# 1 AMIDR: A Complete Pulse Method for Measuring Cathode Solid-State Diffusivity

# 2 **Supporting Information**

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### 14 **Proofs**

- 15 Proof S1: Start of Pulse Bound for an Ensemble of Geometries with Varying Diffusion Length
- For pulses that stop near their start, flux is uniform for both small and large geometries.
- Using equation 6 and the definitions  $Q = \Delta q_{tot} I^{-1} D_c r^{-2}$  and  $\tau = \Delta q / \Delta q_{tot}$  returns an equation
- 18 for calculating the  $\Delta q_{tot}$  for a single geometry that has stopped its pulse near its start,

$$\Delta q_{tot} = \frac{2r\sqrt{I\Delta q}}{A\sqrt{\pi D_c}}.$$
 (S1)

- 20 Accordingly, the  $\Delta q_{tot}$  for multiple geometries that have stopped their pulse near its start is
- 21 calculated as

22 
$$\Delta q_{tot} = \sum \Delta q_{tot,j} = \sum \left( \frac{2r_j \sqrt{I_j \Delta q_j}}{A \sqrt{\pi D_c}} \right) = \frac{2 \sum r_j \sqrt{I_j \Delta q_j}}{A \sqrt{\pi D_c}}.$$
 (S2)

- 23 where the j subscript is given for individual geometries. As flux is uniform near the start of a
- 24 pulse, the  $I_i$  and  $\Delta q_i$  per geometry is proportional to a geometry's  $S_i$ , and therefore a geometry's
- 25  $r_i^2$  as

$$I_j = \frac{IS_j}{\sum S_j} = \frac{Ir_j^2}{\sum r_j^2}, \text{ and}$$
 (S3)

$$\Delta q_j = I_j \Delta t = \frac{I \Delta t r_j^2}{\sum r_i^2} = \frac{\Delta q r_j^2}{\sum r_i^2}.$$
 (S4)

28 Entering these into equation S2 gives

$$\Delta q_{tot} = \frac{2\sqrt{I\Delta q}\sum r_j^3}{A\sqrt{\pi D_c}\sum r_j^2}.$$
 (S5)

- Defining  $\bar{r}_s = \sum r_j^3 / \sum r_j^2$  returns equation S1. Therefore,  $\bar{r}_s$  averaged in this manner will
- accurately define the bound for pulses that stop near the start of the pulse.
- 32 Proof S2: End of Pulse Bound for an Ensemble of Geometries with Varying Diffusion Length
- For pulses that stop near their end, flux is proportional to a geometry's  $\tilde{V}/S$ . Using
- 34 equation 7 and the definitions  $Q = \Delta q_{tot} I^{-1} D_c r^{-2}$  and  $\tau = \Delta q / \Delta q_{tot}$  returns an equation for
- 35 calculating the  $\Delta q_{tot}$  for a single geometry that has stopped its pulse near its end,

$$\Delta q_{tot} = \Delta q + \frac{Ir^2}{ABD_c}.$$
 (S6)

- 37 Accordingly, the  $\Delta q_{tot}$  for multiple geometries that have stopped their pulse near its end is
- 38 calculated as

39 
$$\Delta q_{tot} = \sum \Delta q_{tot,j} = \sum \left( \Delta q_j + \frac{I_j r_j^2}{ABD_c} \right) = \Delta q + \frac{\sum I_j r_j^2}{ABD_c}. \tag{S7}$$

- 40 where the j subscript is given for individual geometries. As flux is proportional to a geometry's
- 41  $\tilde{V}/S$  near the end of a pulse, the  $I_j$  per geometry is proportional to a geometry's  $\tilde{V}_j$ , and therefore
- 42 a geometry's  $r_i^3$  as

$$I_j = \frac{I\tilde{V}_j}{\sum \tilde{V}_j} = \frac{Ir_j^3}{\sum r_j^3}.$$
 (S8)

44 Entering this into equation S7 gives

$$\Delta q_{tot} = \Delta q + \frac{I \sum r_j^5}{ABD \sum r_j^3}.$$
 (S9)

