

# 11: ELECTROMAGNETISM

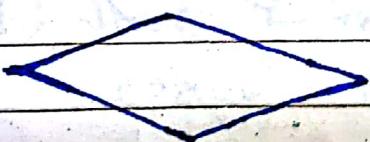
Magnet: The material having property of attracting iron pieces known as magnet.

Types of magnet:

1. Permanent Magnet: It retain their magnetism over a long period of time
2. Temporary Magnet: It retain their magnetism under certain conditions like in the presence of strong magnet or passing electric current around the magnetic material.

Types of magnets on basis of their Shape

1.



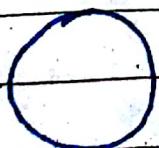
Magnetic Needle

2.



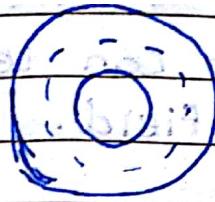
Bar Magnet

3.



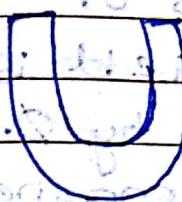
Disc Magnet

4.



**Ring Magnet** (or) **Donut Magnet**

5. **Bar Magnet** : It has two poles (North & South) at its ends.



**Shoe Magnet** : It has two poles (North & South) at its ends.

6. **Neodymium Magnet** : It has two poles (North & South) at its ends.

\* **Properties of Magnet** :

- For every magnet there are two opposite poles (North and South). So the magnet is also known as dipole (dipole means two).

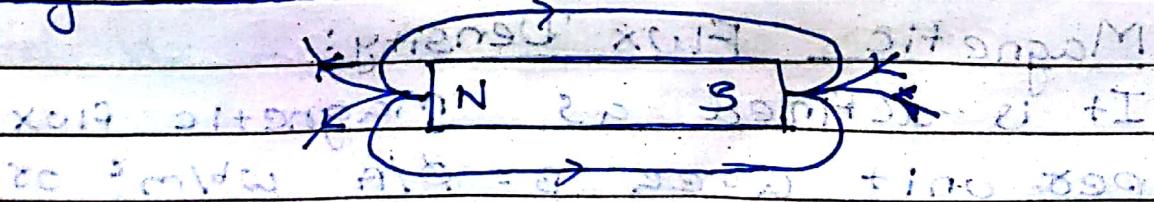
The pole of magnet is always aligned in North

South direction and it is called dipole.

- The lines of force move from North to South outside the magnet and from South to North inside the magnet.

- Like poles repel each other and unlike poles attract each other.

**Magnetic Field:**



- The area around a magnet where the effect of magnetic force produced by magnet can be detected is known as magnetic field.
- \* Magnetic Flux: The total number of magnetic lines of force in magnetic field is known as magnetic flux. It is denoted by  $\Phi$ . Unit of  $\Phi$  is  $\text{weber}$  ( $1 \text{ weber} = 10^8 \text{ magnetic lines}$ ).

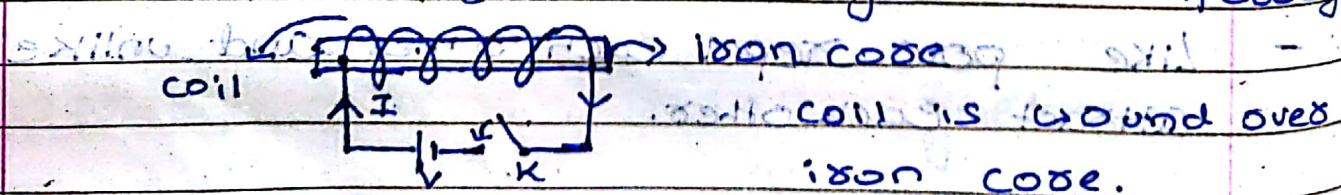
### Characteristics of Magnetic Flux lines:

- They have no physical existence.
- They form close path

~~They never touch/intersect each other~~  
~~lines of magnetic flux form closed loop~~  
~~each other and having same direction repel each other~~

~~lines of magnetic flux closed to each other and having Opp direction attract each other.~~

- \* Electromagnet: An electromagnet is made from a iron core. A wire coil is wound when current is passed through a coil the core is magnetized temporarily.



### Magnetic Flux Density:

It is defined as magnetic flux per unit area  $B = \Phi/A$   $\text{wb/m}^2$  or Tesla (T)

## \* Magneto Motive Force (mmf)

- It is defined as force which establishes flux through circuit.

$$\text{mmf} = N \times I \leftarrow \text{Current}$$

~~Number of turns per unit length~~

- Magneto motive force is analogous to emf in electric circuit.

## \* Magnetic field intensity (H)

- It is defined as mmf per unit length of magnetic flux path and is denoted by H.

$$H = \frac{\text{mmf}}{\text{path length}} = \frac{N \times I}{A} \rightarrow \text{units: A/m}$$

- S.I. unit: A/m

## \* Permeability ( $\mu$ )

- It is the ability of magnetic material to establish magnetic flux throughout.

$$\mu = \frac{\text{Flux}}{\text{Field}}$$

$$\text{where } \mu_0 = 4\pi \times 10^{-7} \text{ H/m}$$

$$H = \frac{B}{\mu_0}$$

## \* Relative permeability ( $\mu_r$ )

- It is defined as the ratio of flux density produced in the material to flux density in vacuum.

## \* Reactance ( $X_L$ )

- It is defined as opposition offered by magnet circuit to the establishment of magnetic flux.

$$X_L = \mu_0 \cdot \mu_r \cdot A \cdot l$$



## \* Permeance

It is define as reciprocal of reluctance.

$$\text{Permeance} = \frac{1}{\text{reluctance}}$$

Permeance is analogous to conductance in electric circuit.

## \* Reluctivity

- Reluctivity or specific resistance is define as reluctance offered by magnetic circuit of  $I=1\text{ mamp}$  &  $A=1\text{ m}^2$ .

