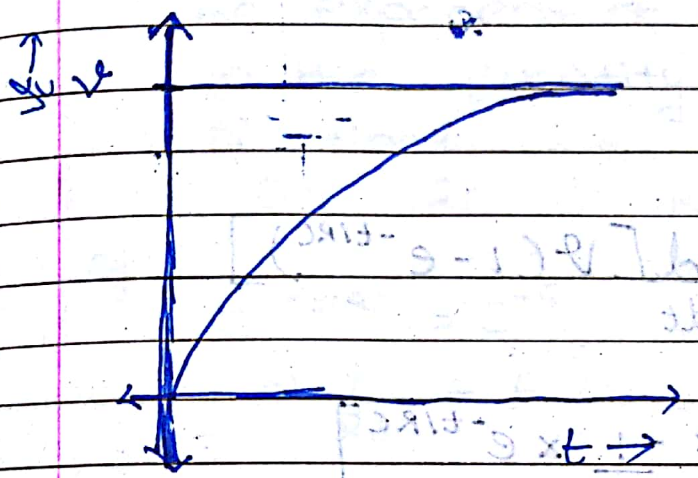
$$\Rightarrow \log_e(v - v_c) = \frac{-t}{RC} + \log_e v$$

$$\Rightarrow \log_e\left(1 - \frac{v_c}{v}\right) = \frac{-t}{RC}$$

$$\Rightarrow 1 - \frac{v_c}{v} = e^{-t/RC}$$

$$\Rightarrow \boxed{\vartheta_c = \vartheta\left(1 - e^{-t/RC}\right)}$$

- Graph of ϑ_c vs t



Variation of charge

$$\vartheta_c = \vartheta\left[1 - e^{-t/RC}\right]$$

$$Q = C\vartheta \Rightarrow \vartheta = \frac{Q}{C}$$

$$\vartheta_c = \frac{q_c}{C}$$

$$\Rightarrow \frac{q_c}{\cancel{C}} = \frac{Q}{\cancel{C}}\left[1 - e^{-t/RC}\right]$$

$$\Rightarrow \boxed{q_c = Q\left[1 - e^{-t/RC}\right]}$$

q_c = Charge on capacitor at any time t

Q = Total charge on Capacitor.

Variation of Current

$$i = \frac{dq}{dt}$$

$$q = C \cdot V_c$$

$$\Rightarrow i = C \cdot \frac{dV_c}{dt}$$

$$\Rightarrow i = C \cdot \frac{d}{dt}\left[V\left(1 - e^{-t/RC}\right)\right]$$

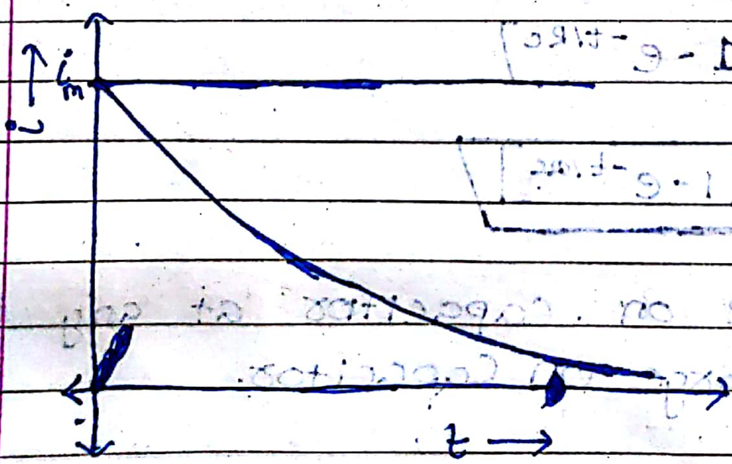
$$\Rightarrow i = C \cdot \left[-V \times \frac{-1}{RC} \times e^{-t/RC}\right]$$

$$\Rightarrow i = \frac{V}{R} \cdot e^{-t/RC}$$

$$\Rightarrow \boxed{i = i_m \cdot e^{-t/RC}}$$

i = current flowing in circuit at any time t

I_m = max current



Time Constant : ($\lambda = RC$)

$$V_c = V\left[1 - e^{-t/RC}\right]$$

In the eq^n, the exponent of e is t/RC

The exponent of e must be a no.

