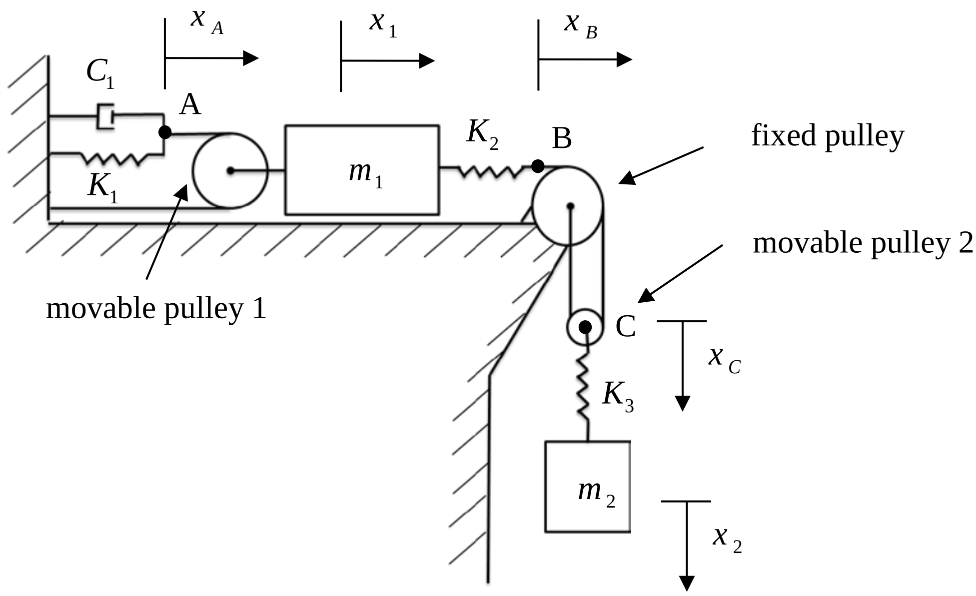
Problem 1 [25 points]

Consider the mechanical system shown below. The assumptions of this system are listed as follows:



Assumptions

- No friction between the block and the surface.
- All cables remain in tension.
- All pulleys are massless and without inertia.
- The system is at the static equilibrium position.

Problem Statement

a) Derive all kinematic constraints.

b) Draw the free body diagrams. Indicate all forces and define forces in terms of x_1, x_2, $\dot{x}_1$, $\dot{x}_2$

c) Derive the governing differential equations of motion.

Problem 1 (continued)

Solution:

Kinematic Constraints

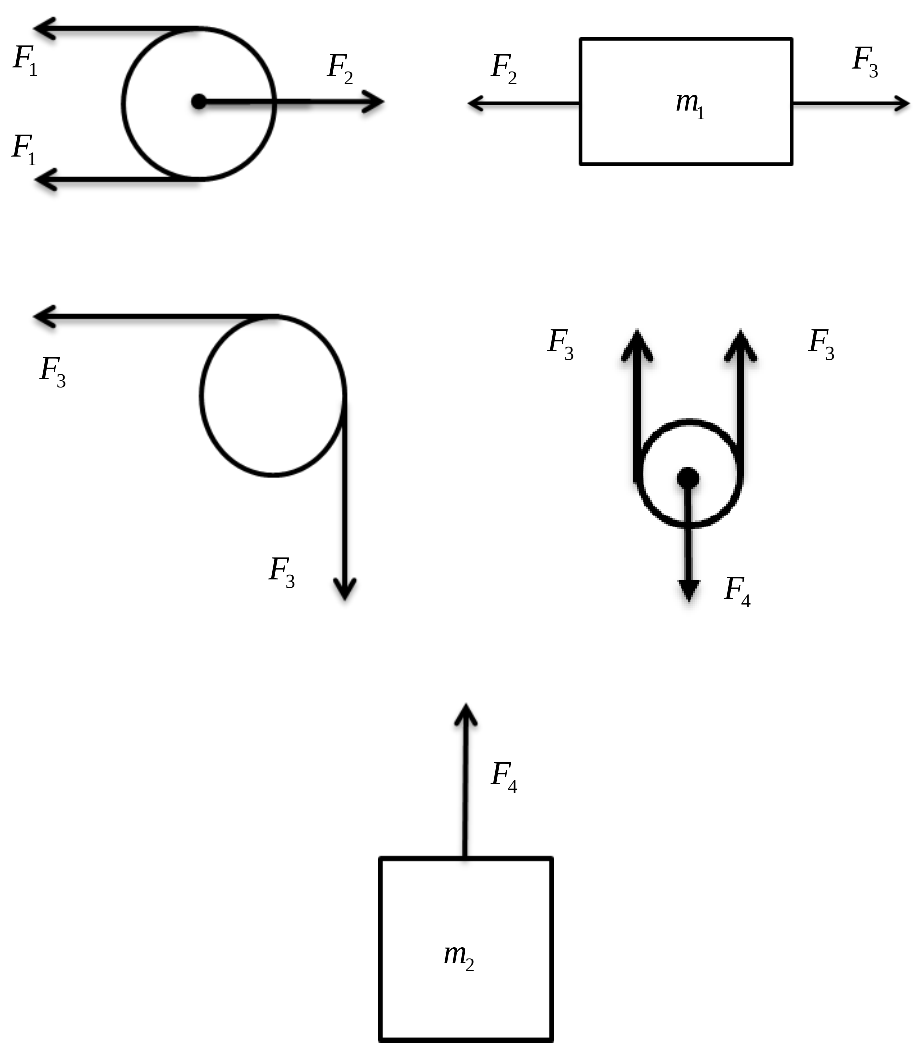
There are two kinematic constraints on the movable pulleys as shown below:

$$x_A = 2x_1, \qquad \text{Eq. 1}$$

$$x_B = 2x_C \qquad \text{Eq. 2}$$

Free Body Diagrams

The free body diagrams are shown as follows (Because the static equilibrium assumption, the weight of mass 2 is not included in the diagram):



The forces F_1 to F_4 and their relationships can be expressed as shown in Eq. 3 to Eq. 7:

$$F_1 = K_1 x_A + C_1 \dot{x}_A = 2K_1 x_1 + 2C_1 \dot{x}_1 \qquad \text{Eq. 3}$$

$$F_3 = K_2(x_B - x_1) \qquad \text{Eq. 4}$$

$$F_4 = K_3(x_2 - x_C) \qquad \text{Eq. 5}$$

$$2F_1 = F_2 = 4K_1 x_1 + 4C_1 \dot{x}_1 \qquad \text{Eq. 6}$$

$$2F_3 = F_4 \qquad \text{Eq. 7}$$

The x_C and x_B can be expressed by x_1 and x_2 by using Eq. 1, Eq. 2, Eq. 4, Eq. 5, and Eq. 7:

$$2F_3 = 2K_2(x_B - x_1) = 2K_2(2x_c - x_1) = F_4 = K_3(x_2 - x_C),$$

$$\rightarrow 2K_2(2x_c - x_1) = K_3(x_2 - x_C)$$

$$\rightarrow x_c = \frac{2K_2 x_1 + K_3 x_2}{4K_2 + K_3} \qquad \text{Eq. 8}$$

$$\rightarrow x_B = 2x_c = \frac{4K_2 x_1 + 2K_3 x_2}{4K_2 + K_3} \qquad \text{Eq. 9}$$

Substitute x_C in Eq. 8 into Eq. 4 and Eq. 7 respectively, we can derive:

$$F_3 = K_2(x_B - x_1) = K_2\left(\frac{4K_2 x_1 + 2K_3 x_2}{4K_2 + K_3} - x_1\right) = -\frac{K_2 K_3}{4K_2 + K_3} x_1 + \frac{2K_2 K_3}{4K_2 + K_3} x_2, \qquad \text{Eq. 10}$$

$$F_4 = 2F_3 == -\frac{2K_2 K_3}{4K_2 + K_3} x_1 + \frac{4K_2 K_3}{4K_2 + K_3} x_2 \qquad \text{Eq. 11}$$

Equations of Motion:

Using Netwon's 2nd law of motion and F_2 F_3 F_4 in Eq. 6 ,Eq. 10, Eq. 11, we can derived the equations of motions as follows:

$$m_1 \ddot{x}_1 = F_3 - F_2 = -\frac{K_2 K_3}{4K_2 + K_3} x_1 + \frac{2K_2 K_3}{4K_2 + K_3} x_2 - 4K_1 x_1 - 4C_1 \dot{x}_1, \qquad \text{Eq. 12}$$

$$m_2 \ddot{x}_2 = -F_4 = \frac{2K_2 K_3}{4K_2 + K_3} x_1 - \frac{4K_2 K_3}{4K_2 + K_3} x_2 \qquad \text{Eq. 13}$$