- Reluctivity or Specific resistance =  $\frac{l}{\mu \times A}$  if  $I=1\text{ mamp}$  and  $A=1\text{ m}^2$

- Reluctivity is analogous to resistivity in electric circuit.

## Relation b/w $B$ & $H$

-  $B$  is directly proportional to  $H$ .

$$B \propto H \Rightarrow \mu_r B = \mu_0 H$$

## \* Classification of Magnetic Materials

Magnetic material are classified according to their relative permeability.

(i) Ferromagnetic: Their relative permeability is greater than unity and depends on field strength. This materials can be easily magnetised.

Eg:- Iron, cobalt, nickel.

ii) Paramagnetic: Relative permeability is greater than 1 and are slightly magnetised.

Eg: Al, Platinum etc.

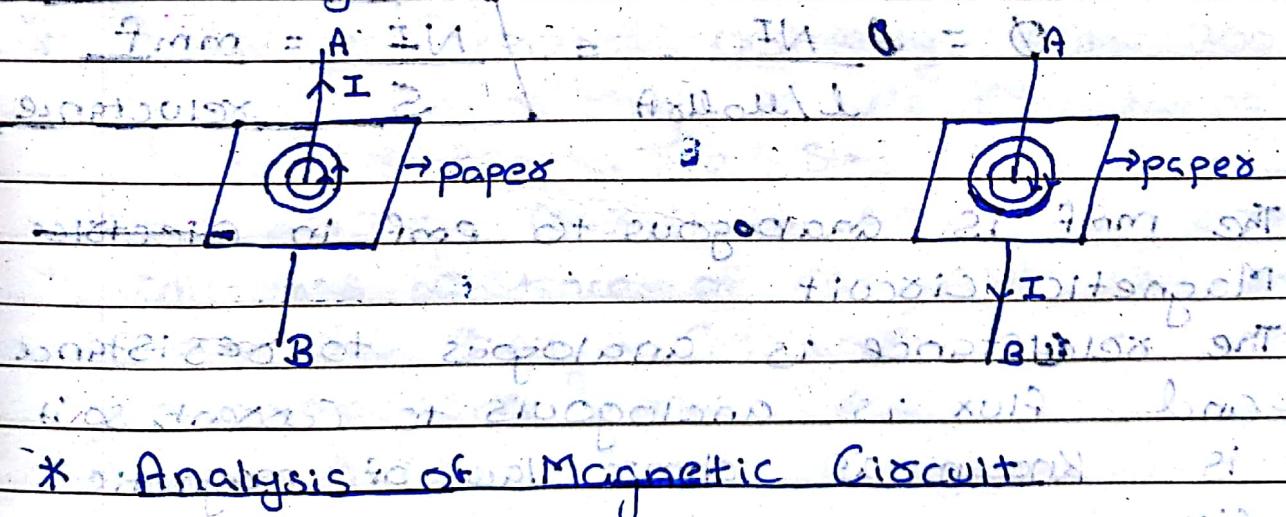
iii) Diamagnetic: Relative permeability is slightly < 1 and have opposite effect of paramagnetic material.

Eg: - Ag, Cu etc.

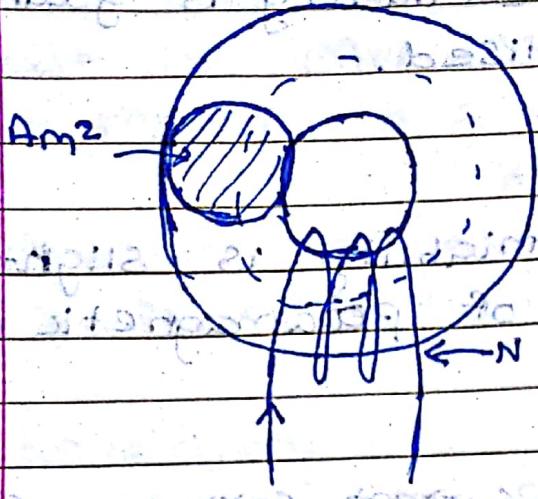
\* Magnetic field due to current carrying conductor

- Direction of magnetic field produced by current carrying conductor can be found using right hand thumb rule.

RHT → Hold the conductor in right hand in such a way that thumb indicates direction of flow of current, and bent fingers indicates direction of magnetic field.



\* A simple magnetic circuit is made up of single magnetic material of iron cob or nickel or aluminium.



$$B = \frac{\Phi}{A}$$

$$\Rightarrow \Phi = B \cdot A \text{ (iii)}$$

$$\Phi = \mu H \cdot A \text{ (iv)}$$

Also,

$$H = \mu_0 M + N I$$

$$\text{Current } I = \frac{V_o}{R} = \frac{\text{emf}}{\text{Resistance}}$$

$$\Phi = \frac{NI}{l/\mu_0 A} = NI \cdot \frac{1}{S} = \frac{NI}{S} \cdot A = \frac{mmf}{S}$$

$S \uparrow \text{reluctance}$

The mmf is analogous to emf in electric circuit.

Magnetic circuit

The reluctance is analogous to resistance and flux is analogous to current, so it is known as ohm's law of magnetic circuit.