So the quantity RC should have dimension of time,

So RC is known as time constant.

eg:

$$e^{-t/RC} = e^{-5}$$

$$\Rightarrow \frac{t}{RC} = 5$$

$$\Rightarrow t = 5RC$$

Note: Higher the ~~time~~ value of RC, higher the time it will take to charge.

Rate of rise of Voltage across capacitor

$$= \frac{dV_c}{dt} = \frac{V}{RC} e^{-t/RC}$$

At t = 0

$$\boxed{\frac{dV_c}{dt} = \frac{V}{RC}}$$

is the rate of rise of Voltage across capacitor

- Practically V_c can never be equal to V because there is some R present in circuit

$v_c = v\left[1 - e^{-t/RC}\right]$; $i = I_m \times e^{-1}$

At $t = RC = \lambda$

$v_c = v \times 0.632$ $\quad$ $i = I_m \times 0.368$

At $t = 2RC = 2\lambda$

$v_c = v \times 0.864$.

At $t = 3\lambda$

$v_c = v \times 0.95$

At $t = 4\lambda$

$v_c = v \times 0.981$

At $t = 5\lambda$

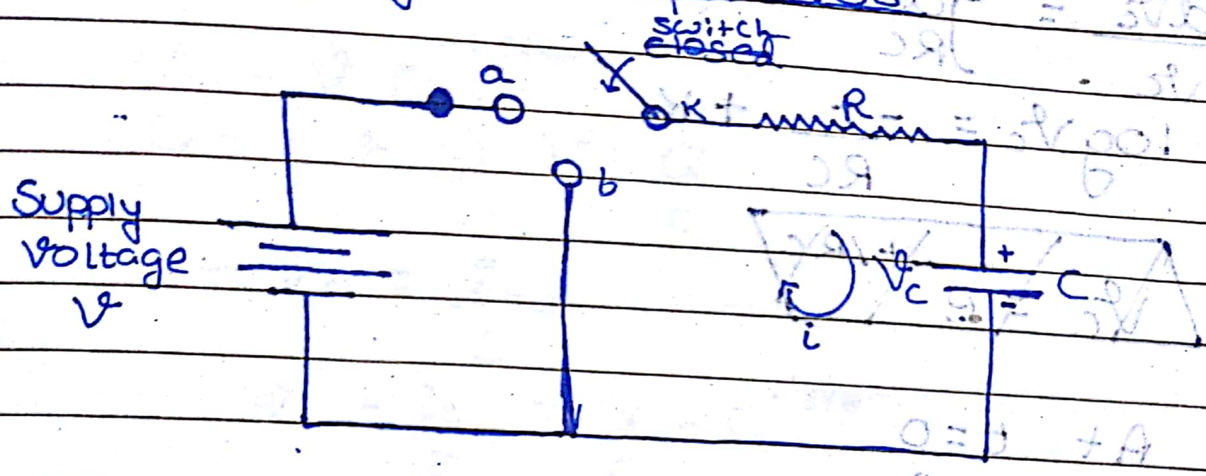
$v_c = v \times 0.993$

As we assume that the v_c most nearer to v can be 99% of v.
therefore time taken by capacitor to charge fully is 5λ.

Time constant is defined as time taken voltage across the capacitor to increase to 63.2% of its final value.

Time constant is defined as time required for the charging current to decrease to 36.8% of its max value.

