

1. Summation Practice

(a)

$$\sum_{k=3}^{n+1} 1 = n - 1$$

(b)

$$\sum_{i=1}^{100} (4 + 3i)$$

$$n(a_1 + \frac{d(n-1)}{2}) \left\{ \begin{array}{l} a_1 = 7 \\ n = 100 \\ d = 3 \end{array} \right\} \implies 100(7 + \frac{3(100-1)}{2}) = 15550$$

(c)

$$\begin{aligned} \sum_{i=1}^{200} (i-3)^2 &= \sum_{i=1}^{200} (i^2 - 6i + 9) \\ &= \sum_{i=1}^{200} i^2 - 6(\sum_{i=1}^{200} i) + \sum_{i=1}^{200} 9 \\ &= \frac{200(200+1)(400+1)}{6} - 6 \left\{ \frac{200(200+1)}{2} \right\} + 9(200) \\ &= 2567900 \end{aligned}$$

(d)

$$\begin{aligned} \sum_{i=10}^{80} (i^3 + i^2) &= \sum_{i=10}^{80} i^3 + \sum_{i=10}^{80} i^2 \\ &= (\sum_{i=1}^{80} i^3 - \sum_{i=1}^9 i^3) + (\sum_{i=1}^{80} i^2 - \sum_{i=1}^9 i^2) \\ &= (\frac{80^2(80+1)^2}{4} - (\frac{9^2(9+1)^2}{4})) + (\frac{80(80+1)(160+1)}{6} - \frac{9(9+1)(18+1)}{6}) \\ &= 10669170 \end{aligned}$$

(e)

$$\begin{aligned} \sum_{j=0}^{n-1} j(j+1) &= \sum_{j=0}^{n-1} j^2 + 2(\sum_{j=0}^{n-1} j) + \sum_{j=0}^{n-1} 1 \\ &= \frac{n(n-1)(2n-1)}{6} + n(n-1) + (n-1) \end{aligned}$$

(f) Create a summation for the following sequence: $2+4+8+16+32+64$

$$\sum_{i=1}^6 2^i$$

(g) Create a summation for the following sequence: $2+6+18+54+162$

$$\sum_{i=1}^5 (3^i - 3^{(i-1)})$$

(h) Create a summation for the following sequence: $(-4)+(-1)+2+5+8+11+14$

$$\sum_{j=1}^7 (3i - 7)$$

2. Order of Growth

(a)

$$\begin{aligned} & \sum_{i=2}^{n-1} \lg i^2 \\ &= 2 \left(\sum_{i=2}^{n-1} \lg(i) \right) \implies \Theta \left(2 \left(\sum_{i=2}^{n-1} \lg(i) \right) \right) = \Theta(\lg i) \end{aligned}$$

(b)

$$\begin{aligned} & \sum_{i=0}^{n-1} \sum_{j=0}^{i-1} (i+j) \\ &= \sum_{i=0}^{n-1} \sum_{j=0}^i (2i+j-1) = \frac{1}{2} \left(\sum_{i=0}^{n-1} n^2 + \dots \right) \implies \Theta(n^3) \end{aligned}$$