A Three Level Global Mean Energy Balance Model

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Abstract

1. Introduction

Because of its complexity, and the limitation of computational resources, attempts to create mathematical models of the behavior of the Earth's atmosphere are of necessity exercises in the judicious selection, exclusion and approximation of various known physical processes (Hartmann 1994). To date, the atmosphere has been characterized on the one hand as a zero-dimensional point in an energy balance model and on the other hand as an extensive three-dimensional, time varying system with many variable constituents, boundary interactions and feedbacks.

Our goal in this paper is to utilize measurements of global mean atmospheric quantities and to extract from these measurements information about how the Earth's atmosphere may be represented from the point of view of energy transfer between broad vertical layers in the system. The system we shall present is extremely simple, being essentially a three-point (or three level) model of the Earth. The model will be steady-state and we shall examine its temperature response to perturbations in the basic forcing, such as would occur through solar or greenhouse enhancement.

The motivation for describing the system as only three layers -- a surface with two atmospheric layers -- is the availability of global data which represent these three levels with fairly high precision. There are now regularly published near-global surface temperature distributions as well as measurements from Microwave Sounding Units (MSUs) which give global coverage for tropospheric and stratospheric temperatures. We have, therefore, sufficient empirical information concerning the atmospheric temperature response for model calibration.

In the simplest case, the Earth's atmosphere may be viewed as a single layer of material consisting of gases and suspended droplets and particles. The constituents of the atmosphere play important roles in the capture and transfer of energy within it and between the layer and the Earth's surface. However, rather than view the atmosphere as a single layer, we have chosen to divide it into three parts. In this way we may represent the atmosphere as having an upper level (i.e. a stratosphere), lower level (i.e. a troposphere) and a surface value. In the real Earth system, the two upper layers account for over 99% of the mass of the atmosphere and exhibit quite distinct characteristics. For example, these two layers are anticipated to undergo opposite temperature trends as greenhouse gas concentrations increase into the future (IPCC 1990) and indeed have responded in this manner in satellite and radiosonde observations (Christy 1995, IPCC 1995). We shall define the surface layer as that layer for which surface temperatures are conventionally reported, i.e. near-surface air over land and sea surface temperatures over oceans. In most representations of the atmosphere in simple energy balance models (EBMs), the troposphere and surface are considered as one because they are typically well-coupled (IPCC 1994).

We shall define the three EBM levels as 1: upper atmosphere, 2: lower atmosphere and 3: surface. Each of these levels shall be considered as a point for which there is only transmission (t, □), reflection (r, □) and absorption (a, □) of radiation (ifrared represented by English letters and solar by Greek letters):

$$a_i + t_i + r_i = 1 \quad \text{(Infrared)}$$

$$\square_i + \square + \square_i = 1 \quad \text{(Solar).}$$

The values of these parameters will be initially represented by a set of constant coefficients dependent upon type (solar or infrared) and level (1, 2 or 3). The solar and infrared radiation energies will not be subdivided into spectral bands. The resulting three-level system contains five degrees of freedom for each radiation type. The complete set of equations, accounting for all of the possible re-reflections and re-absorptions within the system, is given in the Appendix.

To calibrate the system, we begin with the energy balance of each of the levels and assume only radiative transport between the two atmospheric layers.

$$S_1 = L_1$$
$$S_2 = L_2 - HF$$
$$S_3 = L_3 + HF$$

S_i is net solar energy absorbed at level i, L_i is the net infrared emission and HF represents latent and sensible heat fluxes from the surface (level 3) to the lower atmosphere (level 2).

In our three-level energy balance model (EBM3) we shall distinguish the two atmospheric layers by properties observed in the real Earth system. For the lower atmosphere, we shall assume there are strong infrared absorbers so as to represent the effect of clouds and greenhouse gases in the Earth's troposphere. In addition, nearly all of the solar reflection due to atmospheric reflectivity will occur in this layer, again in an effort to account for the real distribution of clouds. We shall consider the upper atmosphere to be almost completely transparent to solar radiation, with some absorption to mimic the role of ozone in the stratosphere.