- Defining  $\bar{r}_e = \left(\sum r_i^5 / \sum r_j^3\right)^{1/2}$  returns equation S6. Therefore,  $\bar{r}_e$  averaged in this manner will
- accurately define the bound for pulses that stop near the end of the pulse.
- 48 Proof S3: Impedance of a Thin, Low Diffusivity Surface Layer
- 49 A more complete form of equation 3 showing *X* in the interior of the geometry is

50 
$$X(\tau) = \tau + \frac{1}{AQ} \left( \frac{1}{B} + \frac{y^2}{2} - \frac{1}{2} - 2 \sum_{i=1}^{\infty} \frac{e^{-\alpha_i^2 Q \tau}}{\alpha_i^2} C(\alpha_i, y) \right), \tag{S10}$$

- where the series  $\alpha_i$  and function  $C(\alpha_i, y)$  are defined for each geometry as
- 52 Planar Sheet:  $\sin(\alpha) = 0$
- 53  $C(\alpha, y) = \frac{\cos(\alpha y)}{\cos(\alpha)},$
- 54 Cylinder:  $J_1(\alpha) = 0$ ,
- $C(\alpha, y) = \frac{J_0(\alpha y)}{J_0(\alpha)},$
- Sphere:  $\alpha \cot(\alpha) 1 = 0$ ,

57 
$$C(\alpha, y) = \frac{\sin(\alpha y)}{y \sin(\alpha)}$$

- y is the relative position along the diffusion path on a scale from 0, the center, to 1, the surface,
- and  $J_0$  is the Bessel function of the first kind of order 0.1 Taking the derivative of X in terms of y
- 60 returns

61 
$$\frac{dX(\tau)}{dy} = \frac{1}{AQ} \left( y - 2 \sum_{i=1}^{\infty} \frac{e^{-\alpha_i^2 Q \tau}}{\alpha_i^2} \frac{dC(\alpha_i, y)}{dy} \right), \tag{S11}$$

- where the series  $\alpha_i$  and function  $C(\alpha_i, y)$  is defined for each geometry as
- Planar Sheet:  $\sin(\alpha) = 0$ ,

$$\frac{dC(\alpha, y)}{dy} = -\frac{\alpha \sin(\alpha y)}{\cos(\alpha)},$$

65 Cylinder: 
$$J_1(\alpha) = 0$$
,

$$\frac{dC(\alpha, y)}{dy} = -\frac{\alpha J_1(\alpha y)}{J_0(\alpha)},$$

Sphere: 
$$\alpha \cot(\alpha) - 1 = 0$$
,

68 
$$\frac{dC(\alpha, y)}{dy} = \frac{\alpha y \cos(\alpha y) - \sin(\alpha y)}{y^2 \sin(\alpha)}.$$

Next, the derivative of X in terms of y near the surface is calculated as

70 
$$\lim_{y \to 1} \frac{dX(\tau)}{dy} = \frac{1}{AQ} \left( 1 - 2 \sum_{i=1}^{\infty} \frac{e^{-\alpha_i^2 Q \tau}}{\alpha_i^2} \lim_{y \to 1} \frac{dC(\alpha_i, y)}{dy} \right), \tag{S12}$$

71 where the series  $\alpha_i$  and function  $C(\alpha_i, y)$  is defined for each geometry as

72 Planar Sheet: 
$$\sin(\alpha) = 0$$
,

$$\lim_{y \to 1} \frac{dC(\alpha_i, y)}{dy} = -\alpha \tan(\alpha) = 0,$$

74 Cylinder: 
$$J_1(\alpha) = 0$$
,

$$\lim_{y \to 1} \frac{dC(\alpha_i, y)}{dy} = -\frac{\alpha J_1(\alpha)}{J_0(\alpha)} = 0,$$

76 Sphere: 
$$\alpha \cot(\alpha) - 1 = 0$$
,

$$\lim_{y \to 1} \frac{dC(\alpha_i, y)}{dy} = \alpha \cot(\alpha) - 1 = 0.$$

Conveniently, under these specific conditions  $\lim_{y\to 1} \frac{dC(\alpha_i, y)}{dy} = 0$  for all geometries. This shows that

