MATH 5131 - Modeling Economic Competition and Cooperation: A Case Analysis of Apple Inc. and Logitech International S.A.

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1 Introduction

In the technology space, firms often navigate a complex interplay of cooperation and competition, even when operating in distinct niches. Apple, a global leader in consumer electronics and computer software, and Logitech, renowned for its computer peripherals, exemplify this dynamic. Despite their differing primary focuses, overlapping market interests compel these companies to occasionally cooperate or compete. For instance, Apple's drive to meet high consumer demand requires substantial investments in research and development to maintain its pace of innovation. This focus on innovation can sometimes divert attention from areas such as affordability, third-party integration, and broader accessibility. Such gaps create opportunities for smaller players like Logitech to thrive by offering value-driven alternatives and filling niches Apple may overlook. This study explores the multifaceted relationship between Apple and Logitech through the lenses of economic cooperation, monopolistic competition, and monopoly, illustrating how firms balance collaboration and rivalry in evolving markets.

2 Modeling Cooperation

$$x' = x(\epsilon_1 - \delta_1 x + \alpha_1 y)$$
$$y' = y(\epsilon_2 - \delta_2 y + \alpha_2 x)$$

Economic cooperation occurs when firms, industries, or nations collaborate to achieve mutual benefits, such as increased efficiency, market expansion, and innovation. There are many ways that firms can cooperate:

- **Joint Ventures**: Two or more firms combine resources for a shared project.

- **Complementary Products**: Firms design products that enhance each other's offerings.
- **Shared Goals**: Firms align on common objectives, such as entering new markets or reducing costs.

Now, how do we apply this system of differential equations to economic cooperation? Consider the following application of each constant:

- x – output level or market share of firm x
- y – output level or market share of firm y
- ε – market demand or potential revenue for each firm
- δ – internal costs of doing business (e.g., marginal cost, self-limiting production effects, labor costs, capital available, etc.)
- α – cooperation parameter, representing joint actions like collusion to stabilize or grow mutual revenues

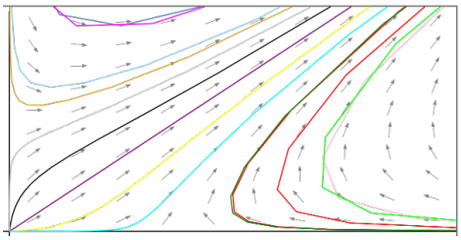

Figure 1: $X' = x(2 - 1.5x + 2.5y), Y' = y(1 - 1.5y + 2.5x)$

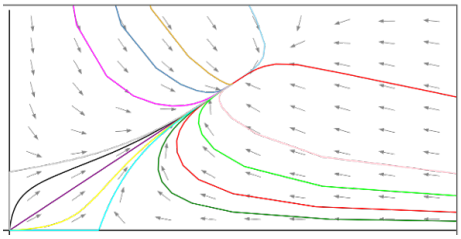

Figure 2: $X' = x(2 - 2.5x + 1.5y), Y' = y(1 - 2.5y + 1.5x)$

Analyzing the equilibrium points in the above figures reveals the behavior of firms in economic cooperation. As discussed in class, there are two sets of equilibrium points depending on the interaction between competition and cooperation coefficients. When internal competition outweighs external cooperation (Scenario 1: $\delta_1\delta_2 - \alpha_1\alpha_2 > 0$), we find an 'unstable node' at the origin, 'saddles'

on both axis, and a 'stable node' in the first quadrant. When the opposite is true (Scenario 2: $\delta_1\delta_2 - \alpha_1\alpha_2 < 0$), we find similar equilibrium points except there is no equilibrium in the first quadrant. As can be seen in Figure 1, this causes X and Y to grow without bound as we move along the solution curves.

Scenario 1 is consistent with economic intuition for cooperation. We observe a stable first quadrant equilibrium point that represents greater profit for both firm X and firm Y than they would obtain alone in the market (the equilibrium points on either axis). Scenario 2, however, is not consistent with any intuition. Profit growing without bound is unrealistic, as consumer demand represents a natural bound on revenue. This is a certainly a limitation of this model if external cooperation outweighs internal competition. However, perhaps consumer demand impacts the calculations for the competition/cooperation constants in a way that external cooperation can never outweigh internal competition. Either way, Figure 1 depicts an economically unrealistic situation.

2.1 Cooperation in Practice

Cooperation between these two firms is most evident when visiting an Apple store or browsing Apple's or Logitech's official websites. Both in-store and online, Apple prominently markets and sells a variety of Logitech computer and tablet peripherals (e.g., keyboards, mice, webcams, styli, keyboard folios, and more) alongside its own products. Some might argue that this collaboration seems counterintuitive, as it could divert revenue away from Apple's own accessories—a perspective that holds some validity. However, we believe this partnership benefits both Apple and Logitech in ways that cannot be fully captured through data alone.