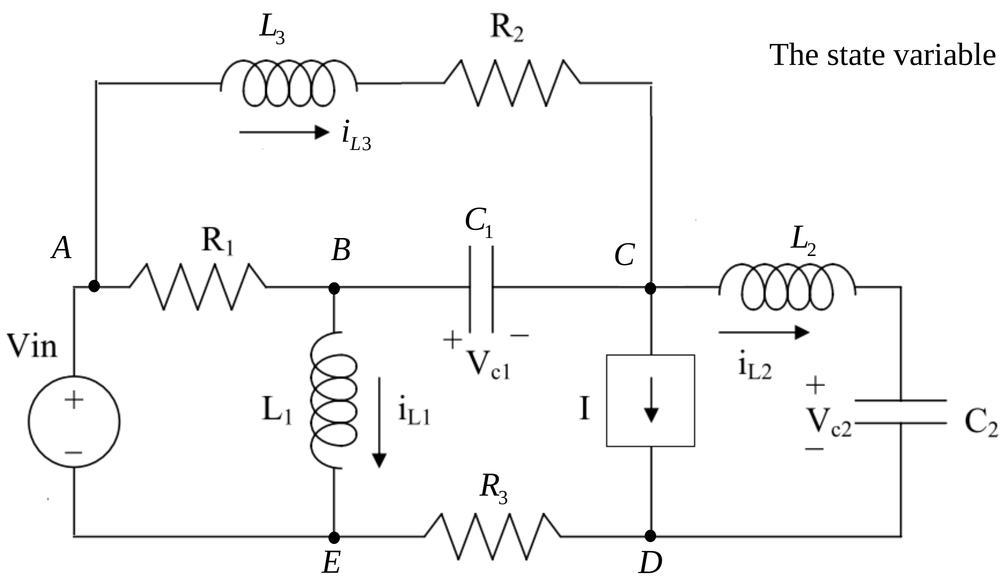
Eq. 12 and Eq. 13 are the governing differential equations of motion of this system.

Problem 2 [30 points]

Consider the electric circuit shown below with a voltage source, "V_{in}" and a current source, "I":

- Let the current flowing through the inductors L_1, L_2 and L_3 be denoted as "i_{L1}", "i_{L2}" and "i_{L3}" respectively.
- Assume the voltage drop across the capacitors, C_1 and C_2 to be "V_{C1}" and "V_{C2}", respectively.

The state variable $z = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \\ z_5 \end{bmatrix} = \begin{bmatrix} V_{C1} \\ V_{C2} \\ i_{L1} \\ i_{L2} \\ i_{L3} \end{bmatrix}$



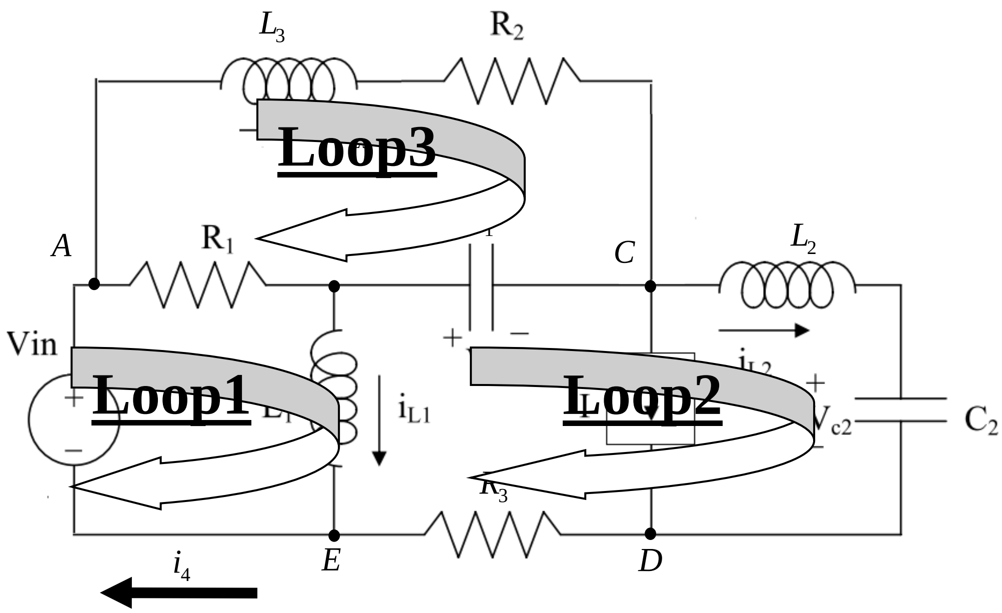
<u>Problem Statement</u>

a) Derive the governing differential equations for the circuit shown above. Indicate the loops or nodes taken for the voltage balance equations or current balance equations.

b) Represent the derived differential equations in state-space matrix form.

Consider the output of the system to be $\begin{bmatrix} i_{R3} \\ V_{C1} \end{bmatrix}$, where i_{R3} is the current through the resistor, "R_3".