Leakage flux: Magnetic flux which does not follow the desired path in magnetic circuit is known as leakage flux. When a current is passed through a coil, magnetic flux  $\Phi$  is set, most of

The flux is set in desire path, but some of the flux is set around the coil. This flux is not utilised for any work. So it is known as leakage flux.

$$\Phi = \Phi_L + \Phi_U$$

leakage flux                          Useful flux

\* Leakage coefficient ( $\lambda$ ) / Hopkinson's leakage coeff.

$$\lambda = \frac{\text{total flux}}{\text{useful flux}} = \frac{\Phi_L + \Phi_U}{\Phi_U}$$

Its value lies between 1.1 - 1.25

Q- A coil is wound uniformly with 300 turns over a scaled ring of  $M_s = 900$  having ring diameter,  $d = 20\text{cm}$ . The steel ring is made up of bars having cross-section of diameter  $2\text{ cm}$ . If the coil has resistance of  $5\Omega$  and is connected to  $250\text{V DC}$  Supply, calculate:

i) The primary moving force

ii) Field intensity in ring

iii) Reluctance of magnetic path

iv) Total flux

v) Permeance in air gap

$$mmf = 300 \times 5 \times 900 \times 8.8 \times 10^{-6}$$

$$= 168 \cdot 0 \text{ mWb} \quad \text{(i)}$$

$$= 168 \cdot 0 \text{ mWb} \quad \text{(ii)}$$

$$ii) H = \frac{NI}{l} = \frac{1500}{2 \times 8} = 2387.5 \text{ A/m}$$

$$iii) S = \frac{l}{2\pi r} = \frac{1}{2\pi \times 8 \times 10^{-3}} = 2 \times 10^{-12} \times 10^{-13} \text{ m}^2$$

$$= 17.684 \times 10^{-5} \text{ AT/Wb}$$

$$iv) \phi = \frac{mmf}{S} = \frac{1500}{17.684 \times 10^{-5}} = 8.48 \times 10^{12} \text{ Wb}$$

$$v) Permeability = \mu = 15.1655 \times 10^{-7} \text{ Vs/AT}$$

## \* Electromagnetic Induction

The process of generation of emf due to production of current by changing the magnetic flux is known as electromagnetic induction.

### \* laws of Electromagnetic induction

i) Faraday's law

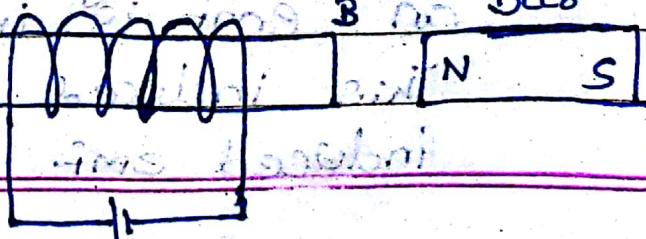
ii) Fleming's right hand rule

iii) Lenz's law

- i) Faraday's law: When a magnetic flux linked with the coil changes, an emf is induced in the coil.
- ii) The Faraday's 2nd law: The emf of induced emf in a coil is proportional to the rate of change of flux.
- $$e = N \frac{d\phi}{dt}$$
- ∴  $e = N \frac{d\phi}{dt}$

- iii) Fleming's right-hand rule: This rule can be applied when the conductor is moving across the field. 1st finger, 2nd finger and thumb of right hand are mutually perpendicular to each other. If 1st finger indicates direction of magnetic field, thumb indicates direction of motion of the conductor; then the 2nd finger will indicate direction of induced emf in the conductor.
- iv) Lenz's law: This law is applied when the flux linked with the coil changes. The induced emf and current flows in such a direction that magnetic field opposes the cause which produces it.

Explanation: According to Lenz's law, if a bar magnet is moved from left to right, then the current flows in such a direction.



When North pole of bar magnet is taken (i) near to the coil, current is flowing in coil in such a way that side b of coil attains North polarity which opposes the movement of bar magnet (ii) When South pole of bar magnet is taken away from the coil, current is flowing in such a way that side b of the coil attains South polarity which again opposes the movement of bar magnet.

### Types of Induced emf:

1. Dynamically Induced emf

2. Statically Induced emf

i) Self induced emf

ii) Mutually induced emf

1. Dynamically Induced emf: The conductor is moving in stationary magnetic field. In such a situation that there is change in flux. This type of emf is known as dynamically induced emf.

e.g.: - i) Generator

2. Statically Induced emf:

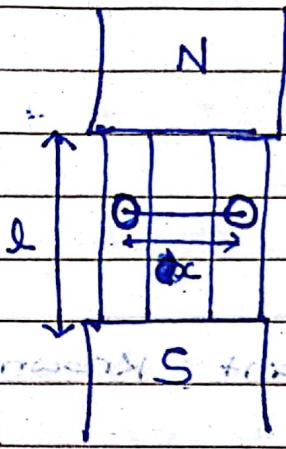
When the conductor is stationary and flux linking with the conductor changes an emf is induced in conductor.

This induced emf is known as statically induced emf.

coil If it can be further classified into:

- i) Self Induced Emf
- ii) Mutual Induced Emf

1.

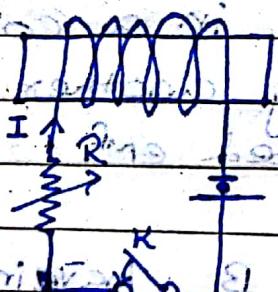


Area covered by conductors  
 $A = l \cdot x$

$$\therefore \phi = B \cdot A$$
$$= B \cdot l \cdot x$$
$$\Rightarrow e = \frac{d\phi}{dt} = B \cdot l \cdot \frac{dx}{dt}$$

$e = Blv$

i) When conductor is stationary and magnetic field changes by changing the value of current.



The emf induced in a coil due to change in flux within the coil is known as

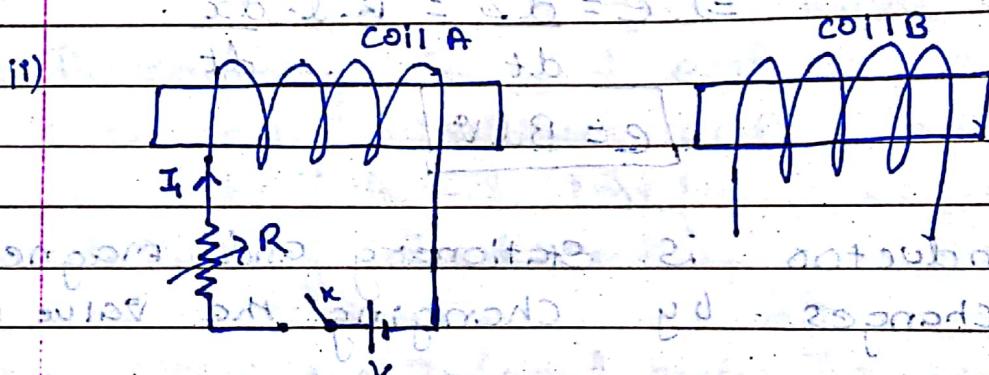
self induced emf.