The quantities of energy absorbed and the manner of their transfer between components of the simple system have been outlined in several studies. We have selected results from six studies (Table 1) to serve as a guide for our constraints outlined below. In Fig. 1 we show the two results presented in Kiehl and Trenberth (1996) which represent values identified as traditional and values which are more recent (and controversial, see Stephens 1995). These latter values are derived from information which suggests the atmosphere (especially through clouds) absorbs more solar radiation than traditionally assumed (Cess et al. 1995).

a. Surface Temperature and Infrared Emissivity

Since we are attempting to learn something about the Earth, we constrain the system by insisting that the temperature of the surface (T_3) be (Dutton 1995):

$$T_3 = 287.5\text{K} \pm 0.5\text{K} .$$

As we see in Table 1, the average surface infrared emission is about 1.12Q. Given the surface temperature and quantity of energy emitted (where T = 288 K, the traditional value, and Q = 341 W m^{-2}), we calculate the emissivity constant of the surface as 0.979. This is at the top end of the typical range of surface infrared emissivities for various surface types (0.90 to 0.98, Piexoto and Oort 1992, Liou 1992) and so will not be strictly accepted. We shall use the quantity:

$$a_3 = 0.93 \pm 0.01.$$

b. Surface Albedo

In order to produce the proper absorption at the surface and reflection estimated from the top of the atmosphere we shall constrain the surface albedo ($\square_3$) as:

$$\square_3 = 0.87 \pm 0.02.$$

c. Non-radiative Heat Transfer

In keeping with the range of values for HF transfer in Table 1, we shall constrain this non-radiative quantity as:

$$HF = 0.29Q \pm 0.01Q.$$

Atmospheric Temperatures

As experimentation with the system of equations proceeded, we found it necessary to search for those solution sets which produced the coolest values of atmospheric temperature in order that they might be as representative as possible of mid-level layer temperatures to which we have access. For example, the MSU 2R (lower troposphere) reveals a global mean temperature of 269K, or the average which occurs for the 1000-400 hPa layer. The MSU 4 (lower stratosphere) global average is 211K which is average for the 100-50 hPa layer. The final result (see below Table 2) is within 3 K of the troposphere, but 22 K warmer than the lower stratosphere.

To determine the values of the remaining radiation coefficients we derived the expressions for the quantities which are represented in Table 1. This allowed further constraints on the total system and a narrowing of the range of the coefficient values. Without constrants, the model contains 10 degrees of freedom in the radiation coefficients, as well as unknown temperatures for the two atmospheric layers (though we restricted the coefficients and temperatures as described above).

With Q as the average incoming solar flux, we settled on the following values of absorption and emission which satisfied the constraints and goals of this effort (rounded to 0.001).

$$\begin{aligned}
S_1 + S_2 &= 0.201Q \\
HF &= 0.287Q \\
L_1 + L_2 &= 0.488Q \\
S_3 &= 0.499Q \\
L_3 &= 0.212Q
\end{aligned}$$

All values fall within the range listed in the previous studies (Table 1) except L_3 which is 0.002 above the largest value. This result is our optimal case as it provides the atmospheric temperatures which are closest to those in our observations, though the "stratospheric" layer is much warmer than initially anticipated. The results of this steady state solution of EBM3 will be denoted as $EBM3_o$.

The transfer of energy is indicated in Fig. 2 for $EBM3_o$ along with values from Hartmann (1994) which are separated between troposphere and stratosphere. We see excellent agreement in the quantities, the largest difference being in the about of radiation emitted directly to space from the surface. As Kiehl and Trenberth (1996) note, this is a particularly difficult value to assess because the atmosphere tends to intercept and reradiate at the same frequencies as are emitted from the surface, so there is difficulty in separating surface from atmospheric emissions.

