Q1 Part 2.6

a. The sample space is binary 0,..,7 or:

000	0
001	1
010	2
011	3
100	4
101	5
110	6
111	7

b. Out of the sample space, we see that only two options out of the total 8 are above the value of 5, therefore:

$$\frac{2\ valid\ options\ \ above}{8\ total\ options}$$
$$= \frac{2}{8} = \frac{1}{4}$$

c. Out of the sample space listed in part a, we see that there are 4 options, including 3,4,5,6 which make up half of the sample space:

$$\Pr[3 + 6] = \frac{4}{8} = \frac{1}{2}$$

Q1 Part 2.26

a. The Algebra of Events of the diagram is:

$$B \cup A \cap (C \cup D)$$

b. The probability that links C, D, or both are OK are:

$$\Pr[\sim C \cup \sim \mathrm{D}] = 0.5^2 = 0.25$$
$$\Pr[C \cup \mathrm{D}] = 1 - \Pr[\sim C \cup \sim \mathrm{D}] = 1 - 0.25 = 0.75$$

c. The probability of S communicating with R is:

$$\Pr[SR] = \{B \cup A \cap (C \cup D)\} - \{[A \cap (\mathrm{C} \cup \mathrm{D})] \cap \mathrm{B}\}$$
$$= \{0.5 + 0.5 * (0.75)\} - \{0.5 * 0.375\}$$
$$= 0.875 - 0.1875$$
$$= 0.6875$$

The reason we subtract the highlighted portion is since we would be double counting some cases if we didn't, therefore we must subtract the cases in which we do so.

d. If S fails to communicate with R, we need:

$$\Pr\,[\sim B | \sim SR]$$

We can get this using Baye's rule as:

$$\Pr[\sim SR | \sim B] = 1 - 0.375 = 0.625$$

Now:

$$\Pr[\sim B | \sim SR] = \frac{\Pr[\sim SR | \sim B] * \Pr\,[\sim \mathrm{B}]}{\Pr\,[\sim SR]}$$
$$= \frac{0.625 * 0.5}{0.3125} = 1.0$$

We see that B fails 100 % of the time when S and R fails to communicate with each other. This process can once again be done for A, C, and D and we get that A will fail 80% of the time, and finally, C and D fail 60% of the time. This can also be shown by making a truth table and finding which make the Algebra of Events equation false.

Q1 Part 2.42

a. To find p, we must first find $\Pr[1r|0s] = \Pr[Or|1s] = p$

   Using Baye's Rule:

$$\Pr[1r|0s] = \frac{\Pr[0s|1r] * \Pr\,[1r]}{\Pr\,[0s]}$$
$$= \frac{0.001 * 0.5}{0.5}$$
$$0.01 \quad = p$$

b. If we use the same formula as shown in part a,

$$\Pr[1r|0s] = \frac{\Pr[0s|1r] * \Pr\,[1r]}{\Pr\,[0s]}$$

$$p = \frac{0.001 * \Pr\,[1r]}{0.2}$$
$$\Pr[1r] = 200p$$

   Using the same formula to find $\Pr\,[0r|1s]$

$$\Pr[0r|1s] = p = \frac{0.001 * \Pr\,[0r]}{0.8}$$
$$\Pr[0r] = 800p$$

   Finally, to get p, we can: $1 = \Pr[0r] + \Pr[1r] = 1000p$

$$p = 0.001$$

c. $\Pr[0s|0r] = 1 - p = 0.999$

Question 2

When the game ends the scores are as follows:

Alice – 3
Bob – 5
Carol – 3

In order for someone to win the game, they must reach a score of 6. So, if we show every possible situation, in the format of a tree, and every time someone wins 6 we stop continuing down that path, we are able to see the probability of winning. Each time Bob wins we end that path with a blue check and since Alice and Carol have both won 3 games, we can say that both people have the same chance of winning the game. We can denote Alice or Carol winning with a black check