Discharging of Capacitor



Discharging:

At $t=0$

$v_c = v$

$$-i = \frac{V_c}{R} = \frac{v}{R} = -I_m$$

i] Discharging Voltage v_c:

By KVL:

$$0 - iR - v_c = 0$$

$$\Rightarrow iR + v_c = 0$$

$$\Rightarrow v_c = -iR$$

Here $i = \frac{dq}{dt}$

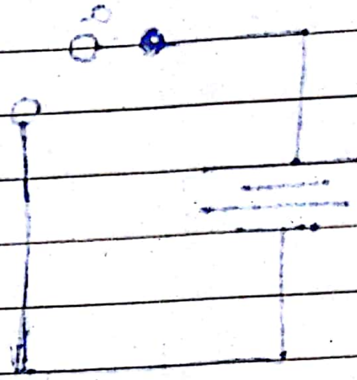
and $q = C \cdot v_c$

$$\Rightarrow i = C \cdot \frac{dv_c}{dt}$$

$$\Rightarrow v_c = -C \cdot R \cdot \frac{dv_c}{dt}$$

$$\Rightarrow \int \frac{dv_c}{v_c} = -\int \frac{dt}{RC}$$

$$\Rightarrow \log v_c = -\frac{t}{RC} + K$$



$$\Rightarrow \boxed{v_c = e^{-t/RC}}$$

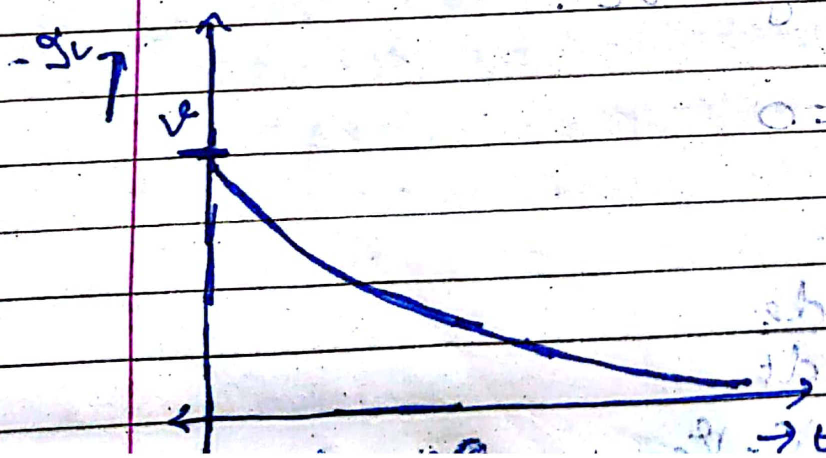
At $t = 0$

$v_c = v$

$\Rightarrow K = \log v$

$$\Rightarrow \log \frac{v_c}{v} = -\frac{t}{RC}$$

$$\Rightarrow \boxed{v_c = v \cdot e^{-t/RC}}$$



Variation of Charge

$$V_c = V \cdot e^{-t/RC}$$

$$Q = CV \Rightarrow V = \frac{Q}{C}$$

$$\Rightarrow V_c = \frac{q_c}{C}$$

$$\Rightarrow -\frac{q_c}{\cancel{C}} = \frac{Q}{\cancel{C}} \cdot e^{-t/RC}$$

q_c = charge on capacitor at any time t

Q = total charge on capacitor.