79 the change in concentration over diffusion path position near the surface is always simply

$$\lim_{y \to 1} \frac{dX}{dy} = \frac{1}{AQ'} \tag{S13}$$

- for all geometries and all  $\tau$ . This equation being true for all  $\tau$  is essential for a thin, low  $D_c$  surface layer to act as a resistor independent of time.
- A thin, low  $D_c$  surface layer will generate a relative change in surface concentration,  $X_{s,l}$ , in addition to the  $X_s$  related to the bulk diffusion of the active material. As long as this surface layer is thin enough,  $X_{s,l} = \lim_{z \to 1} \frac{dX}{dy} H = \frac{H}{AQ}$  where H = h/r is the relative thickness and h is the thickness (cm) of the surface layer. This can be converted out of dimension-less convention to generate a change in surface concentration at the surface layer,  $\Delta c_{s,l}$ , as

$$\Delta c_{s,l} = \frac{Irh}{AzF\tilde{V}D_c}.$$
 (S14)

Next, this can be converted into a surface layer overpotential,  $\eta_l$ , using dE/dc of the surface layer as

$$\eta_l = -\frac{dE}{dc} \frac{Irh}{AzF\tilde{V}D_c}.$$
 (S15)

where positive I is now treated as current driving positive ions to leave the active material, the typical convention for working electrodes. Finally, this overpotential can be treated as a resistor given V = IR to give a surface layer resistance,  $R_I$ , as

$$R_l = -\frac{dE}{dc} \frac{rh}{AzF\tilde{V}D_c}.$$
 (S16)

Similar to equation 13, this is dependent on both dE/dc and  $D_c$  which will be unique to the phase of the surface layer.

# **Additional List of Symbols**

 $\Delta c$  in addition to  $\Delta c_s$  at the surface layer (mol cm<sup>-3</sup>) 99  $\Delta c_{s,l}$  $C(\alpha,z)$ 100 geometric function 101 h surface layer thickness (cm) 102 Н relative *h* 103  $I_i$  I of an individual particle (A) 104  $\Delta q$  of an individual particle (C)  $\Delta q_i$ r of an individual particle (cm) 105  $r_i$ 

106  $R_l$  surface layer  $R(\Omega)$ 

107  $S_i$  S of an individual particle (cm<sup>2</sup>)

 $\tilde{V}_j$   $\tilde{V}$  of an individual particle (cm<sup>3</sup>)

109 X relative  $\Delta c$ 

108

113

115

116

117

118

119

120

121

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123

110  $X_{s,l}$  X in addition to  $X_s$  at the surface layer

111 y relative position along diffusion path

112  $\eta_l$  surface layer  $\eta$  (V)

# **Experimental**

114 Scanning Electron Microscopy (SEM) Imaging and Particle Sizing

The SEM images were taken with ThermoFisher Scientific AxiaSEM (USA). The materials were added on a conductive carbon tape adhered to a stub for SEM imaging. A Secondary Electron detector was used in high-vacuum mode, and the images were collected at an accelerated voltage of 5 kV and a current of 12 pA.

An image containing at least 200 particles such as Figure S1a was selected for each material and the particles were given boundaries by hand with a paintbrush tool. These boundaries are then recognized by a simple image analysis software, ImageJ, and the particles individualized such as in Figure S1b and assigned an area using the scale defined by the SEM. The radii of the particles are then calculated as  $r = \sqrt{A/\pi}$  and averaged using the capacity-

weighted, geometric mean as  $\bar{r} = 10^{\frac{\sum \log r_j * r_j^3}{\sum r_j^3}}$ . Additionally,  $\bar{r}_s = \sum r_j^3 / \sum r_j^2$  and  $\bar{r}_e = 10^{\frac{\sum \log r_j * r_j^3}{\sum r_j^3}}$ .

 $(\sum r_j^5/\sum r_j^3)^{1/2}$  for calculating the start and end bounds considering diffusion length variation are also calculated to check and ensure that the particle size distribution is not too wide. Table S1 shows that all active materials analyzed by SEM had relatively similar particle sizes and limited size variation.