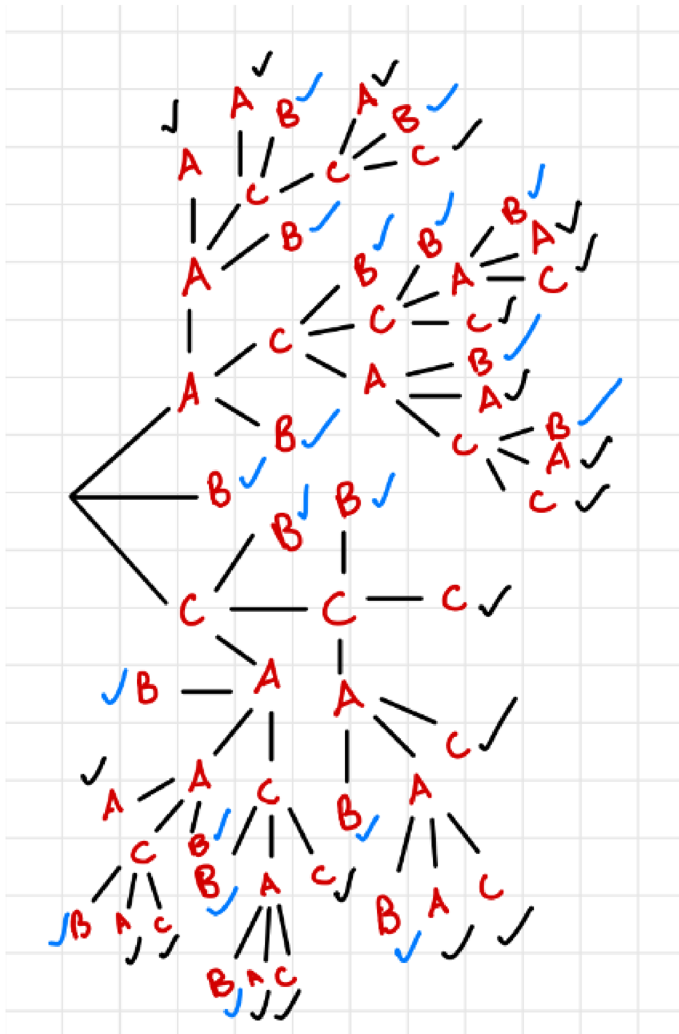

Figure 1: Tree of all possible outcomes

Each time we go deeper in the tree, we have to multiply the number of times someone wins at this move by an extra $\frac{1}{3}$

After going through each branch, we get the following equation for finding the probability of Bob winning.

$$1\left(\frac{1}{3}\right) + 2\left(\frac{1}{3}\right)^2 + 4\left(\frac{1}{3}\right)^3 + 6\left(\frac{1}{3}\right)^4 + 6\left(\frac{1}{3}\right)^5 = \frac{65}{81}$$

Therefore the probability of Bob winning the game is approximately 80.25%

Since the Alice and Carol have both won the same amount, we can say:

$$\Pr[A] = \Pr[C] = \frac{1 - \Pr\,[B]}{2} = \frac{8}{81}$$

So Alice and Carol both have approximately a 9.876% chance of winning the game.

So if there were a theoretical \$100 prize pool, \$80.25 should go to Bob, and Alice and Carol should both receive \$9.88

Question 3

a)

We first need to find the probability of being accepted in system A.
We first must denote the following terms:

$$\Pr[+] = \text{probability of acceptance}$$
$$\Pr[-] = \text{probability of rejection}$$
$$\Pr[\sim L] = \text{probability of being an intruder}$$
$$\Pr[L] = \text{probability of being a legit user}$$

Now the probability of being accepted in system A is:

$$P[+] = P[L] * 0.92 + P[\sim L] * 0.1 = 0.9118$$

And the probability oof being rejected in system A is:

$$P[-] = P[L] * 0.08 + 0.9 * P[\sim L] = 0.0882$$

Now we can determine the probability of someone being a legit user when they are accepted.

$$\Pr[L|+] = \frac{\Pr[+|L] * \Pr[L]}{\Pr[+]} = \frac{0.92 * 0.99}{0.9118} = 0.9989$$

Now toe probability of someone being a legit user and rejected:

$$\Pr[L|-] = \frac{\Pr[-|L] * \Pr[L]}{\Pr[-]} = \frac{0.08 * 0.99}{0.0882} = 0.897$$

We now repeat this same process for system B:

$$P[+] = P[L] * 0.999 + P[\sim L] * 0.20 = 0.991$$
$$P[-] = P[L] * 0.001 + 0.8 * P[\sim L] = 0.00899$$

So, the probability if someone's a legit user when they are accepted:

$$\Pr[L|+] = \frac{\Pr[+|L] * \Pr[L]}{\Pr[+]} = \frac{0.999 * 0.99}{0.991} = 0.998$$

And the probability if someone's a legit user when they are rejected:

$$\Pr[L|-] = \frac{\Pr[-|L] * \Pr[L]}{\Pr[-]} = \frac{0.001 * 0.99}{0.00899} = 0.110$$

b)

Using the same Pr[+] and Pr[-] for each system as we calculated in part a, we now must determine the probabilities if someone is an intruder.

For system A:

$$\Pr[\sim L|+] = \frac{\Pr[+|\sim L] * \Pr[\sim L]}{\Pr[+]} = \frac{0.10 * 0.01}{0.9118} = 0.0011$$

$$\Pr[\sim L|-] = \frac{\Pr[-|\sim L] * \Pr[\sim L]}{\Pr[-]} = \frac{0.90 * 0.01}{0.0882} = 0.102$$

For system B:

$$\Pr[\sim L|+] = \frac{\Pr[+|\sim L] * \Pr[\sim L]}{\Pr[+]} = \frac{0.20 * 0.01}{0.991} = 0.002$$

$$\Pr[\sim L|-] = \frac{\Pr[-|\sim L] * \Pr[\sim L]}{\Pr[-]} = \frac{0.80 * 0.01}{0.00899} = 0.8899$$

c)

It is wise if they would choose system B. This is because it has a much lower chance of rejecting a legit user compared to A. It also has a much larger chance of rejecting an intruder making it a much safer alternative.