04-655-Assignment 2: Graph Theory, Constraint Satisfaction, and Knowledge Representation

The objective

The objective of this assignment is to help students practice translating real-world problems into formal representations used in artificial intelligence. By working with graph theory, constraint satisfaction problems, and knowledge representation, you will learn to identify variables, domains, and constraints, represent relationships as graphs, and express knowledge using propositional and first-order logic. The goal is to strengthen your ability to move from informal descriptions to precise, machine-readable models that support automated reasoning and problem solving.

Part 1: Graph Theory and Constraint Satisfaction Problem

Question 1

[10pts] Define in your own words:

1. Domain
2. Variable
3. Constraint
4. State
5. Forward checking
6. Backtracking
7. Arc Consistency
8. Constraint Graph
9. Most Constrained Variable
10. Least Constraining Value

Question 2

[10pts] Give a precise formulation for the following as a constraint satisfaction problem:

1. **Class Scheduling**
   There is a fixed number of professors and classrooms, a list of classes to be offered, and a list of possible time slots for classes. Each professor has a set of classes that he or she can teach.

2. **Sudoku Puzzle**
   Consider a standard 9 × 9 Sudoku. Represent the puzzle as a CSP with cells, rows, columns, and 3 × 3 boxes.

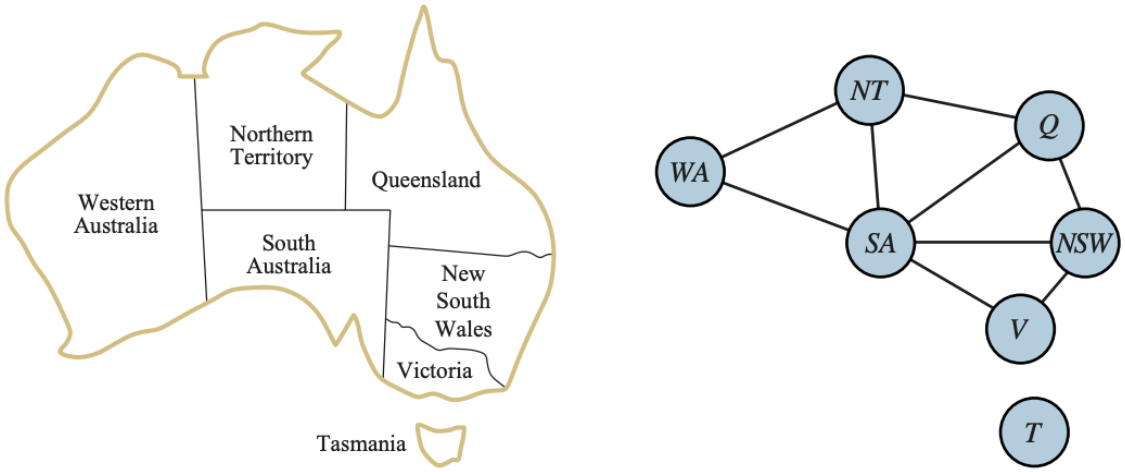
3. **Cryptarithmetic Puzzle**
   Use the equation CRASH + HACKER = REBOOT. Treat each distinct letter as a variable with an appropriate numeric domain and include the necessary arithmetic and distinct-digit constraints.

4. **Continuous Meeting Scheduling**
   Three one-hour meetings must be scheduled between 8:00 a.m. and 6:00 p.m. with no overlaps. Formulate this as a CSP.

5. **Cloud-Resource Allocation for Data-Processing Jobs**
   A cloud provider has a finite number of CPU cores, GPUs, and memory units.
   - Jobs arrive with requirements such as 4 CPU + 1 GPU + 8 GB RAM.
   - Total allocated resources cannot exceed available capacity at any time.
   - High-priority jobs must be placed before low-priority ones.
     Define variables for job start times and resource assignments, with capacity and priority constraints.

Question 3

[10 pts] We are given the task of coloring each region in the map of Australia, either red, green, or blue, in such a way that no two neighboring regions have the same color.



(a) (b)

1. Formulate this problem as a CSP
2. How many solutions are possible?
3. How many solutions are there if four colors are allowed?
4. How many solutions are there if two colors are allowed?

Question 4

Implement the Australia map coloring problem (refer to the map in the question above) as a Constraint Satisfaction Problem (CSP). Use the starter notebook provided: `AI_4_Eng_Ass2_Graph_&_CSP.ipynb`. Your task is to complete the `#TODO` sections in the notebook.