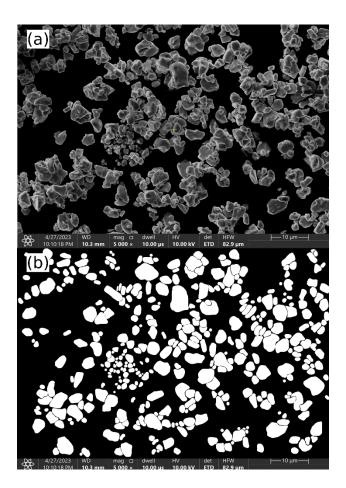


Figure S1: NMC811 example of a scanning electron microscope (SEM) image (a) before and (b) after particle individualization.

Table S1: Results of Particle Size Averaging

Material	$ar{r}$	$ar{r}_{\!\scriptscriptstyle S}$	$ar{r}_e$	Pulse Start Q-shift $(\bar{r}_{S}/\bar{r})^{2}$	Pulse End Q-shift $(\bar{r}_e/\bar{r})^2$
NMC640	0.784 μm	0.737 μm	0.871 μm	0.884	1.23

NMC811	1.736 μm	1.607 μm	1.947 μm	0.857	1.26
NM9505	1.318 μm	1.173 μm	1.596 μm	0.792	1.47

### Electrode Construction

For low mass loading electrodes, cathode active material (Zhenhua NMC640, Umicore NMC811, Zhenhua NM9505), Super-S carbon black, and polyvinylidene fluoride (PVDF) were combined in a ratio of 84:8:8 by weight. Then, N-methyl-2-pyrrolidone (NMP) was added as a solvent to form a 33% solids weight slurry after mixing in a planetary mixer (Mazerustar) for 300 s. The slurries were then coated onto Al foil using a 38  $\mu$ m notch bar upon a smooth glass plate and dried in an oven at 120 °C for 1 hour, resulting in coatings with partially visible Al foil and mass loadings of 0.6 – 2.4 mg cm<sup>-2</sup> depending on the active material used. Next, the electrode sheets were calendared at a pressure of ~2000 atm and punched into 1.275 cm diameter discs.

## 3-Electrode Coin Cell Assembly

The reference electrode, a 50  $\mu$ m diameter Au wire with a 7  $\mu$ m thick polyimide insulation (Goodfellow Cambridge Ltd., United Kingdom), was cut into lengths of 2 – 3 cm. Each end was then cut or stripped with a scalpel under an optical microscope at 50X. The exterior end was stripped on one side as shown in Figure S2a for a length of ~4 mm to later ensure contact with solder used to connect to a Ni contact tab. The interior end was cut by getting the scalpel underneath the polyimide coating as if to begin stripping and then rotated downward and cut at a 30 – 60° angle. This ensured a small interface with the electrolyte without risk of the metallic end being sealed off by the polyimide coating which can happen if just the wire is simply cut at a 90° angle. Then the middle of the wire was secured between a 2325-type coin cell casing top cap clipped to a polypropylene gasket. Next, the cap assembly, coin cell

casing bottom can, spacer, spring, and positive electrode were dried in a 110 °C vacuum antechamber overnight before being entered into a glovebox. Inside the glovebox, the coin cell was assembled upside down with the cap assembly first as show in Figure S2b. The negative electrode, a lithium metal foil, was placed in contact with the spacer before being covered by a 20 µm separator (Celgard 2300), the interior reference electrode end, 75 µl of 1.5 M LiPF<sub>6</sub> in ethylene carbonate (EC):dimethyl carbonate (DMC) electrolyte (1:1 v/v), and another separator in that order. Lastly, the cathode and can were placed on top and the cell crimped and sealed at 300 atm pressure with the exterior reference electrode end placed on top of the cap to avoid shearing of the wire. Lastly, after removing from the glovebox, the exterior reference electrode end was soldered into a folded Ni contact tab and covered and protected by dried epoxy (Loctite EA-1C) as shown in Figure S2c.