Variation of Current

$$0 = V_c + iR$$

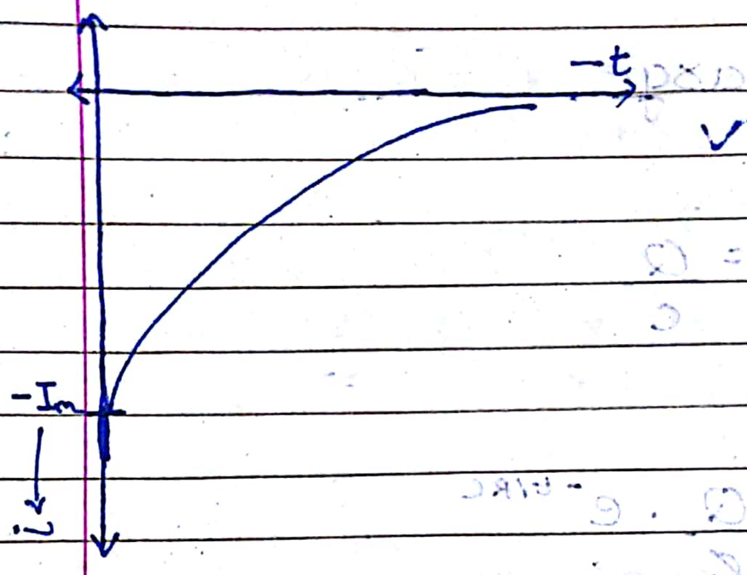
$$iR = -V_c$$

$$\Rightarrow i = -\frac{V_c}{R}$$

$$= -\left[\frac{V \cdot e^{-t/RC}}{R}\right]$$

$$i = \frac{-V}{R} \cdot e^{-t/RC}$$

$$\Rightarrow \boxed{i = -I_m \cdot e^{-t/RC}}$$



iv] Rate of decay of voltage across Capacitor

$$\Rightarrow \frac{dV_c}{dt} = \frac{-V}{RC} \cdot e^{-t/RC}$$

At $t = 0$

$$\boxed{\frac{dV_c}{dt} = \frac{-V}{RC}}$$

Q- Capacitor is charged to a D.C Source through a resistor of $10^6 \Omega$. If in 1 Sec the potential diff across the capacitor reaches to 80% of final value. Calculate the C of capacitor.

At $t = 1$

$$\frac{8}{10} \times V = V\left[1 - e^{-1/RC}\right]$$

$$e^{-1/RC} = \frac{2}{10}$$

$\Rightarrow \frac{\cancel{1}}{10^6 \times C} = \cancel{1}.609$

$\Rightarrow C = 0.62 \times 10^{-6}$ F

Q- An 8×10^{-6} F capacitor is connected in series with $0.5 \times 10^6\ \Omega$ across 200 volt DC supply
Calculate

i] $\lambda = RC =$

ii] Initial charging current

iii] Time taken for the potential difference across the capacitor to increase to 160 volt.

iv] The current and the potential difference across the capacitor in 4 seconds after it is connected to supply

i] $\lambda = RC = 4$ sec

ii] $i = i_m \cdot e^{-t/RC}$

$\Rightarrow i = i_m = \frac{V}{R} = \frac{200}{5 \times 10^5}$

$i = 4 \times 10^{-4}$

iii] $16\cancel{0} = 20\cancel{0}\left[1 - e^{-t/RC}\right]$

$4 = 20 \cdot e^{-t/RC}$

$\Rightarrow \cancel{1}.609 = \frac{\cancel{t}}{4}$ $\Rightarrow$

$t = 6.437$

4. $v_c = v\left[1 - e^{-4/4}\right]$

$\Rightarrow v_c = 126.42$ V

$i = I_m \cdot e^{-t/RC}$

$= 4 \times 10^{-4} \times 0.367$

$= 1.47 \times 10^{-4}$ A

Q- 10 µF Capacitor in series with 1 MΩ resistor is connected across 100V supply determine

i) Time const of circuit

ii) Initial value of charging current

iii) Initial rate of rise of voltage across capacitor

iv) Voltage across capacitor after time $t = \lambda$.

v) The circuit current at this time.

vi) The voltage across capacitor 3 sec after closing the switch

vii) Time taken for the capacitor voltage to reach 60 volt.

i) $\lambda = 10$

ii) $i_0 = I_m = \dfrac{100}{10^6} = 10^{-4}$ A

iii) $\dfrac{dv_c}{dt} = 10$

iv) $v_c = v\left[1 - e^{-1}\right] \Rightarrow v_c = 6.32\text{ V} \times 10$

v) $i = I_m \cdot e^{-1} = 3.67 \times 10^{-6}$ A

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