MIDTERM: TOPIC 2

In this problem, consider the motion of a particle in three different potential wells:

1. A double-well potential, which is a very common model for systems with multiple equilibrium points:

$$U_1(x) = 2\pi^2\left(\frac{x^4}{4} - \frac{x^2}{2} + \frac{1}{4}\right)$$

2. A square/inverse-square potential (which might look familiar to those of you who took classical mechanics last year, as this is the effective 1D potential corresponding to a very common 2D system):

$$U_2(x) = \pi^2\left(\frac{1}{2x^2} + \frac{x^2}{2} - 1\right)$$

3. A quartic/inverse-square potential, which will allow us to find which features are common to all three potentials, and which features are unique to each potential:

$$U_3(x) = \frac{2\pi^2}{3}\left(\frac{1}{2x^2} + \frac{x^4}{4} - \frac{3}{4}\right)$$

In all three cases, you may assume the mass of the particle is $m = 1$.

Use the Runge Kutta method to model the motion of the particle to verify or refute the following claims:

- In all three cases, if a particle starts off with a velocity $v = 0$ at a position close to the minimum of the potential, the period of oscillation is approximately 1. *Hint*: The most accurate way to do this is to run your simulation for a long time and look at the Fourier transform of your result.
- In *at least one* of these potentials, starting the particle far from the minimum of the potential will NOT change the fundamental frequency of oscillation (i.e., the lowest positive frequency).
- In *at least one* of these potentials, starting the particle far from the minimum of the potential will change the fundamental frequency of oscillations, and if the energy is "small" the shift Δ is proportional to the energy of the particle E. Note that you can find the frequency either by measuring the period from a plot of $x(t)$, or by looking at the Fourier transform. If you run the simulation for a long time, the peaks of the Fourier transform will be very sharp.
- In all three potentials, at low energies $\langle x \rangle - 1 \propto E$.

Now add damping and driving to the problem. In addition to the forces derived from the potentials above, add two more terms to the force: a damping force $-\gamma v$ and a driving force $F_0 \cos \omega t$.

Now verify or refute the following claims:

- In the absence of a driving force (for example, if you set $F_0 = 0$), if you start the particle off away from equilibrium the motion will be an oscillation with exponentially decaying amplitude, and the time that it takes for the amplitude to decay to half its initial value is inversely proportional to γ.
- In the presence of a *small* driving force, after an initial period of complicated motion, it will settle into a state of steady oscillations at the same frequency as the driving force, and with an amplitude that is proportional to the amplitude of the driving force.

WHAT TO TURN IN:

- A report that describes the evidence for or against each assertion above. Note: don't just say that Figure 1 supports such-and-such assertion is correct! You must *demonstrate* that you understand *why* Figure 1 supports the assertion. For example, "Figure 1 shows that when Quantity 1 is doubled, Quantity 2 is quadrupled. This shows that Quantity 1 is proportional to the square Quantity 2; in log-log space, the slope of the line in Figure 1 is +2."
    - Include in your report as many figures and tables as necessary to support your conclusions. All graphs and tables should be properly labeled and described in the text.
    - A portion of your grade *will* be based on correct writing style: grammar, punctuation, and word usage matter!
    - Although there is no strict minimum or maximum length for your report, it is *almost* certainly going to be more than 2-3 pages in length (especially if you embed figures and tables).
    - Use 10-point font (something fairly standard like Times New Roman, Arial, Calibri, etc.), 1-inch margins on all sides, single line spacing.
    - Reports should be in Word, LaTeX, etc., and submitted in PDF format.
- One or more Jupyter notebooks showing how you generated plots, performed calculations, etc.

Compile *all* files into a single folder and compress it into a .zip format. Please upload **ONE .zip FILE TO BLACKBOARD**. I am not picky about the naming convention of this file, but it should be something obvious that includes your name and the midterm project you selected. For example, if I selected this topic for my midterm, I might name my file: **BBinder_Midterm2.zip**.

Midterm projects are due **OCTOBER 13 by 9am**. Late work *will* be penalized by -10% per calendar day late, and no midterms will be accepted after one week (October 20).