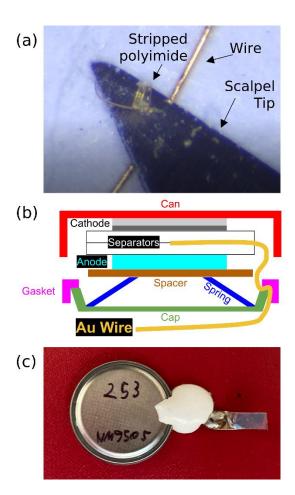


Figure S2: Custom reference electrode coin cells. (a) Stripping of the wire under optical microscope with a scalpel, (b) cross section schematic of the 3 electrode coin cell assembly, (c) appearance of a 3 electrode coin cell after epoxy has set.

## Reference Electrode Lithiation

The reference electrode was lithiated against the cathode at 150 nA for 1 hour by a BioLogic VMP-3e and allowed to relax for > 1 hour until the reference electrode's potential settled at ~0.31 V vs. Li<sup>+</sup>/Li<sup>0</sup>. Because I was so small the change in the cathode's SOC was negligible.

### Cell and Wire Shielding

Figure S3 shows the steps of the cell shielding arrangement. The BioLogic VMP-3e was selected for AMIDR testing as well and comes with wires partially shielded by a shielded cable up to ~10 cm near their ends. These wires were attached to a BioLogic CCH-1 4-point

connection coin cell holder where the coin cell was held. First, the coin cell holder and all wires except the ground wire were sealed inside a nitrile glove with the open end tied around the shielded cable and the end of the ground wire with a twist tie. The ground wire was then attached to a sheet of aluminum foil which was wrapped entirely around the nitrile glove and tightly sealed around the shielded cable. Then, the aluminum foil was sealed inside a second nitrile glove with the open end again tied around the shielded cable with a twist tie. Lastly, the cells were tested in a metallic temperature box which provided an extra layer of shielding.

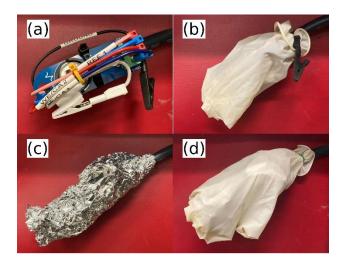


Figure S3: Shielding steps: (a) Uncovered cell holder wrapped into a tight form factor, (b) cell holder wrapped in a first layer of insulating nitrile glove with ground wire out, (c) cell holder wrapped completely with aluminum foil attached to the ground wire, (d) cell holder wrapped in a second layer of insulating nitrile glove.

# AMIDR Protocols

All cells began with a 1.5 cycle formation protocol consisting of a C/20 charge to 4.2 V, a C/20 discharge to 3.0 V, another C/20 charge to 4.2 V, and a 15 minute V hold at 4.2 V. After this, pulse protocols were applied with the parameters given by and in the order of Table S2. In between protocols are unrecorded rest steps with longer than the prescribed rest times often up to a day or two. The standard protocol was used for NMC640 and NMC811 materials. The beta protocol was used for NM9505 early in development of AMIDR, but otherwise the high

resolution protocol was used for NM9505 to get better detail to compensate for its higher variance dq/dV.

Table S2: Pulse Protocol Parameters

Test	Parameters	4.2-3.95 V	3.95-3.7 V	3.7-3.5 V	3.5-3.7 V	3.7-3.95 V	3.95-4.2 V
Standard	V Limit Spacing	25 mV		25 mV	10 mV	25 mV	
	Rate	C/	40	C/120	C/300	C/	40
	Rest Time	1 hour		4 hour	4 hour	1 hour	
Beta	V Limit Spacing	25 mV		25 mV	25 mV	25 mV	
	Rate	C/	40	C/120	C/120	C/40	
	Rest Time	1 hour		4 hour	4 hour	1 hour	
High Res	V Limit Spacing	12-13 mV	12-13 mV	12-13 mV	10 mV	12-13 mV	12-13 mV
	Rate	C/80	C/80	C/240	C/300	C/80	C/80
	Rest Time	1 hour	1 hour	4 hour	4 hour	1 hour	1 hour