Apple recognizes its challenges in appealing to consumers who prioritize affordability, such as students. For this demographic, Apple's "premium" price tags often render its products inaccessible. Moreover, while Apple and Logitech both compete in the computer peripherals market, Logitech has a broader international consumer base, whereas Apple's peripherals predominantly cater to its U.S. audience. In this segment, Apple does not hold a global lead. Logitech, by contrast, focuses primarily on computer peripherals, dedicating substantial resources to developing innovative, efficient, and user-friendly products that often outpace competitors.

By allowing Logitech to integrate its products into the Apple ecosystem, both companies expand their consumer bases and increase revenue. To address concerns that this collaboration might cannibalize Apple's own peripheral sales, we argue that excluding Logitech products would narrow Apple's consumer base, ultimately hindering both its market reach and financial growth.

3 Modeling Monopolistic Competition

$$x' = x(\beta_1 - d_1 x - c_1 y)$$
$$y' = y(\beta_2 - d_2 y - c_2 x)$$

Monopolistic competition is a type of imperfect competition where many firms sell products that are similar but not identical. This type of competition has a few defining characteristics:

- **Product Differentiation**: Each firm offers a unique product (e.g., branding, quality, features, location, etc).
- **Many Sellers**: Multiple firms competing in the market.
- **Free Entry and Exit**: Firms can enter or exit the market with relative ease, depending on profitability.
- **Non-Price Competition**: Firms compete using advertising, product quality, and branding instead of just prices.

Like in cooperation, we can apply the above system to monopolistic competition in the following way:

- x – output level or market share of firm x
- y – output level or market share of firm y
- β – market demand or potential revenue for each firm; costs, profit margin, product differentiation
- d – internal costs of doing business (e.g., marginal cost, self-limiting production effects, labor costs, capital available, etc.)
- c – external competition parameter, representing cost differentiation, economies of scale, consumer demand, etc.

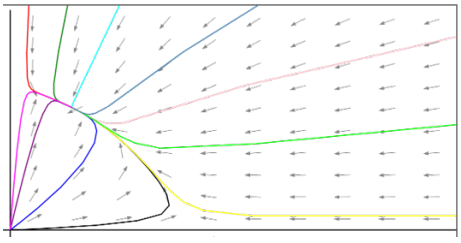

Figure 3: $X' = x(10 - 6x - 4y), Y' = y(10 - 5y - 3x)$

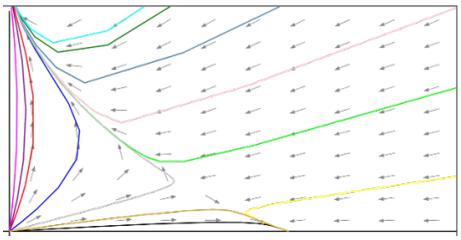

Figure 4: $X' = x(10 - 4x - 6y), Y' = y(10 - 3y - 5x)$

Analyzing the equilibrium points in the above figures reveals the behavior of firms in economic competition. Like in cooperation there are two sets of equilibrium points depending on the interaction between internal and external competition coefficients. When internal competition outweighs external competition (Scenario 1: $d_1d_2 - c_1c_2 > 0$), we find an 'unstable node' at the origin, 'saddles' on both axis, and a 'stable node' in the first quadrant. When the opposite is true (Scenario 2: $d_1d_2 - c_1c_2 < 0$), we find an 'unstable node' at the origin, a 'stable node' on either axis, and no equilibrium point in the first quadrant. Unlike in cooperation, as we move along solution curves in scenario 2, profit breakdown converges to either axis depending on starting values.

Unlike cooperation, our models have economic meaning for both scenarios. Scenario 1 represents a stable market in which two firms can share profits. This is the general form of economic competition. Scenario 2 describes a market in which two firms cannot coexist. This can be seen in economic theory with markets such as economies of scale.

3.1 Competition in Practice

Despite their collaboration, Apple and Logitech remain fierce competitors in the computer and tablet peripherals market. Both companies develop and market their own products that target overlapping consumer needs, creating direct competition in areas such as keyboards, mice, and styli. Apple, for instance, has invested heavily in developing its own ecosystem-specific accessories, such as the Magic Keyboard, Magic Mouse, and Apple Pencil, all of which are designed to integrate seamlessly with its devices. These products often appeal to Apple's loyal customer base, who value a unified brand experience, and serve as high-margin revenue generators for the company.