Explanation: Consider a coil having n turns connected across supply voltage to

Switch: If the current through coil is changed by changing the value of  $R$  then the flux linking with the coil also changes. This change in flux induces emf in coil which is known as Self induced emf.

$$e = L \frac{di}{dt}$$

where  $L$  is constant & known as Self inductance.



The emf induced in a coil due to change of flux produced by another coil is known as mutually induced emf.

Explanation:

Consider coil B having  $N_2$  turns is placed near coil A having  $N_1$  turns. When the switch  $K$  is closed, current  $i$  flows through coil A which produces flux  $\phi_{11}$ . Out of this flux  $\phi_{11}$ , most of the flux stay  $\phi_{12}$  links with coil B. If the current through coil A is changed by changing the value of

variable resistor  $R$  with changes in flux  $\Phi$  linking with coil  $B$ . This change in flux in coil  $B$  induces emf in coil  $B$ . This emf is known as mutually induced emf.

$$e \propto \frac{di}{dt}$$

$$e = M \frac{di}{dt}$$

where  $M = \text{constant}$  is known as mutual inductance.

### \* Comparison of dynamically induced & statically induced emf.

- The magnetic field is stationary and conductor is moving across the field.
- Dynamically      Statically  
The magnetic field is stationary and conductor is stationary  
stationary and conductor and magnetic field changes is moving across the by changing the value of field.
- The direction of induced emf is determined by Fleming's right hand rule.
- The mag of emf is given by  $e = Blvs \sin \theta$
- Generators works on principle of dynamically induced emf.
- Transformers works on principle of statically induced emf.

Self Inductance: The property of a coil due to which it opposes the change of current flowing through the coil is known as self inductance.

$$e = N \frac{d\phi}{dt}$$

$$= N \cdot \phi \times \frac{di}{dt}$$

$$e = N \cdot \phi \times \frac{di}{dt} \quad \text{--- (1)}$$

$$e = L \cdot \frac{di}{dt} \quad \text{--- (2)}$$

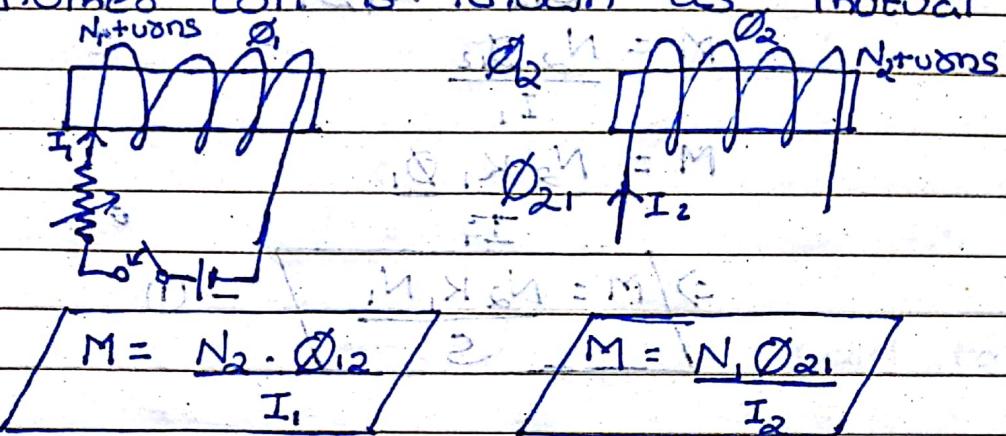
$$\Rightarrow L = \frac{N\phi}{I}$$

$$L = \frac{N^2 \phi}{S} \quad \text{--- (3)}$$

$$B = \mu_0 \frac{N}{l}$$

## Mutual Inductance (M)

- The property of a coil due to which change of current in one coil induces emf in another coil is known as mutual inductance.



For 100% flux link,  $M = N_1 N_2 A \mu$

$$\Phi_{12} = \Phi_1$$

$$\Rightarrow M = \frac{N_2 \cdot \Phi_1}{I_1}$$

Also,  $\Phi_1 = \frac{N_1 I_1}{S}$

$$\Rightarrow M = \frac{N_1 N_2 I_1}{S}$$

$$\Rightarrow M = \frac{N_1 N_2}{S} \cdot I_1$$

### \* Coefficient of Coupling



$$\Phi_1 = N_1 I_1 \quad (M) \text{ such that } K_1 \leq 1$$

S

$$M = N_2 \cdot N_1 \cdot K_1$$

$$\text{similarly } M = N_2 \cdot N_1 \cdot K_1 \text{ so } M \text{ is constant in} \\ \text{rotating system so } \Phi_{21} = K_2 \Phi_{12} \text{ constant in}$$

$$M = \frac{N_2 \Phi_{12}}{I_1}$$

$$M = \frac{N_2 K_1 \Phi_{12}}{I_1}$$

$$\Rightarrow M = \frac{N_2 K_1 N_1}{S} \quad \text{--- (1)}$$

$$\frac{N_2 K_1 N_1}{S} = M$$

$$\Phi_{21} = K_2 \Phi_{22} \quad K_2 \leq 1$$

$$M = \frac{N_1 \Phi_{21}}{I_2}$$

$$= \frac{N_1 K_2 \Phi_{22}}{I_2} \quad S = M \quad \text{--- (2)}$$

$$M = \frac{N_1 N_2 K_2}{S} \quad \text{--- (2)A}$$

Multiplying eqn ① and ②

$$M^2 = \frac{N_1^2 \cdot N_2^2 \cdot K_1 \cdot K_2}{S^2}$$

Assuming  $K_1 = K_2 = K$

$$\Rightarrow M^2 = \frac{(N_1 N_2 K)^2}{S^2} \Rightarrow R = \sqrt{K_1 K_2}$$

$$\Rightarrow M = \frac{N_1 N_2 K}{S} / \text{unit } \times \text{Henry/weber} = \text{Ampere}$$

Also,  $E_{\text{link}} = \Phi_0 \times \cos \theta$

$$M^2 = \frac{N_1^2}{S} \cdot \frac{N_2^2}{S} \cdot K^2$$

$$M^2 = \frac{L_1 \cdot L_2}{S} \cdot K^2$$

$$\Rightarrow K^2 = \frac{M^2}{L_1 \cdot L_2}$$

Note: When  $K=1$  the coils are said to be  
tightly coupled.  $M = K$  (iii)  
And when  $K=0$  the coils are said to be  
magnetically isolated from each other.