The protocols used are available in the package containing the AMIDR program. These protocols can be lightly modified for different materials, but care should be taken to not disturb a few key elements that are essential to allow the AMIDR program to properly read and analyze the results. Each pulse is composed of a series of CC steps sampling every 0.001 s for 1 s, 0.01 s for 9 s, 0.1 s for 90 s, 1 s for 900s, and 10 s indefinitely. Additional steps could be added to a pulse, but it was not expected that a pulse would last longer than 10,000 s. The AMIDR program automatically averages 100 recorded datapoints together so that each step contributes 9 (or 10 for the first step) high voltage resolution datapoints for analysis. If additional datapoints are recorded, the datapoints would achieve better voltage resolution, but the recorded files would also be larger. Each rest period is composed of a long rest step and two short 0 A CC steps. When a rest step is applied, this means that the galvanometer of the cell tester is physically disconnected, whereas a 0 A CC step means that the galvanometer is connected to the cell, and 0

A of current is aimed for, but a small amount of current is still recorded. The rest step is therefore desired to prevent unexpected change in SOC during long rest periods. However, when the galvanometer is reconnected at the transition from a rest step to a CC step, it introduces a small amount of static charge to the cell which can impact the measured voltage. Therefore, two 0 A CC steps are included. One to allow the static charge to dissipate, and a second to capture a high resolution relaxed cell voltage before a pulse. The AMIDR program automatically recognizes these different steps and analyzes them accordingly. It's important to note that at the start of the beta protocol the issue with reconnection of galvanometer producing static charge was not realized and the results include unaccounted for  $\Delta V$  at the beginning of the pulses. This error was fixed at ~4.0 V during discharge, but pulses before this had somewhat erroneous  $D_c$  and very erroneous R measured.

## AMIDR Analysis Parameters

All cell data was analyzed with the single pulse AMIDR program with default settings. The pulse data analyzed was the measured potential of the active material versus the reference electrode while the results are labeled with the active material potential versus the lithium metal anode. All pulses were fit to the spherical model and used the inputs listed in Table S3 to calculate  $D_c$ , Li saturation, and  $D_t^*$ . The specific capacities of low mass loading cells are hard to measure due to small and inaccurate active mass measurements and amplified formation side reactions. Therefore, the recorded specific capacities are linearly adjusted to match the specific capacities of regular mass loading cells at certain voltages. The active material density value used to calculate  $\rho_d$  was 4.9 gcm<sup>-3</sup> for all materials as any error in this value is inconsequential on a logarithmic scale.

Table S3: Fitting and Calculation Parameters

Material	Radius	Temperature	Theoretical Fully Desaturated Capacity	Regular Loading Datapoint 1		Regular Loading Datapoint 2	
NMC640	0.784 μm	303.15 K	279 mAh/g	74 mAh/g	3.8 V	148 mAh/g	4.1 V
NMC811	1.736 μm	303.15 K	275.5 mAh/g	70 mAh/g	3.7 V	178 mAh/g	4.1 V
NM9505	1.318 μm	303.15 K	275 mAh/g	66 mAh/g	3.7 V	149 mAh/g	4.0 V

X-ray Diffraction (XRD) and Ni in Li Layer Calculation

The powder XRD spectra were obtained using a Bruker D8 diffractometer with a Cu target X-ray tube and a diffracted beam monochromatic. The spectra were collected over a scattering angle range of 15° to 70° in 0.02° steps at 3 s per step. The lattice parameters and cation-mixing rates were determined from refinements made with the Rietveld structure refinement software Rietica using the R-3m space group.

Powder XRD was accomplished primarily to determine  $Ni_{Li}$  to compare to  $D_c$  results. Figure S4 shows the spectra results and Table S4 shows the results of the Reitveld refinement.