Logitech, meanwhile, leverages its position as a specialist in computer peripherals to challenge Apple's offerings with innovative, high-performing, and more affordable alternatives. Logitech's MX Master series mice, for example, are often regarded as superior to Apple's Magic Mouse in terms of ergonomics and features. Similarly, Logitech's keyboards and keyboard folios cater to a broader audience, including professionals and students seeking cross-platform compatibility and budget-friendly options. Logitech's ability to consistently

deliver value-driven and feature-rich products pressures Apple to enhance its peripherals to remain competitive.

Furthermore, Logitech's global reach gives it an advantage in markets where Apple struggles to compete due to its premium pricing strategy. While Apple dominates in regions like the U.S., Logitech's widespread availability and competitive pricing make it a formidable competitor internationally. This dynamic forces both companies to continuously innovate, ensuring that their product offerings remain compelling to consumers. Ultimately, the competition between Apple and Logitech drives technological advancement and gives consumers a diverse range of options tailored to varying needs and budgets.

4 Modeling Monopoly

$$x' = x(\beta_1 - d_1 x)$$
$$y' = y(\beta_2 - d_2 y - c_2 x)$$

A monopoly is a market structure in which a single firm dominates the entire market, producing and selling a unique product with no close substitutes. The firm has significant market power, allowing it to set prices and control supply without facing competition. The following are the key characteristics of this market structure:

- **Single Seller**: Only one firm supplies the entire market, making it the sole producer.
- **No Close Substitutes**: The product offered is unique, and consumers have no close alternatives.
- **High Barriers to Entry**: Significant obstacles, such as legal restrictions, high startup costs, or control of essential resources, prevent other firms from entering the market.
- **Price Maker**: The monopolist has the ability to set prices, constrained only by consumer demand.
- **Lack of Competition**: No rivals exist in the market, leading to reduced pressure for innovation or efficiency.

The application of the system to monopoly applies the same definitions of constants, but the key systematic difference is the lack of external competition for firm x.

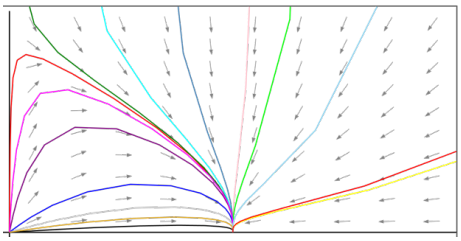

Figure 5: $X' = x(8 - 4x), Y' = y(10 - 3y - 5x)$

Analyzing the equilibrium points of this model reveals how firms interact in a monopoly. Given a monopoly for firm X, equilibrium points are an 'unstable node' at the origin, a 'saddle on the Y axis, and a 'stable node' on the X axis.

This scenario describes exactly what economics tells us happens in a monopoly. The monopolistic firm dominates the market, and barriers to entry are so high that no other firms can compete.

4.1 Monopoly in Practice

Apple's quasi-monopoly in the iPad keyboard case market is driven by its control over both the hardware and software ecosystems. The Magic Keyboard exemplifies this dominance, leveraging Apple's proprietary technology and tight integration with iPadOS to offer unparalleled functionality. Features like the floating cantilever design, precise trackpad support, and seamless compatibility make it the go-to choice for consumers seeking a premium, first-party experience.

This level of control allows Apple to dictate pricing and maintain a significant competitive edge. The high level of integration between the Magic Keyboard and iPadOS ensures a smooth user experience that third-party manufacturers, including Logitech, cannot fully replicate due to Apple's restrictions on proprietary features and APIs. Consequently, Logitech, while a significant competitor, is limited to offering alternatives like the Combo Touch, which cater primarily to budget-conscious or price-sensitive consumers.

Apple's dominance in this market creates barriers for competitors, as Logitech and others must operate within the constraints of Apple's ecosystem. In the short run, this ensures Apple can command premium prices and capture the majority of market share. Logitech, meanwhile, survives by carving out a niche, appealing to those who prioritize affordability over full integration. However, this dynamic perpetuates Apple's monopoly-like power, making it challenging for competitors to pose a significant threat or disrupt its market leadership.

5 Analysis and General Conclusions

Economic cooperation, competition, and monopoly are certainly oversimplifications of the interactions between firms in the real world. However, we believe that our models do a good job of corroborating the extremes predicted by economists. Consumer demand bounds revenue, so cooperation leads to an equilibrium that improves profits for both firms. Competition produces either a competitive equilibrium or drives all revenue to one firm. Finally, monopoly produces a market in which barriers to entry are so high that one firm dominates regardless of the starting scenario.