Q- Flux of  $10^{-3}$  WB is produced by coil A when it carries current of 4 A. Another coil B produces flux of  $1.5 \times 10^{-3}$  WB having the same current. Coil A has 800 turns and coil B has 1200 turns. The coils are kept such that 80% of the flux produced by coil A links with coil B. Calculate (v)

- Self-Inductance of each coil (vi)
- Mutual Inductance (vii)
- Coefficient of coupling
- % of flux produced by coil B that links with coil A.

$$\text{i) } L_1 = \frac{800 \times 800}{I_1} N_1 \times \emptyset_1$$

$$= \frac{800 \times 10^{-3}}{4} = 2 \times 10^{-1} \text{ H}$$

$$L_2 = \frac{1200 \times 1.5 \times 10^{-3}}{4} = 4.5 \times 10^{-1} \text{ H}$$

$$\text{ii) } M = \frac{1200 \times 8}{10} \times 10^{-3} = 24 \times 10^{-2} \text{ H}$$

$$\text{iii) } K = \frac{M}{\sqrt{L_1 + L_2}} = \frac{24 \times 10^{-2}}{3 \times 10^{-1}} = 8 \times 10^{-1} = 8 \text{ H}$$

~~$$\text{iv) } A \text{ ( ) } K = \sqrt{K_1 K_2}$$~~

~~$$0.8 = \sqrt{K_1 K_2}$$~~

~~$$K_2 = \frac{24 \times 10^{-2} \times 4}{1200 \times 800 \times 200} = 16 \text{ H}$$~~

~~$$\text{iv) } 0.8 = \sqrt{0.8 \times K_2}$$~~

$$\Rightarrow K_2 = 0.8^2 = 0.64 \text{ H}$$

$$\Rightarrow 80 \times 1.6 \text{ H}$$

Q- Coil A and B with 50 and 500 turns resp are wound side by side on a core of cross-section  $50 \text{ cm}^2$  and mean length of 1.2 m. Estimate

- $L$  of each coil.
- $M$  between the coils.
- EMF induced in coil A if the current in coil B increases steadily from 0.5 A to 1.0 A in 0.01 sec. Assume  $\mu_0 = 1000 \text{ N/A}$

$$\text{i) } L_1 = \frac{N_1^2 \mu_0 S}{s} = \frac{50^2 \times 1000 \times 50 \times 10^{-4}}{1.2} = 1.04 \times 10^3 \text{ H}$$

$$S = \frac{1.2}{10^3 \times 8.85 \times 10^{-12} \times 50 \times 10^{-4} \times 4 \pi \times 10^{-7}} = 1.04 \times 10^7 \text{ m}^2$$

$$= 1.04 \times 10^7 = 2.71 \times 10^0 = 1.9 \times 10^5 \text{ H}$$

$$L_1 = 50 \times 50 = 9.22 \times 10^{-8} = 13.15 \times 10^{-3} \text{ H}$$

$$M = 1.9 \times 10^5 \times 0.5 \times 10^{-3} = 1.9 \times 10^2 \text{ H}$$

$$L_1 + L_2 = 9.22 \times 10^{-8} + 13.15 \times 10^{-3} = 13.15 \text{ H}$$

$$\text{ii) } M = \frac{N_1 N_2}{s} = \frac{50000}{1.2} = 9.22 \times 10^{-7} \times 13.15 \times 10^{-3} \text{ H}$$

$$\text{iii) } \frac{di}{dt} = \frac{5}{10^{-2}} = 5 \times 10^2 \Rightarrow E = 65.5 \text{ V}$$

$$\Rightarrow E = 9.22 \times 10^{-7} \times 5 \times 10^2 = 46.1 \times 10^{-5} \text{ V}$$

$$Q- N=250$$

$$I_1 = 2 \text{ A}$$

$$\text{FLUX} = 10.3 \text{ mWb} = 10.3 \times 10^{-4} \text{ Wb}$$

$$\therefore dI_1 = 2 \text{ A} \quad dt = 2 \text{ msec}$$

$$\text{Voltage induced} = 63.75 \text{ V} = Emf$$

$$K = 0.85$$

~~Find (i) self inductance of 1st coil  
(ii) mutual inductance  
(iii) no. of turns in 2nd coil~~

~~$$L_1 = \frac{250 \times 250}{10^3} = 6.25 \times 10^{-3} \text{ H}$$~~

~~$$63.75 = L_2 \times 10^3$$~~

~~$$L_2 = 63.75 \times 10^{-3}$$~~

~~$$63.75 \times 10^{-3} = N_2 \times$$~~

~~$$N_2 = 250 \times 3 \times 10^{-4} = 37.5 \times 10^{-3} \text{ H}$$~~

$$Emf = M \frac{di}{dt} \Rightarrow M = \frac{63.75}{10^3} = 63.75 \times 10^{-3}$$

~~$$M = N_1 N_2 K = 250 \times 250 \times 63.75 \times 10^{-3} = 39.075 \times 10^{-3} = M$$~~