Table 1, last column, lists the energy amounts for various components of the budget for $EBM3_o$ using the constraints above. The corresponding radiation transfer coefficients for $EBM3_o$ and, for reference, the two cases provided in Kiehl and Trenberth satisfied by versions of EBM3, are given in Table 2. For the latter two cases, we required a surface temperature of 287.5±0.5K and that all budget quantities be in agreement with those listed in Table 1 and Fig. 1 (K&T1, K&T2). Note that the atmospheric temperatures of the latter two reference cases are much warmer than for $EBM3_o$ with more realistic troposphere and stratosphere constraints.

Perturbation Experiments

With coefficients now fixed as constants, we determined the temperature response of solar forcing anomalies. For a +2% (-2%) change in Q, the system's temperature response for the three levels (surface, troposphere, stratosphere) was +1.42, (-1.45), +1.35 (-1.36) and +1.16 (-1.18) respectively. The tropospheric values are the same as those indicated in Rind and Lacis (1993) calculated for a surface from a 1-D radiative-convective model.

In a second experiment, we increased the atmospheric infrared absorption coefficients a_1 (0.0960 to 0.0977) and a_2 (0.7400 to 0.7442) to match the estimated EGE forcing of 1.92 W m^{-2} over the past 90 years (IPCC 1990). The infrared transmission coefficients were reduced accordingly. The surface, tropospheric and stratospheric responses were +0.38, +0.13 and -0.24°C respectively. The surface response was very close to the +0.36°C result of Dutton (1995) for his "no-feedback" case based on the simple Stefan-Boltzmann law:

$$\frac{\partial F}{\partial T} = 4\sigma T^3 = \frac{4F}{T}, \ \ where\ F = \sigma T^4 .$$

For doubling of greenhouse gases, the enhanced forcing has been calculated to be 4.37 W m^{-2} (IPCC 1990). In this case the increases for the $EBM3_o$ infrared absorption coefficients were 0.0960 to 0.1009 for a_1 and 0.7400 to 0.7492 for a_2. The temperature responses (surface first) were +0.87, +0.31 and -0.66°C. Dutton reports +0.81°C for T_{sfc} from the equation above.

Various model results documented in the IPCC assessments indicate that the deep-layer atmosphere will experience a greater response than that calculated from direct forcing. In other words, components of the Earth system will change as anomalous direct forcing is applied. With EBM3, for example, additional water vapor is expected to enter the atmosphere when anomalously warmed, which would impact the system as an increase in the infrared absorption coefficient a_2. This dependency of a_2 on T_2 is a feedback mechanism not accounted for in $EBM3_o$. Rather than address this particular feedback issue, we shall consider a more fundamental problem for $EBM3_o$, that of the magnitude of the tropospheric temperature response.

Magnitude of the tropospheric temperature response

The results of the perturbation experiments above indicate that when $EBM3_o$ is forced, the magnitude of the surface response is greater than that of the troposphere. However, we know from observations (Christy 1995) and model simulations that the global mean tropospheric temperature fluctuates with greater variance than that of the surface. Much of the Earth's surface is ocean and therefore subject to relatively small variations in temperature whether considering near-surface air or the material surface itself. The standard deviation (°C) of monthly (annual) global average temperatures for the surface are 0.146 (0.106) while values for MSU T_{2R} are 0.197 (0.160). In addition, many GCM simulations point to a greater tropospheric temperature response than observed at the surface (IPCC 1995).

The approach we chose to enhance the tropospheric variability to a more realistic value was to allow HF to be linearly dependent on the temperature difference between the surface and the troposphere or ΔT_{32} (= $T_3 - T_2$) . In this case we define:

$$HF = 0.287Q + \square(\Delta T_{32} - \Delta T_{3_o2_o})$$

where $\Delta T_{3_o2_o}$ (16.44°C) is the temperature difference in $EBM3_o$ and $\square$ is a constant. In this way, heat is transfered to the troposphere if the surface warms more than the atmosphere. Physically, this formulation allows for the reduction in the vertical instability produced in the perturbation experiments whenever the surface warms more that the lower atmosphere.