Problem 2 (continued)



Solution:

State variables: v_{C1}, v_{C2}, i_{L1}, i_{L2}, i_{L3}

System inputs: $\begin{bmatrix} V_{in} \\ I \end{bmatrix}$

System ouput: $\begin{bmatrix} i_{R3} \\ V_{C1} \end{bmatrix}$

Apply KCL at node D, the current through R_3 :

$$i_{R3} = I + i_{L2}, \qquad \text{Eq. 1}$$

Apply KCL at node E, the current through into voltage source:

$$i_4 = i_{L1} + i_{R3} = i_{L1} + I + i_{L2}, \qquad \text{Eq. 2}$$

Apply KCL at node A, the current through R_1 :

$$i_{R1} + i_{L3} = i_4, \qquad \text{Eq. 3}$$

$$i_{R1} = i_{L1} + I + i_{L2} - i_{L3}$$

Next, apply KVL on loop 1,

$$V_{in} - R_1 i_{R1} - L_1 \frac{di_{L1}}{dt} = 0\,,$$

$$\rightarrow L_1 \frac{di_{L1}}{dt} = Vin - R_1 I - R_1 i_{L1} - R_1 i_{L2} + R_1 i_{L3} \qquad \text{Eq. 5}$$

Apply KCL at Node B:

$$i_{R1} = i_{L1} + C_1 \frac{dV_{C1}}{dt} = 0\,,$$

$$\rightarrow C_1 \frac{dV_{C1}}{dt} = i_{R1} - i_{L1} = I + i_{L2} - i_{L3} \qquad \text{Eq. 6}$$

Apply KVL on loop2:

$$L_1 \frac{di_{L1}}{dt} - V_{C1} - L_2 \frac{di_{L2}}{dt} - V_{C2} - R_3 i_{R3} = 0\,,$$

$$\rightarrow L_2 \frac{di_{L2}}{dt} = Vin - R_1 I - R_1 i_{L1} - R_1 i_{L2} + R_1 i_{L3} - V_{C1} - V_{C2} - R_3 i_{L2} - R_3 I$$

$$\rightarrow L_2 \frac{di_{L2}}{dt} = Vin - (R_1 + R_3) I - R_1 i_{L1} - (R_1 + R_3) i_{L2} + R_1 i_{L3} - V_{C1} - V_{C2} \qquad \text{Eq. 7}$$

Apply KVL on loop3:

$$-L_3 \frac{di_{L3}}{dt} - R_2 i_{L3} - (-V_{C1}) - (-R_1 i_{R1}) = 0\,,$$

$$\rightarrow L_3 \frac{di_{L3}}{dt} = V_{C1} + R_1 I + R_1 i_{L1} + R_1 i_{L2} - (R_1 + R_2) i_{L3} \qquad \text{Eq. 8}$$

The current through $C_2 = i_{L2}$

$$C_2 \frac{dV_{C2}}{dt} = i_{L2}\,, \qquad \text{Eq. 9}$$

State Space Representation:

The state variable z is defined as $z = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \\ z_5 \end{bmatrix} = \begin{bmatrix} V_{C1} \\ V_{C2} \\ i_{L1} \\ i_{L2} \\ i_{L3} \end{bmatrix}$.

By using Eq. 5 to Eq. 9, the state space form of this system is shown as follows:

$$\dot{z} = \begin{bmatrix} 0 & 0 & 0 & \frac{1}{C_1} & \frac{-1}{C_1} \\ 0 & 0 & 0 & \frac{1}{C_2} & 0 \\ 0 & 0 & \frac{-R_1}{L_1} & \frac{-R_1}{L_1} & \frac{R_1}{L_1} \\ \frac{-1}{L_2} & \frac{-1}{L_2} & \frac{-R_1}{L_2} & \frac{-(R_1+R_3)}{L_2} & \frac{R_1}{L_2} \\ \frac{1}{L_3} & 0 & \frac{R_1}{L_3} & \frac{R_1}{L_3} & \frac{-(R_1+R_2)}{L_3} \end{bmatrix} z + \begin{bmatrix} 0 & \frac{1}{C_1} \\ 0 & 0 \\ \frac{1}{L_1} & \frac{-R_1}{L_1} \\ \frac{1}{L_2} & \frac{-(R_1+R_3)}{L_2} \\ 0 & \frac{R_1}{L_3} \end{bmatrix} \begin{bmatrix} V_{in} \\ I \end{bmatrix}$$

Output $y = \begin{bmatrix} iR_3 \\ V_{C1} \end{bmatrix} = \begin{bmatrix} I + i_{L2} \\ V_{C1} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} z + \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_{in} \\ I \end{bmatrix}$

Problem 3 [20 Points]

A) Use the Laplace Transform to find the solution $y(t)$ to the following differential equation:

$$\ddot{\boldsymbol{y}}(\boldsymbol{t}) - \mathbf{4}\dot{\boldsymbol{y}}(\boldsymbol{t}) + \mathbf{4}\boldsymbol{y}(\boldsymbol{t}) = \mathbf{3}\boldsymbol{e}^{-\mathbf{2}\boldsymbol{t}}$$

with initial conditions $y(0) = 0$ and $\dot{y}(0) = 1$.