~~$$N_2 K = 63.75 \times 10^{-3} \times R \sqrt{L_1 L_2}$$~~

~~$$63.75 \times 10^{-3} \times 2 = 2 \times b$$~~

$$\Rightarrow \left( 63.75 \times 10^{-3} \right)^2 \times 2 \times 100 \times 2 \times 10^{-3} = 150 \times 10^{-3} \text{ H}$$

$$L_2 = 150 \times 10^{-3} \text{ H}$$

$$150 \times 10^{-3} = \cancel{150} \times 10^{-3} \text{ H}$$

$$M = N_2 \times \Phi_{12}$$

$$\Rightarrow N_2 = \frac{63.75 \times 10^{-3} \times 2 \times 10^{-3}}{0.85 \times 10^3} = 800 \text{ turns}$$

Q- Coil has 480 turns, mean length of 30cm. and crosssection area of  $5 \text{ cm}^2$ . Calculate i) the inductance of coil

ii) Avg EMF induced if current of 4A is reversed in 60ms

SOL:

$$N = 480, l = 30 \times 10^{-2} \text{ m}, A = 5 \times 10^{-4} \text{ m}^2$$

$$\text{i) } L = \frac{N^2 \times \mu_0 \times l}{l} \times A$$

$$V_{\text{EMF}} = 480 \times 480 \times 4 \pi \times 10^{-7} \times 5 \times 10^{-4} \times 3 \times 10^{-1}$$

$$= 482.54 \times 10^{-6}$$

$$\text{ii) } \ell = 482.54 \times 10^{-6} \times \frac{4}{60 \times 10^{-3}} = 63.38 \times 10^{-4}$$

$$f_{\text{SI}} = 37.5$$

Q - 2 identical coils A & B (consisting of 1500 turns each lies in parallel plane such that 70% of the flux produced by current in coil A links with coil B. Current of 4A flowing in coil A produces flux of 0.04 mwb in coil A. calculate

- i). L of each coil
- ii) M
- iii) if the current in coil A changes from 4A to -4A in 0.021 sec what will be the emf induced in coil B.

$$i) L_A = \frac{1500 \times 0.04 \times 10^{-3}}{4} = 1.05 \times 10^{-3}$$

$$\text{Also, } L_A = L_B = 15 \text{ mH} = L$$

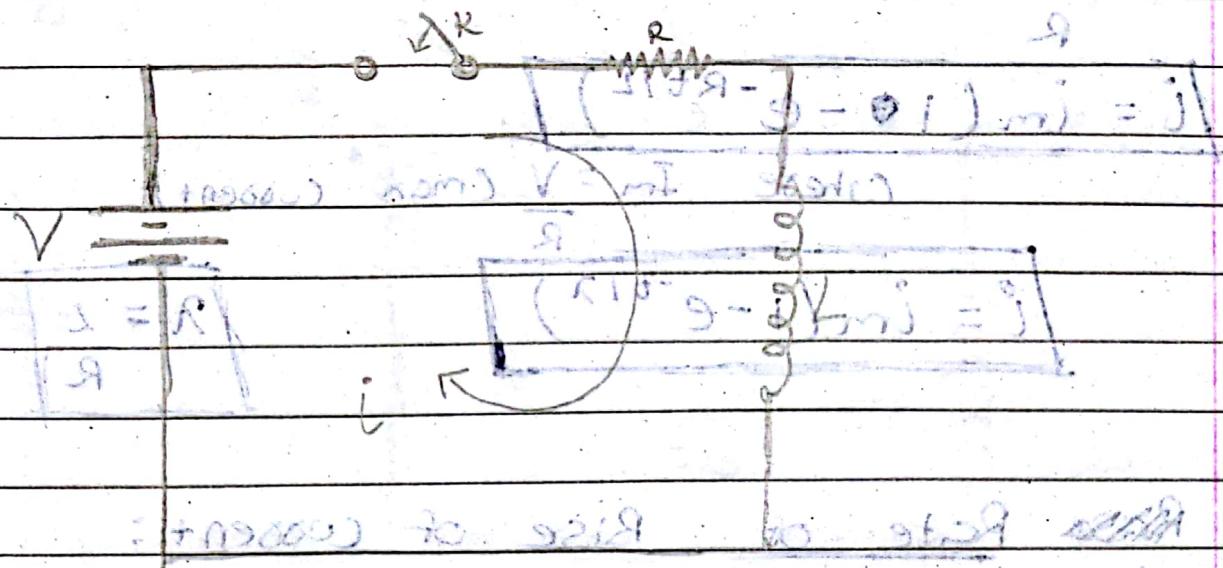
$$\Rightarrow M = K \sqrt{L_1 L_2} = KL = 0.7 \times 15 \times 10^{-3}$$

$$M = 10.5 \times 10^{-3} \text{ H} = 1.05 \text{ mH}$$

$$E_{\text{ind}} = \frac{0.5 \times 10^{-3}}{2 \times 10^{-2}} \times 10.5 \times 10^{-3} \times 4 = 4.2 \text{ V}$$

$$E_{\text{ind}} = 9 \text{ V}$$

## Rise of current in an Inductive circuit



$$\text{At } t=0, i=0 \quad \therefore V = iR + L \frac{di}{dt} = 0$$

By KVL  $\boxed{V - iR - L \frac{di}{dt}} = 0$

$$\boxed{V - iR - L \frac{di}{dt}} = 0$$

$$V = iR + L \frac{di}{dt}$$

$$V - iR = L \frac{di}{dt}$$

$$\frac{-R \frac{di}{dt}}{V - iR} = \frac{dt}{L}$$

$\Rightarrow$  integrating both sides we get

$$\log(V - iR) = -\frac{Rt}{L} + C$$

$$\text{At } t=0, i=0$$

$$C = \log V$$

$$\Rightarrow 1 - \frac{iR}{V} = e^{-\frac{Rt}{L}}$$

$$i = \frac{V}{R} (1 - e^{-Rt/L})$$

$$i = i_m (1 - e^{-Rt/L})$$

where  $i_m = \frac{V}{R}$  (max current)

$$i = i_m (1 - e^{-t/\lambda})$$

$$\lambda = \frac{L}{R}$$

Rise Rate of Rise of current:

$$\frac{di}{dt} = +i_m x + \frac{R}{L} e^{-t/\lambda}$$

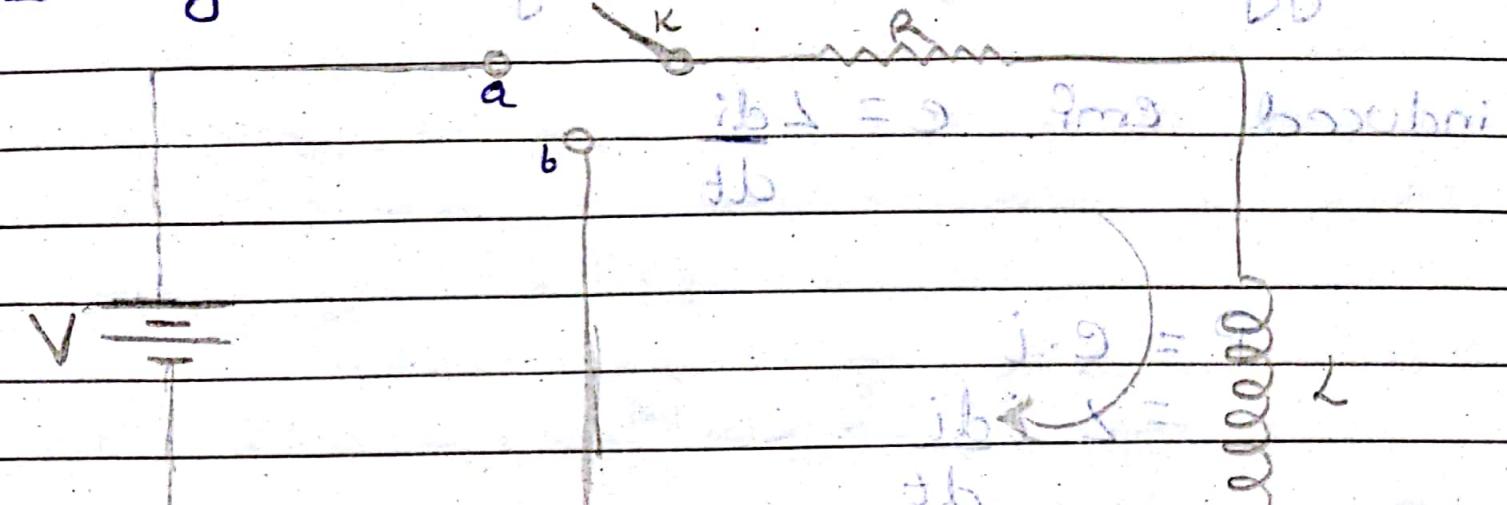
$$= \frac{R \cdot i_m}{L} x e^{-t/\lambda}$$

$$At t=0, i=0 \Rightarrow Rj = V$$

$$\frac{di}{dt} \Big|_{t=0} = i_m \cdot R$$

$$= \frac{V}{L}$$

# Decay in current in inductive circuit



$$i_{bA} = 9 \text{ Amperes}$$

at  $t=0$

$$i_{bK} = 9$$

$$i_b = 9$$

the Gaussian loops at such areas

$$\text{At } t=0, i = i_m = \frac{V}{R}, i_{bA} = i_b$$

$$\text{By KVL, } i_b i_L = v_b - E$$

$$V_0 = iR + L\frac{di}{dt}$$

$$\Rightarrow \frac{di}{i} = \frac{R-R}{L} dt$$

$$\Rightarrow \log i = -\frac{Rt}{L} + C$$

At  $t=0$ ,  $i_m = 9$

$$C = \log i_m$$

$$\Rightarrow -\frac{Rt}{L}$$

$$\Rightarrow i = i_m e^{-\frac{Rt}{L}}$$

$$\boxed{i = i_m e^{-\frac{Rt}{L}}}$$

## \* Energy Stored in magnetic fields

$$\text{induced emf } e = L \frac{di}{dt}$$

$$P = e \cdot i \\ = L i \frac{di}{dt}$$

• work done in short interval  $dt$

$$dw = L i \frac{di}{dt} \cdot dt \quad i = i_0 e^{rt} \quad i_0 = 0$$

$$\Rightarrow dw = L i \frac{di}{dt} \quad i_0 e^{rt} + R i = 0$$

$$\Rightarrow \boxed{W = \frac{1}{2} L I^2}$$

Rise of current:  $i = i_0 e^{rt}$

Q- Coil having resistance of  $2\Omega$  and inductance of  $12H$  is connected to a supply voltage of  $100V$ . Find

- Rate of Change of current at the instant closing the switch
- Final steady value of current
- Time constant
- Time taken for the current to reach value of  $4A$

$$i) \frac{di}{dt} = \frac{V}{L} = \frac{100}{12} = 50 \text{ A/sec}$$

frequency  $f = \frac{1}{T} = \frac{1}{\sqrt{\frac{L}{C}}} = \sqrt{\frac{1}{12 \times 10^{-6}}} = 25.83 \text{ Hz}$

ii)  $\omega = 2\pi f = 2\pi \times 25.83 = 160.2 \text{ rad/sec}$

~~iii)  $i = i_0 e^{-\frac{t}{RC}}$  where  $i_0$  is initial current~~

~~between  $t=0$  and  $t=0.5$  sec,  $i = 20$~~

~~to AC, side of inductor  $i = 20 \text{ A}$~~

$$i_0 e^{-\frac{t}{RC}} = 20 \Rightarrow 20 = i_0 e^{-\frac{0.5}{12 \times 10^{-6}}} \Rightarrow i_0 = 20 e^{0.5 / (12 \times 10^{-6})}$$

$$= 20 e^{0.5 / (12 \times 10^{-6})} = 20 e^{0.5 / (12 \times 10^{-6})} = 20 e^{0.5 / (12 \times 10^{-6})}$$