Table 1. Amounts of energy gained or lost from the atmosphere and earth system as a fraction of the average solar radiation striking the top of the atmosphere as estimated from five studies.

	NAS	Liou	IPCC	Schn.	K&T1	K&T2	$EBM3_o$
Solar energy absorbed in Earth System	0.70	0.70	0.70	0.70	0.687	0.687	0.700
Solar absorbed at surface	0.51	0.44	0.45	0.45	0.491	0.433	0.499
Solar absorbed in atmosphere	0.19	0.26	0.25	0.25	0.196	0.254	0.201
Solar reflected by atmosphere	0.26	0.24	0.25	0.25	0.225	0.237	0.258
Solar reflected by surface	0.04	0.06	0.05	0.05	0.088	0.076	0.042
Net heat lost from surface	0.51	0.44	0.45	0.45	0.491	0.433	0.499
Surface heat loss through latent heat fluxes	0.23	0.23	0.23	0.24	0.228	0.228	0.230
Surface heat loss through sensible heat fluxes	0.07	0.06	0.04	0.05	0.070	0.044	0.057
Surface IR emissions transmitted directly to space	0.06		0.14	0.04	0.117	0.117	0.043
Sfc IR emissions not directly transmitted to space	0.15		0.04	0.12	0.088	0.088	0.169
Net IR energy emitted from surface	0.21	0.15	0.18	0.16	0.161	0.161	0.212
Total energy emitted by atmosphere to space	0.64		0.56	0.66	0.570	0.570	0.657
IR emission from surface	1.16	1.14	1.14	1.04	1.140	1.140	1.052

NAS: National Academy of Sciences (1975), Liou: Liou (1992), IPCC: IPCC 1994, Schn.: Schneider (1990), K&T1: Kiehl and Trenberth (1996, from traditional estimates), K&T2: Trenberth and Kiehl (1996, recent, more controversial estimates), EBM3: Three level Energy Balance Model of this proposal.

Table 2. Coefficients of absorption, transmission and reflection for EBM3 where Greek letters represent shortwave and English are longwave for the three cases representing the last three columns of Table 1.

SOLAR	Kiehl & Trenberth 1	Kiehl & Trenberth 2	$EBM3_o$	INFRARED	Kiehl & Trenberth 1	Kiehl & Trenberth 2	$EBM3_o$
α_1	0.029	0.026	0.020	a_1	0.074	0.366	0.096
τ_1	0.819	0.816	0.942	t_1	0.919	0.431	0.899
ρ_1	0.152	0.158	0.038	r_1	0.007	0.203	0.005
α_2	0.162	0.226	0.171	a_2	0.577	0.762	0.740
τ_2	0.733	0.660	0.584	t_2	0.103	0.204	0.045
ρ_2	0.105	0.114	0.245	r_2	0.320	0.034	0.215
α_3	0.770	0.754	0.868	a_3	0.992	0.995	0.926
ρ_3	0.230	0.246	0.132	r_3	0.008	0.005	0.074
						w/o HF	
T_1 (K)	243.51	243.98	233.76			T_1	233.7
T_2	277.50	285.94	271.12			T_2	269.5
T_3	287.07	287.55	287.56			T_3	308.5

Appendix

The three level model utilizes Greek letters for solar transmission, reflection and absorption (τ, ρ, α) and English letters for the corresponding infrared coefficients. The direct infrared emission is $A_i = a_i \sigma T_i^4$ for level i.