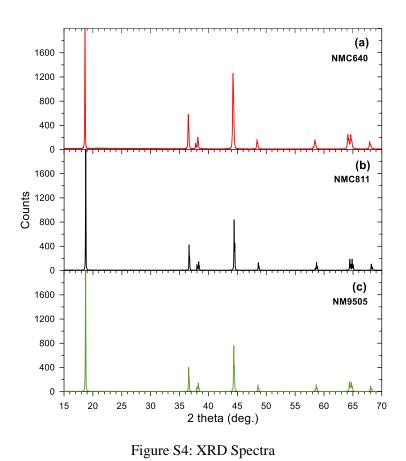


Table S4: Reitveld Refinement

Material	Lattice Parameter a	Error	Lattice Parameter s	Error	%Ni in Li layer	Error	Bragg R- Factor
NMC640	2.8817 Å	6.66E-5	14.264 Å	5.73E-4	7.45%	0.1166%	5.87
NMC811	2.8719 Å	5.78E-5	14.189 Å	2.94E-4	2.20%	0.1431%	3.02
NM9505	2.8766 Å	5.64E-5	14.190 Å	2.81E-4	1.86%	0.1183%	2.72

# **Intermediate Analysis Results**

Figure S5 shows an example of a plot that AMIDR first produces after combination of test files from individual protocols. If a test file has been forgotten to be included, it will be apparent in this plot. In addition, AMIDR also produces the plots shown in Figure S6 which show the relaxed potentials of cells undergoing AMIDR in terms of charge vs. discharge. While some cells showed better matching than others, all these cells were considered acceptable.

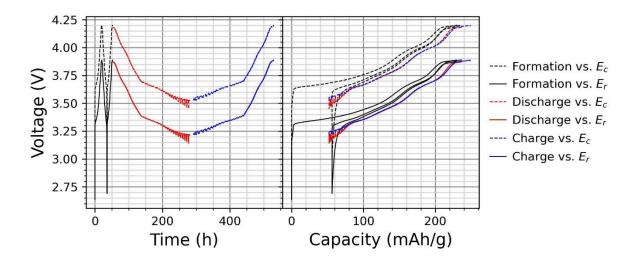


Figure S5: Example of complete test summary of a single cell.

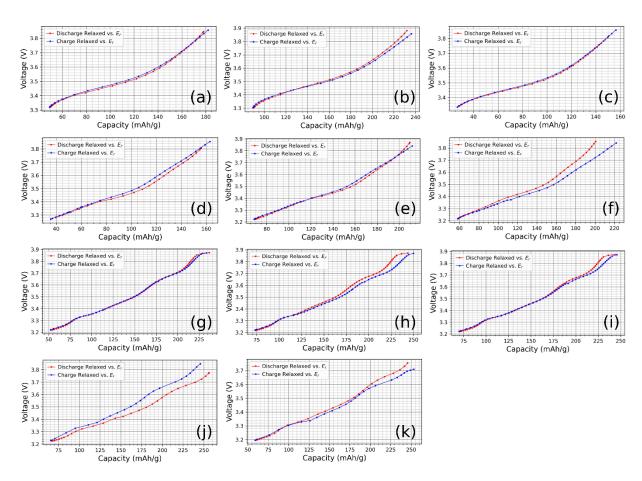
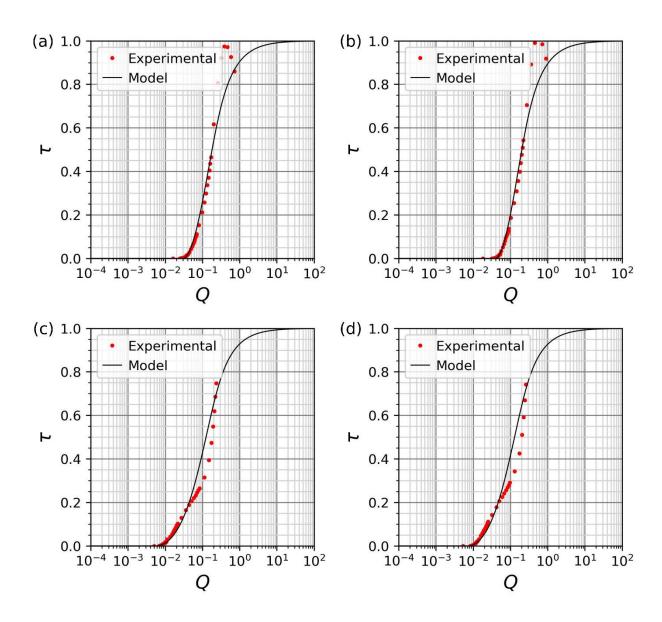


Figure S6: Matching Charge vs. Discharge relaxed potentials of (a-c) NMC640, (d-f) NMC811, (g-i) NM9505 with high resolution protocol, (j-k) NM9505 with beta protocol.