B) The differential equations for a certain dynamic system are given below:

$$2\ddot{x}(t)y(t) - \dot{x}(t)\left[2 - y(t)\right] + 3y(t)^2 + 2x(t)\sin[\dot{y}(t)] = 27$$
$$\ddot{y}(t) + 3\dot{y}(t)\cos[x(t)] + \dot{y}(t)x(t) + 2x(t) = 5u(t)$$

a) Determine the equilibrium/operating point(s) of the system. Is/are the equilibrium/operating point(s) unique? Justify your answer.

b) Linearize the given differential equations about the equilibrium/operating point(s) as obtained in part (a). Choose the equilibrium value of the input '*u(t)*' as u_0.

c) Put the resulting linearized equations of motion into state-space matrix form with 'x(t)' as the output.

Problem 3 (continued)

Solution A):

Taking the Laplace transform of the differential equation:

$$[s^2L\{y\} - sy(0) - y'(0)] - 4[sL\{y\} - y(0)] + 4L\{y\} = L\{\boldsymbol{3e^{-2t}}\}$$

Substituting the initial conditions:

$$(s^2 - 4s + 4)Y(s) = \frac{3}{s+2} + 1$$

Rearranging:

$$Y(s) = \frac{3}{(s+2)(s-2)^2} + \frac{1}{(s-2)^2}$$

The partial fraction expansion of the first term:

$$\frac{3}{(s+2)(s-2)^2} = \frac{a}{s+2} + \frac{b}{s-2} + \frac{c}{(s-2)^2}$$

Solving:

$$3 = (a)(s-2)^2 + b(s-2)(s+2) + c(s+2)$$

Equating coefficients:

$$s^2: \;\; 0 = a + b$$
$$s^1: \;\; 0 = -4a + c$$
$$s^0: \;\; 3 = 4a - 4b + 2c$$

Solving:

$$a = \tfrac{3}{16} \qquad\qquad b = -\tfrac{3}{16} \qquad\qquad c = \tfrac{3}{4}$$

Producing:

$$\frac{3}{(s+2)(s-2)^2} = \left(\frac{3}{16}\right)\left(\frac{1}{s+2}\right) - \left(\frac{3}{16}\right)\left(\frac{1}{s-2}\right) + \left(\frac{3}{4}\right)\frac{1}{(s-2)^2}$$

The Inverse Laplace of the first term:

$$L^{-1}\left\{\frac{3}{(s+2)(s-2)^2}\right\} = \frac{3}{16}e^{-2t} - \frac{3}{16}e^{2t} + \frac{3}{4}te^{2t}$$

The Inverse Laplace of the second term:

$$L^{-1}\left\{\frac{1}{(s-2)^2}\right\} = te^{2t}$$

Final Solution:

$$y(t) = \frac{3}{16}e^{-2t} - \frac{3}{16}e^{2t} + \frac{7}{4}te^{2t}$$

Solution B):

a) [5 points] Determine the equilibrium/operating point(s) of the system. Is/are the equilibrium/operating point(s) unique? Justify your answer.

$$3y_0^2 = 27 \Rightarrow y_0 = \pm 3$$

$$x_0 = \frac{5u_0}{2}$$

The equilibrium point is not unique.

b) [10 points] Linearize the given differential equations about the equilibrium/operating point(s) as obtained in part (a). Choose the equilibrium value of the input '*u(t)*' as u_0.

$$\cancel{2\ddot{x}_0 y_0} + 2\left.\frac{\partial \ddot{x}(t)y(t)}{\partial \ddot{x}(t)}\right|_{x_0,y_0,u_0} \Delta\ddot{x}(t) + 2\left.\frac{\partial \ddot{x}(t)y(t)}{\partial y(t)}\right|_{x_0,y_0,u_0} \Delta y(t) - \cancel{\dot{x}_0\left[2-y_0\right]} - \left.\frac{\partial \dot{x}(t)\left[2-y(t)\right]}{\partial \dot{x}}\right|_{x_0,y_0,u_0} \Delta\dot{x}(t)$$

$$-\left.\frac{\partial \dot{x}(t)\left[2-y(t)\right]}{\partial y}\right|_{x_0,y_0,u_0} \Delta y + 3y_0^2 + \left.\frac{\partial 3y(t)^2}{\partial y}\right|_{x_0,y_0,u_0} \Delta y(t) + \cancel{2x_0 \sin[\dot{y}_0]} + \left.\frac{\partial 2x(t)\sin[\dot{y}(t)]}{\partial x}\right|_{x_0,y_0,u_0} \Delta x(t)$$

$$+\left.\frac{\partial 2x(t)\sin[\dot{y}(t)]}{\partial \dot{y}}\right|_{x_0,y_0,u_0} \Delta\dot{y}(t) = 27$$

By the equilibrium condition in part (a)

$$2y_0\Delta\ddot{x}(t) - (2-y_0)\Delta\dot{x}(t) + \cancel{3y_0^2} + \cancel{2\sin[\dot{y}_0]\Delta x(t)} + 2x_0\cos[\dot{y}_0]\Delta\dot{y}(t) = \cancel{27}$$