<u>Level 1 (Upper Atmosphere)</u>

Shortwave absorption, S_1:

$$\alpha_1 Q\left[1 + \frac{\tau_1}{(1-\rho_1\rho_2)(1-\rho_2\rho_3) - \tau_2\rho_3\tau_2\rho_1}\{\rho_2(1-\rho_2\rho_3) + \tau_2\rho_3\tau_2\}\right]$$

Long-wave net loss, L_1:

$$2A_1 - a_1\Big[A_2 + \frac{r_2(1-r_2r_3)}{(1-r_1r_2)(1-r_2r_3) - t_2r_3t_2r_1}\Big\{A_1 + A_2r_1 + \frac{t_2r_1(A_3 + A_2r_3)}{1-r_2r_3}\Big\} +$$

$$\frac{t_2(1-r_1r_2)}{(1-r_1r_2)(1-r_2r_3) - t_2r_3t_2r_1}\Big\{\frac{t_2r_3(A_1 + A_2r_1)}{(1-r_1r_2)} + A_3 + A_2r_3\Big\}\Big]$$

<u>Level 2 (Lower Atmosphere)</u>

Short wave absorption, S_2:

$$\frac{Q\tau_1\alpha_2(1-\rho_2\rho_3 + \tau_2\rho_3)}{(1-\rho_1\rho_2)(1-\rho_2\rho_3) - \tau_2\rho_3\tau_2\rho_1}$$

Long wave net loss, L_2:

$$2A_2 - \frac{a_2}{(1-r_1r_2)(1-r_2r_3) - t_2r_3t_2r_1}\left[(A_1 + A_2r_1)(1-r_2r_3 + t_2r_3) + (A_3 + A_2r_3)(1-r_1r_2 + t_2r_1)\right]$$

<u>Level 3 (Surface)</u>

Short Wave absorption, S_3:

$$\frac{\alpha_3 Q\tau_1}{(1-\rho_1\rho_2)(1-\rho_2\rho_3) - \tau_2\rho_3\tau_2\rho_1}\left[\tau_2(1-\rho_2\rho_3) + \tau_2\rho_2\rho_3\right]$$

Long wave net loss, L_3:

$$A_3 = a_3\Big[A_2 + \frac{t_2}{(1 - r_1 r_2)(1 - r_2 r_3) - t_2 r_3 t_2 r_1}\left\{(A_1 + A_2 r_1)(1 - r_2 r_3) + t_2 r_1 (A_3 + A_2 r_3)\right\}$$
$$+ \frac{r_2}{(1 - r_1 r_2)(1 - r_2 r_3) - t_2 r_3 t_2 r_1}\left\{t_2 r_3 (A_1 + A_2 r_1) + (1 - r_1 r_2)(A_3 + A_2 r_3)\right\}\Big]$$

From the definitions above we may derive the following for Table 2:

Solar quantities:

Absorbed at surface: S_3

Absorbed in atmosphere: $S_1 + S_2$

Reflected by atmosphere:

$$Q\left[r_1 + \frac{t_1^2 r_2 (1 - r_2 r_3)}{(1 - r_1 r_2)(1 - r_2 r_3) - t_2 r_3 t_2 r_1} \right]$$

Reflected by surface:

$$Q\left[\frac{t_1^2 t_2^2 r_3}{(1 - r_1 r_2)(1 - r_2 r_3) - t_2 r_3 t_2 r_1} \right]$$

Infrared quantities:

Net heat lost from the surface: $L_3 + LH + SH$

Surface heat lost through latent fluxes: LH

Surface heat lost through sensible fluxes: SH

Surface emission transmitted directly to space:

$$\frac{t_1 t_2 a_3 \sigma T_3^4}{(1 - r_1 r_2)(1 - r_2 r_3) - t_2 r_3 t_2 r_1}$$

Surface emissions not directly transmitted to space:

L_3 - (emissions directly to space)

Total energy emitted by atmosphere to space:

$$A_1[1 + t_1(r_2(1 - r_2 r_3)) + t_2^2 r_3] + t_1 A_2 [1 + r_1 r_2 (1 - r_2 r_3) + t_2 r_3 (1 - r_1 t_2)]$$

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