$$iii) t = \frac{1}{\omega} = \frac{1}{20} = 0.05 \text{ sec}$$

$$9. RCE = 20 \cdot [e^{-\frac{t}{RC}} - 1] \Rightarrow 20 = 20 \cdot [e^{-\frac{0.05}{12 \times 10^{-6}}} - 1]$$

$$\frac{5}{4} e^{-\frac{0.05}{12 \times 10^{-6}}} - 1 = 0$$

$$iv) i = 5(1 - e^{-\frac{t}{RC}}) \Rightarrow e^{-\frac{t}{RC}} = 0.2$$

$$\frac{t}{RC} = \ln 0.2 = -\ln 0.2 = -0.693$$

$$t = 0.693 \times 12 \times 10^{-6} = 0.0083 \text{ sec}$$

$$t = 0.0083 \text{ sec}$$

Q- D.C. Voltage of 120V is applied to a coil having  $R = 8\Omega$  and  $L = 12 \text{ H}$ . Calculate:

i) Value of Current 0.3 seconds after switching on the supply.

ii) With the current having reached the final value

how much time it would take for the current to reach value of 6A after switching on the supply.

$$E.O. = (R + L)$$

$$i) I = \frac{120}{8} (1 - e^{-\frac{0.3}{1.5}}) = 2.71 \text{ A}$$

$$E.O. = R + L$$

$$ii) 6 = \frac{120}{8} e^{-\frac{t}{1.5}} \Rightarrow t = 1.37 \text{ sec}$$

Q- Circuit of resistance  $R$  and inductance  $L$  has direct voltage of  $230V$ , after switching on the current in the circuit, it was found  $5A$  when  $t=0.3$  sec. After the current had reached its final value, the circuit was suddenly short circuited. The current was again found to be  $5A$  at point  $t=3$  sec after short circuiting the coil.

Find the values of  $R$  and  $L$ .

$$V = 230V$$

$$5 = 230 [1 - e^{-0.3/R}] \quad \cancel{R} \quad \cancel{5 = 230 \cdot e^{-0.3/R}}$$

$$1 - e^{-0.3/R} = \frac{5}{230} \quad \cancel{(1 - e^{-0.3/R})^2 = \frac{25}{529}} \quad \cancel{0.01}$$

$$1 - e^{-0.3/R} = \frac{-3}{23} \quad \cancel{e^{-0.3/R} = \frac{20}{23}}$$

$$1 - e^{-0.3/R} = \frac{-3}{23} \quad \cancel{e^{-0.3/R} = \frac{20}{23}}$$

$$\log(1 - e^{-0.3/R}) = -0.3 \quad \cancel{R = 0.3 / 0.01}$$

$$R = \frac{0.3}{0.01}$$

$$1 - e^{-0.3/R} = \frac{20}{23}$$

$$230$$

$$\log\left(\frac{230 - 5R}{230}\right) = -0.3$$

10000.730 ~~9000~~ 8500+1 very (iii)

+19000.70 9000 to 8700 relation (vi)

$$1 - e^{-0.3/\lambda} = e^{-0.3/\lambda}$$
$$\Rightarrow e^{-0.3/\lambda} = \frac{1}{2}$$

$$\frac{1}{2} = 0.6936$$

$$102 \quad 3PPP.6 = \frac{1}{2} + \epsilon$$

$$\Rightarrow \lambda = 0.4328 = \frac{L}{R}$$

$$C_1 \times R = R \frac{L}{R}$$

~~PRO~~  
$$S = \frac{230}{R} [1 - e^{-0.3/0.432}]$$

$$S = \frac{230}{R \times 2} \Rightarrow R = \frac{23^2}{1.61887} \quad (ii)$$

$$10000 = \Rightarrow X = 0.01887 \quad (vi)$$

$$L = 9.954 \text{ H}$$

Q- Relay has  $R = 300\Omega$  and it is switched on to 100V DC Supply. If the current reaches 63.2% of its final steady value in 2 m sec. Determine

i)  $\lambda =$

ii) Inductance of circuit

iii) Final steady value of current

iv) Initial rate of rise of current

$$\text{iii) } I_m = \frac{V}{R} = \frac{100}{300} = 0.3333 \text{ A}$$

$$\text{iv) } 0.632 = 1 - e^{-t/\lambda}$$

$$e^{-t/\lambda} = 0.368$$

$$\Rightarrow t = 0.9996$$

$$\lambda = \frac{t}{2 \times 10^3} = 0.0004998 \text{ sec} = 0.4998 \text{ ms}$$

$$\Rightarrow \lambda = 2 \times 10^{-3}$$

$$0.999672$$

$$\lambda = 2 \text{ m sec} = 2$$

ii)  $L = 0.6 \text{ H}$

$$\text{iv) } \frac{di}{dt} = \frac{100 - 63.2}{2 \times 10^{-3}} = 166.67 \text{ A/sec}$$

coil having  $L = 76.4 \text{ H}$  and  $R = 8 \Omega$  is connected to constant 200V supply. How long will it take for the voltage across resistor to reach 100V.

$$i = \frac{V}{R} = \frac{100}{8} = 12.5 \text{ A}$$

$$100 = 200 \left( 1 - e^{-\frac{t}{T}} \right) \Rightarrow e^{-\frac{t}{T}} = \frac{1}{2} \Rightarrow t = 0.693 \text{ ms}$$

Ans: 0.693 ms

The nature of decay, looking from 200V DC supply is not indicated until the current reaches 0.35A which occurs 4ms after the relay circuit is closed. This time being the time const of circuit.

Calc. L and R for rise time  $\Delta i/dt$

$$0.35 = 200 \left( 1 - e^{-\frac{4}{T}} \right)$$

$$R = 361.21 \Omega$$

$$\frac{2}{R} = 4 \times 10^{-3} \Rightarrow L = 0.838 \text{ H} \quad L = 1.444 \text{ H}$$

$$\frac{di}{dt} = \frac{V}{L} = \frac{200}{1.444} = 138.50 \text{ A/Sec}$$