Now substitute $y_0 = \pm 3$ and $x_0 = 5u_0/2$

$$\Rightarrow 6\Delta\ddot{x}(t) + \Delta\dot{x}(t) + 5u_0\Delta\dot{y}(t) = 0 \text{ and}$$

$$\Rightarrow -6\Delta\ddot{x}(t) - 5\Delta\dot{x}(t) + 5u_0\Delta\dot{y}(t) = 0$$

Similarly for the second equation,

$$\cancel{\ddot{y}_0} + \Delta\ddot{y}(t) + \cancel{3\dot{x}_0 y_0} + \left.\frac{\partial 3\dot{y}(t)\cos[x(t)]}{\partial \dot{y}}\right|_{x_0,y_0,u_0} \Delta\dot{y}(t) + \left.\frac{\partial 3\dot{y}(t)\cos[x(t)]}{\partial x}\right|_{x_0,y_0,u_0} \Delta x(t) + \cancel{\dot{y}_0 x_0}$$

$$+\left.\frac{\partial \dot{y}(t)x(t)}{\partial \dot{y}}\right|_{x_0,y_0,u_0} \Delta\dot{y}(t) + \left.\frac{\partial \dot{y}(t)x(t)}{\partial x}\right|_{x_0,y_0,u_0} \Delta x(t) + 2x_0 + 2\Delta x(t) = 5(u_0 + \Delta u(t))$$

By equil. condition in (a)

$$\Delta\ddot{y}(t) + 3\cos[x_0]\Delta\dot{y}(t) - \cancel{3\dot{y}_0\sin[x_0]\Delta x(t)} + x_0\Delta\dot{y}(t) + \cancel{2x_0} + 2\Delta x(t) = \cancel{5u_0} + 5\Delta u(t)$$

$$\Rightarrow \Delta\ddot{y}(t) + \left(3\cos[\frac{5u_0}{2}] + \frac{5u_0}{2}\right)\Delta\dot{y}(t) + 2\Delta x(t) = 5\Delta u(t)$$

c) [5 Points] Put the resulting linearized equations of motion into state-space matrix form with 'x(t)' as the output.

Rearranging the linearized eqns we get,

$$\Delta\ddot{x}(t) = (5\Delta\dot{x}(t) - 5u_0\Delta\dot{y}(t))\frac{1}{6}$$

$$\Delta\ddot{y}(t) = -\left(3\cos[\frac{5u_0}{2}] + \frac{5u_0}{2}\right)\Delta\dot{y}(t) - 2\Delta x(t) + 5\Delta u(t)$$

$$\begin{bmatrix}\dot{x}\\ \ddot{x}\\ \dot{y}\\ \ddot{y}\end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & \frac{5}{6} & 0 & \frac{-5u_o}{6} \\ 0 & 0 & 0 & 1 \\ -2 & 0 & 0 & 3\cos[\frac{5u_0}{2}] + \frac{5u_0}{2} \end{bmatrix}\begin{bmatrix}x\\ \dot{x}\\ y\\ \dot{y}\end{bmatrix} + \begin{bmatrix}0\\0\\0\\5\end{bmatrix}u(t)$$

y=[1,0,0,0] $\begin{bmatrix}x\\ \dot{x}\\ y\\ \dot{y}\end{bmatrix}$+0

Problem 4 [25 Points]

A dual-DC motor-driven lifting system is shown below. The wheel with inertia ‘ J_1 ’ and radius ‘ r ’ is connected to the output shaft of DC motor 1, where the damping coefficient of the shaft bearing is ‘ B_1 ’. The wheel with inertia ‘ J_2 ’ and radius ‘ r ’ is connected to the output shaft of DC motor 2, where the damping coefficient of the shaft bearing is ‘ B_2 ’. The wheels wind up the cable and pull the mass ‘ m ’. Assume the pulleys are massless and that the shaft bearings of the pulley are frictionless. The DC motor circuits can be modeled as depicted. The idealized motors are governed by the relationships:

$$e_{bemf1} = K_{e1}\dot{\theta}_1, \qquad T_1 = K_{t1} i_{L1}, \qquad e_{bemf2} = K_{e2}\dot{\theta}_2, \qquad T_2 = K_{t2} i_{L2}$$

Assume that the inputs to the system are the voltages V_a and V_b. Also assume that all the elements are ideal and linear, and neglect gravity.

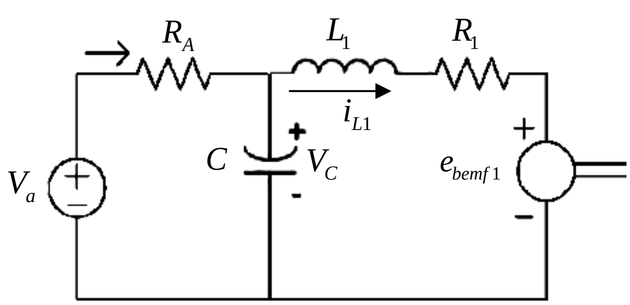
<u>Problem Statement</u>

a) Derive the governing differential equations of the electrical circuit of DC motor 1 and motor 2.