Figure S7 shows fits that were removed due to having too much change in dq/dV and Figure S8 shows fits that were removed due to having too little  $\tau$ . Out of the remaining fits, Figure S9 shows those with the worst fit error and Figure S10 shows a random sample. Some of the fits with too little  $\tau$  may look acceptable, however, they began when the active material was not completely relaxed. The worst fitting acceptable fits are not perfect, but they are close enough to give reasonable results.



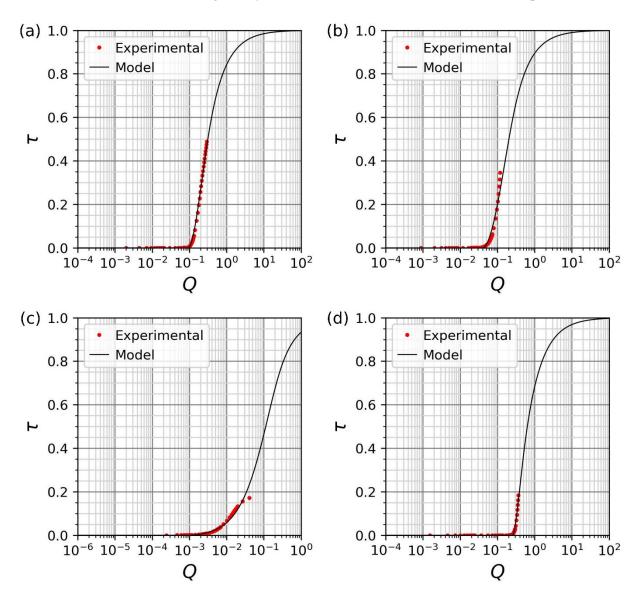


Figure S8: Examples of fits with not enough  $\tau$  in order of decreasing  $\tau$ . This occurs in (a, c) discharge and (b, d) charge. All cells show this error at low SOC.

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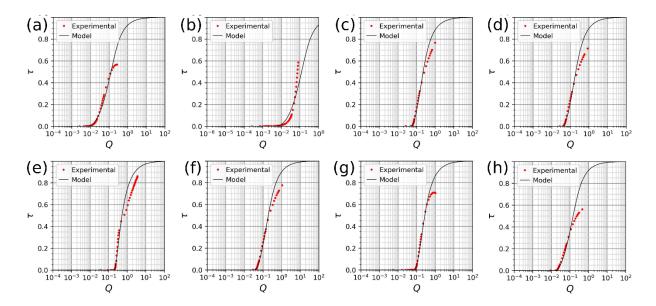


Figure S9: Examples of the highest fit error fits out of acceptable fits for (a-b) NMC640, (c-d) NMC811, (e-f) NM9505 with high resolution protocol, and (g-h) NM9505 with beta protocol. Both (a,c,e,g) discharge and (b, d, f, h) charge fits are shown.

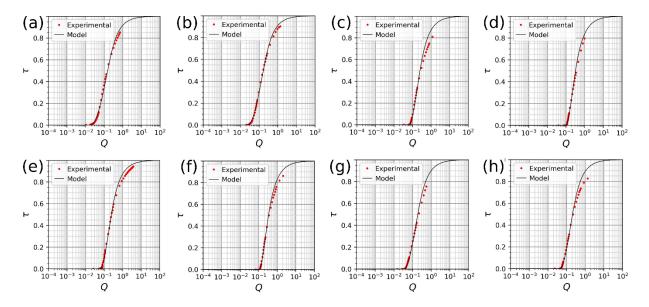


Figure S10: Examples of randomly selected, acceptable fits for (a-b) NMC640, (c-d) NMC811, (e-f) NM9505 with high resolution protocol, and (g-h) NM9505 with beta protocol. Both (a,c,e,g) discharge and (b, d, f, h) charge fits are shown.

### References

1. S. Atlung, K. West, and T. Jacobsen, *J. Electrochem. Soc.*, **126**, 1311–1321 (1979).