1. [10 pts] Run your implementation, filling in the *#TODO* parts, and report the solutions and number of steps for each CSP algorithm.

The output will be in this format

```
Degree Centrality: {'WA': 2, 'NT': 3, 'SA': 5, 'Q': 3, 'NSW': 3, 'V': 2, 'T': 0}
Euler Path Exists?
Hamiltonian Path Exists?

--- Backtracking ---
Solution: {'WA': 'Red', 'NT': 'Green', 'SA': 'Blue', 'Q': 'Red', 'NSW': 'Green', 'V': 'Red', 'T': 'Red'}
Steps: 11

--- Forward Checking ---
Solution: {'WA': 'Red', 'NT': 'Green', 'SA': 'Blue', 'Q': 'Red', 'NSW': 'Green', 'V': 'Red', 'T': 'Red'} Steps: 7

--- AC-3 + Backtracking ---
Solution: {'WA': 'Red', 'NT': 'Green', 'SA': 'Blue', 'Q': 'Red', 'NSW': 'Green', 'V': 'Red', 'T': 'Red'}
Steps: 9
```

2. [5 pts] What is the Hamiltonian path? Did the problem have it? Explain your answer.
3. [5 pts] What is the Euler path? Did the problem have it? Explain your answer.
4. [10 pts] Discuss the difference between forward checking, backtracking, and AC-3.
5. [5 pts] Reduce the domain colors to 2, and report your findings.

Part 2: Knowledge representation

Question 5

[10pts] Define in your own words and provide an example on each of the following:

1. Tautology

2. Contradiction
3. Sufficient Condition
4. Necessary Condition
5. Well-Formed Formula (wff)

Question 6

[5pts] Answer the following two (unrelated) questions.

1. Consider a knowledge base containing just two sentences: P(a) and P(b). Does this knowledge base entail $\forall x\ P(x)$? Explain your answer in terms of models.
2. Is the sentence $\exists x,y \quad x=y$ valid? Explain.

Question 7

[5pts] Which of the following are correct and explain?

1. False $\models$ True.
2. True $\models$ False.
3. $(A \land B) \models (A \Leftrightarrow B)$.
4. $A \Leftrightarrow B \models A \lor B$.
5. $A \Leftrightarrow B \models \neg A \lor B$.

Question 8

[15pts] Decide whether each of the following sentences is valid, unsatisfiable, or neither. Verify your decisions using truth tables or the equivalence rules.

1. $Smoke \Rightarrow Smoke$
2. $Smoke \Rightarrow Fire$
3. $(Smoke \Rightarrow Fire) \Rightarrow (\neg Smoke \Rightarrow \neg Fire)$
4. $Smoke \lor Fire \lor \neg Fire$
5. $((Smoke \land Heat) \Rightarrow Fire) \Leftrightarrow ((Smoke \Rightarrow Fire) \lor (Heat \Rightarrow Fire))$

Submission Guidelines:

- **Format:** Submit your work as a single PDF file. Include both written responses (Part 1 and Part 2) and outputs/screenshots from your Python notebook (Part 1, Question 4).
- Ensure that your answers are clearly labeled according to the question numbers.
- **Notebook:** Along with the PDF, also submit your completed Jupyter Notebook file (`AI_4_Eng_Ass2_Graph_&_CSP.ipynb`) with all code cells executed and outputs visible.

- **Report:** For Part 1, Question 4, include a brief report of your results (solutions, number of steps, Hamiltonian/Euler paths, discussions, and findings from domain reduction).
- **Marks Distribution**: **Part 1** carries **65 marks**, **Part 2** carries **35 marks**.
- **Deadline: Wednesday, October 8**. Late submissions will be **penalized at -10%** per day up to **2 days,** after which you will receive a mark of **ZERO (0)**.
- **File Naming:** Name your files as **`YourAndrewID_Assignment2.pdf`** and **`YourAndrewID_Assignment2.ipynb.`**
- **Academic Integrity:** All work must be completed individually. Plagiarism or unacknowledged code copying will not be tolerated. Cite any sources you consult. The use of Generative AI is highly discouraged. Therefore, any use of Generative AI **MUST** be disclosed and the prompts submitted alongside your submissions, using the naming conventions `YourAndrewID_Assignment2_prompts.pdf.`
- You will **submit a .zip file** (named as `YourAndrewID_Assignment2.zip`) made up of all the files mentioned above.