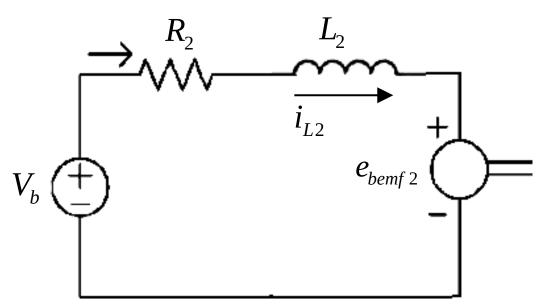
b) Derive the kinematic constraint between x_A, θ_1, and θ_2.

c) Draw the free body diagrams of the movable pulley and the two wheels. Indicate the forces and torques and define them in terms of i_{L1}, i_{L2}, y, θ_1, θ_2, $\dot{\theta}_1$ and $\dot{\theta}_2$.

d) Derive all governing differential equations of motion.

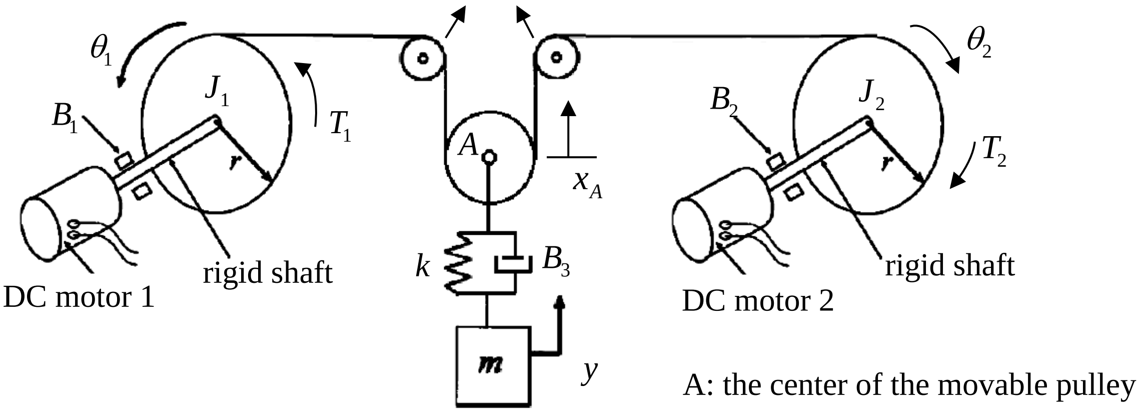


electrical circuit of DC motor 1



electrical circuit of DC motor 2

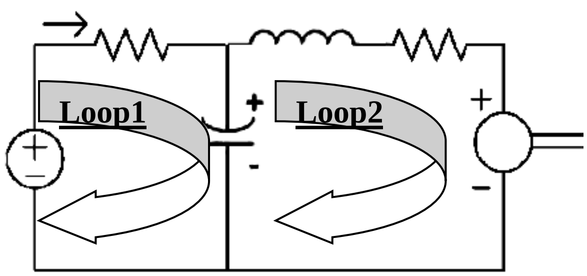
fixed pulleys



Problem 4 (continued)

Solution:

<u>*Electrical Parts*</u>



For DC motor 1, apply KVL on Loop1,

$$V_a - R_A i_{RA} - V_C = 0$$
$$\rightarrow i_{RA} = \frac{V_a - V_C}{R_A} \qquad \text{Eq. 1}$$

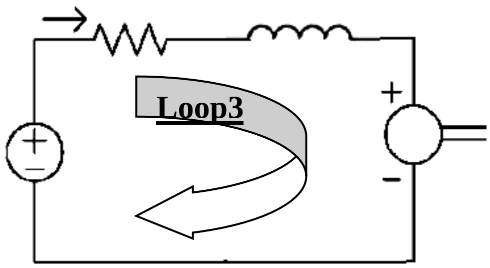
Apply KCL on the node above the capacitor,

$$i_{RA} = \frac{V_a - V_C}{R_A} = C\frac{dV_c}{dt} + i_{L1}$$
$$\rightarrow C\frac{dV_c}{dt} = \frac{V_a - V_C}{R_A} - i_{L1} \qquad \text{Eq. 2}$$

Apply KVL on Loop2,

$$V_C - L_1\frac{di_{L1}}{dt} - R_1 i_{L1} - k_{e1}\dot{\theta}_1 = 0$$

$$\rightarrow L_1 \frac{di_{L1}}{dt} = V_C - R_1 i_{L1} - k_{e1}\dot{\theta}_1 \qquad \text{Eq. 3}$$

