Final Exam Recitation | MEEN 364 | 12/9/16

Outline
- System type
- PID control
- Pole placement

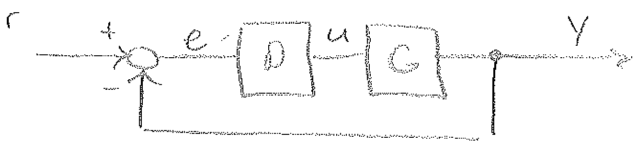
System type (F+P) refers to the degree of the polynomial that the system can reasonably track.

Eg.
A type 1 system can "reasonably" track a polynomial of degree 1, i.e.

$r(t) = t \quad , \quad R(s) = \frac{1}{s^2}$

Error constants describe e_{ss} for each type, named after a position control system

Ex. Unity Feedback w/ Generic $D(s)$ + $G(s)$



$\frac{Y}{R} = \frac{DG}{1+DG}$

1. Find $E(s)$ in terms of $R(s)$

$E(s) = R(s) - Y(s) = R - \left[\frac{DG}{1+DG}\right]R$

$= \left[1 - \frac{DG}{1+DG}\right]R = \frac{1+DG-DG}{1+DG}R = \frac{1}{1+DG}R$

$e_{ss} = \lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s) = \lim_{s\to 0} s\left[\frac{1}{1+DG}\right]R$

2. Input Degree 0 , $r(t) = 1(t) \rightarrow R(s) = \frac{1}{s}$

$e_{ss} = \lim_{s\to 0} \cancel{s}\left[\frac{1}{1+DG}\right]\frac{1}{\cancel{s}} = \frac{1}{1+K_p}$

If $DG\big|_{s=0}$ is non-zero constant (no pole at origin)

Then Type 0 ,

we call $DG\big|_{s=0} = K_p$

3. Input: Degree 1, $r(t) = t$, $R(s) = \frac{1}{s^2}$

$$e_{ss} = \lim_{s \to 0} s \left[\frac{1}{1+DG} \right] \frac{1}{s^2} = \lim_{s \to 0} \frac{1}{s + DGs} = \frac{1}{K_v}$$

If $DGs\big|_{s=0}$ is non-zero constant (ie, one pole at origin)

Then system is Type 1, we call $DGs\big|_{s=0} = K_v$

I.e., for systems of this form, the number of poles of $L(s) = D(s)G(s)$ at the origin corresponds to system type

Note: This is not applicable when sensor dynamics are present ($H(s)$). In that case

1. Find $E(s)$ in terms of $R(s)$
2. Apply FVT for poly inputs

Steady-State Error

Type	Step	Ramp	Parabola
0	$\frac{1}{1+K_p}$	∞	∞
1	0	$\frac{1}{K_v}$	∞
2	0	0	$\frac{1}{K_a}$

Motor Position Control

$$G(s) = \frac{1}{s(\tau s+1)} \qquad \tau = 0.1\ s$$

Design a controller (P and/or I and/or D) for

a) Type 2 (i.e., track a parabola (accel input) with constant error)

b) $e_{ss} < \frac{1}{5}$ for $R(s) = \frac{1}{s^3}$

c) Zeros at $s = -1 \pm j$

1. Assume PID (can always make gain zero if unnecessary)

$$D(s) = K_P + \frac{K_I}{s} + K_D s = \frac{K_P s + K_I + K_D s^2}{s}$$

2. Type ?

$$L(s) = D(s)G(s) = \frac{1}{s(\tau s+1)} \frac{K_P s + K_I + K_d s^2}{s}$$

Two poles at origin $\rightarrow$ Type 2 (Need integrator)

3. Error Constant ?

$$e_{ss} = \frac{1}{K_a} \qquad K_a = \lim_{s\to 0} D(s)G(s)s^2$$

$$K_a = \lim_{s\to 0} \frac{K_P s + K_I + K_d s^2}{\cancel{s^2}(\tau s+1)} \cancel{s^2} = K_I$$

from (b), $e_{ss} < \frac{1}{5} \rightarrow K_a > 5 \rightarrow \boxed{K_I > 5}$

4. Zeros

$$G_{cl}(s) = \frac{DG}{1+DG} = \frac{\frac{1}{s(\tau s+1)} \frac{K_P s + K_I + K_D s^2}{s}}{1 + \frac{1}{s(\tau s+1)} \frac{K_P s + K_I + K_D s^2}{s}}$$

$$G_{cl}(s) = \frac{K_D s^2 + K_P s + K_I}{s^2(\tau s + 1) + K_D s^2 + K_P s + K_I} = \frac{K_D s^2 + K_P s + K_I}{\tau s^3 + (1 + K_D)s^2 + K_P s + K_I}$$

$$b_d(s) = [s - (-1 + j)][s - (-1 - j)] = s^2 + 2s + 2$$

$$\boxed{\frac{K_P}{K_D} = 2} \qquad \boxed{\frac{K_I}{K_D} = 2}$$

Let $K_I = 6 \Longrightarrow K_D = 3,\ K_P = 6$

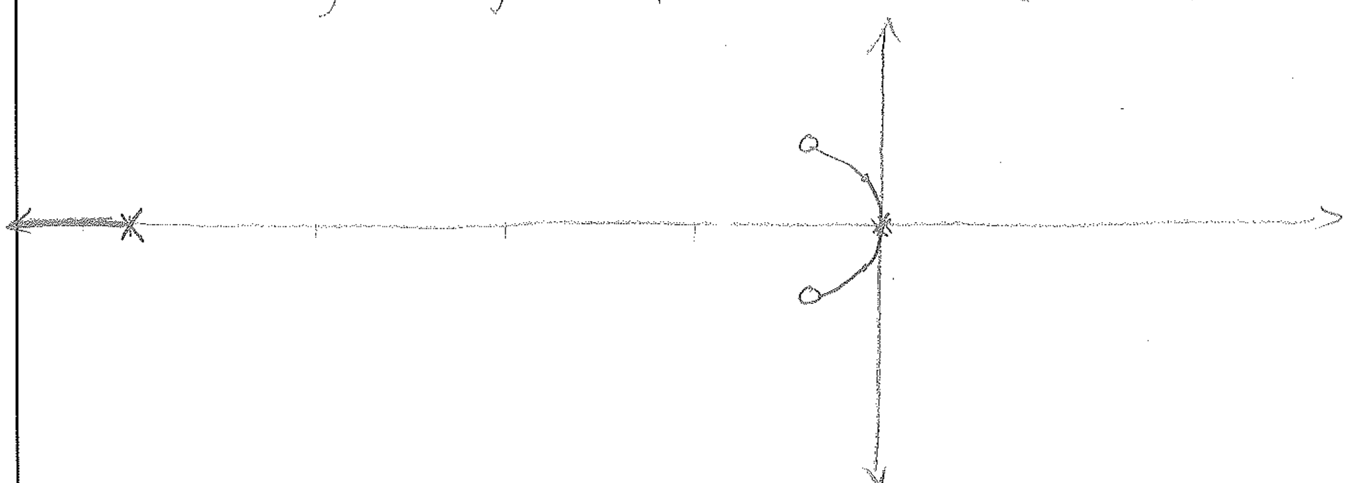
Root Locus

Complex solutions to the equation:

$$1 + KL(s) = 0$$

for K varying from $0 \rightarrow \infty$

For system above, root locus shows the result of increasing all gains by the same factor K



Z: $-1 \pm j$

P: $0,\ -10$

$\angle asy = 180^\circ$

CoA $= -8$

☆ Note: to satisfy (a) - (c), $K_I > 5$

$$\boxed{K > 0.833}$$