Physical Climatology (AES 630) Homework 2

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1.1: Use the data in Table 2.3 to calculate emission temperatures for all planets. The actual energy emission from Jupiter corresponds to an emission temperature of about 124 K; how must you explain the difference between this temperature and the number you obtain?

With the following equation, the emission temperature of a planet can be calculated using the mean orbital distance $\bar{d}$ of the planet and its mean albedo α_p, given solar luminosity L_0.

$$T_e = \sqrt[4]{\frac{L_0(1-\alpha_p)}{16\pi\bar{d}^2\sigma}} \tag{1}$$

Planet	Distance (10^6 km)	Albedo	Emission Temp. (K)
Mercury	58	0.1	437.42
Venus	108	0.65	253.13
Earth	150	0.29	256.34
Mars	228	0.15	217.49
Jupiter	778	0.52	102.06
Saturn	1430	0.47	77.17
Uranus	2878	0.5	53.61
Neptune	4510	0.4	44.82

If the observed emission temperature of Jupiter is 124 K, then it is about 22 K warmer than expected if solar irradiance is the only source of heat. Either internal processes or non-solar flux must be responsible for the discrepancy. Jupiter is known to have an incredibly powerful magnetic dynamo, which is responsible for bright aurorae at its poles when interacting with solar and interplanetary ion "winds;" The consequent upper-atmospheric heating may contribute to the higher observed temperatures. Friction from the gravity-driven internal dynamics of Jupiter may also increase its temperature beyond the energy imparted by the Sun.

1.2:	Calculate the emission temperature of Earth if the solar luminosity is 30% less; use today's albedo and earth-sun difference.

Applying Equation 1 with $L_0' = .7\, L_0 = .7 \cdot 3.9 \times 10^{26}\,\mathrm{W} = 2.73 \times 10^{26}\,\mathrm{W}$

$$T_e = \sqrt[4]{\frac{L_0'(1-\alpha_p)}{16\pi \bar{d}^2 \sigma}} = \sqrt[4]{\frac{2.7 \times 10^{26}\,\mathrm{W}(1-.29)}{16\pi \times 150 \times 10^6\,\mathrm{km}^2 \sigma}} = 233.47\,\mathrm{K}$$

Thus if the solar luminosity were 30% of its original value, the emission temperature of Earth would decrease from 256.34 K to 234.47 K.

1.4:	Using the model in Figure 2.3, calculate the surface temperature if the insolation is absorbed in the atmosphere rather than the surface.

If all the solar insolation were absorbed in the atmosphere rather than transmitted to the surface, then the observed emission temperature of the planet would equal the emission temperature of the atmosphere, which must be the same as the solar insoaltion assuming an energy balance.

$$\frac{S_0}{4}(1-\alpha_p) = \sigma T_E^4 = \sigma T_A^4$$

As previously calculated, the emission temperature of Earth $T_E \approx 256.34\,\mathrm{K}$. Since the atmospheric layer is opaque, it is the only source of radiation towards the surface. Thus, $T_{sfc} = T_E = 256.34$ is the surface emission temperature.

1.7:	A mountain range at $45°N$ is oriented East-West and has a N/S slope faces at 15°. Calculate the insolation per unit surface area on its South and north Faces at noon on the summer and winter solstices. Compare the differences in insolation between the surfaces in each season with the seasonal variation per-face. Ignore eccentricity and atmospheric absorption.

Neglecting orbital radius anomaly and atmospheric effects, the surface insolation with latitude ϕ, declination δ, and surface slope with respect to the normal solar zenith ψ are given by the formula $Q = S_0 cos(\theta_s')$. At noon, the elevation-scaled solar zenith angle is $\theta_s' = \theta_s + \psi = \phi - \delta + \psi$. The resulting modified surface insolation at the North ($\psi = -15°$) and South ($\psi = 15°$) faces are as follows:

ϕ (deg)	δ (deg)	ψ (deg)	Q_{noon} $\mathrm{W\,m^{-2}}$
45	23.45	15	1092.54
45	-23.45	15	155.13
45	23.45	-15	1351.12
45	-23.45	-15	809.92

Figure 1: Solar insolation at Noon. Top: North face in Summer/Winter; Bottom: South face in Summer/Winter.

At the North face, the insolation at Noon during the Winter solstice is only about 14.2% of the insolation on the same surface at the Summer solstice. In contrast, the Winter solstice insolation to the South side is about 60% of the summer insolation. Thus, the North slope experiences a much greater breadth of insolation amounts between seasons. As is to be expected, the South slope insolation is higher on both solstice days, however the North Summer insolation is 80.9% the South Summer insolation, while the North Winter insolation is only 19.2% the South Winter insolation.

1.8: If Ringworld's diameter is the same as Earth's orbit, it orbits a star with the same luminosity as the sun, and it has an albedo $A = 0.3$, what is the emission temperature of the sunlit side? Consider high-conductance and low-conductance to the opposite side. Explain why the temperature is different than Earth's. What radius would the ribbon need to have in order to reproduce Earth's emission temperature?

Equation 1 assumes the ratio of a planet's exposed area to its total surface area is $R = \frac{1}{4}$. Ringworld, however, has half of its surface area fully exposed normal to the star's irradiance (the other half being the opposite side of the ribbon), so with R as the ratio of exposed to total area, Ringworld's emission temperature is modeled by...

$$T_e = \sqrt[4]{\frac{L_0 R(1-\alpha_p)}{4\pi \bar{d}^2 \sigma}} \tag{2}$$

With $\alpha_p = 0.3$ and assuming high conductance (ie equal thermal emission from both sides, so that $R = \frac{1}{2}$):

$$T_e = \sqrt[4]{\frac{L_0(.5)(1-.3)}{4\pi \cdot 1.5\times 10^{11}\,\mathrm{m}^2\sigma}} = 303.77\,\mathrm{K}$$

If ringworld were a perfect insulator, the ratio of exposed to total (emitting) surface area would be $R = 1$. Thus, the emission temperature would be...

$$T_e = \sqrt[4]{\frac{L_0(1-.3)}{4\pi \cdot 1.5\times 10^{11}\,\mathrm{m}^2\sigma}} = 361.24\,\mathrm{K}$$

Aside from a small difference in surface albedo, the emission temperature is different between the two "planets" due to the increase in surface area exposed to irradiance with respect to total emitting surface area. In other words, per unit area the average solar zenith angle at any point in time is higher. Setting the interior

$$\frac{R_e(1-\alpha_e)}{d_e^2} = \frac{R_r(1-\alpha_r)}{d_r^2} \Rightarrow \frac{.25(1-.29)}{(1.5\times 10^{11})^2} = \frac{1(1-.3)}{d_r^2}$$

$$\Rightarrow d_r = 2.9788\times 10^{11}\,\mathrm{m}$$

Thus the non-conducting Ringworld would have to orbit at around 2.9788×10^{11} m to have the same sun-facing surface emission temperature as Earth.