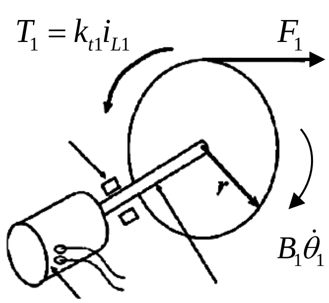


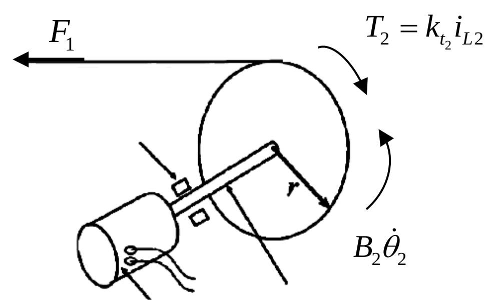
For DC motor 2, apply KVL on Loop3,

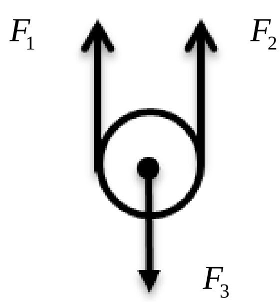
$$V_b - L_2 \frac{di_{L2}}{dt} - R_2 i_{L2} - k_{e2}\dot{\theta}_2 = 0$$
$$\rightarrow L_2 \frac{di_{L2}}{dt} = V_b - R_2 i_{L2} - k_{e2}\dot{\theta}_2 \qquad \text{Eq. 4}$$

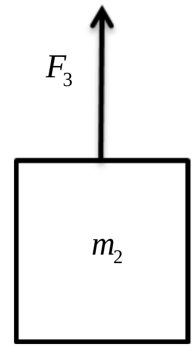
Mechanical Parts-Free Body Diagrams

The free-body diagrams are shown below:









The force F_3 can be expressed as shown in Eq. 5:

$$F_3 = k(x_A - y) + B_3(\dot{x}_A - \dot{y}) \qquad \text{Eq. 5}$$

Kinematic Constraint of the Movable Pulley

The constraint of the movable pulley is shown below:

$$r\theta_1 + r\theta_2 = 2x_A \qquad \text{Eq. 6}$$

The F_3 in Eq. 5 thus can be exrpessed as:

$$F_3 = k(x_A - y) + B_3(\dot{x}_A - \dot{y}) = k(\frac{r}{2}(\theta_1 + \theta_2) - y) + B_3(\frac{r}{2}(\dot{\theta}_1 + \dot{\theta}_2) - \dot{y}) \qquad \text{Eq. 7}$$

Equations of Motion

First, take the Euler and Newton equations of the movable pulley:

$$\sum M = 0 \rightarrow F_1 r = F_2 r \text{ , } \rightarrow F_1 = F_2 \qquad \text{Eq. 8}$$

$$\sum F = 0 \rightarrow F_1 + F_2 = F_3 \rightarrow F_1 = F_2 = \frac{F_3}{2}$$

$$\rightarrow F_1 = F_2 = \frac{F_3}{2} = \frac{k}{2}(\frac{r}{2}(\theta_1 + \theta_2) - y) + \frac{B_3}{2}(\frac{r}{2}(\dot{\theta}_1 + \dot{\theta}_2) - \dot{y}) \qquad \text{Eq. 9}$$

Take the Euler equation of Wheel 1:

$$\sum M = J_1\ddot{\theta}_1 = T_1 - B_1\dot{\theta}_1 - F_1 r$$

$$\rightarrow J_1\ddot{\theta}_1 = k_t i_{L1} - B_1\dot{\theta}_1 - \frac{kr}{2}(\frac{r}{2}(\theta_1 + \theta_2) - y) - \frac{B_3 r}{2}(\frac{r}{2}(\dot{\theta}_1 + \dot{\theta}_2) - \dot{y}) \qquad \text{Eq. 10}$$

Take the Euler equation of Wheel 2:

$$\sum M = J_2\ddot{\theta}_2 = T_2 - B_2\dot{\theta}_2 - F_2 r$$

$$\rightarrow J_2\ddot{\theta}_2 = k_t i_{L2} - B_2\dot{\theta}_2 - \frac{kr}{2}(\frac{r}{2}(\theta_1 + \theta_2) - y) - \frac{B_3 r}{2}(\frac{r}{2}(\dot{\theta}_1 + \dot{\theta}_2) - \dot{y}) \qquad \text{Eq. 11}$$

Take the Newton equation of the block:

$$\sum F = m\ddot{y} = F_3 = k(\frac{r}{2}(\theta_1 + \theta_2) - y) + B_3(\frac{r}{2}(\dot{\theta}_1 + \dot{\theta}_2) - \dot{y}) \qquad \text{Eq. 12}$$

Eq. 2, Eq. 3, Eq. 4, Eq. 10, Eq. 11 and Eq. 12 are the governing differential equations of motion of the system.