

Physical Climatology (AES 630) Homework 4

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4.1: If the top 100 m of ocean warms by 5°C during a 3-month summer, what is the average rate of net energy going into the ocean in W m^{-2} ? If the atmosphere warms by 20 °C over the same time, what is the rate of energy flux into the atmosphere?

$$\begin{aligned}\bar{C}_A &= c_p p_s g^{-1} = \frac{1,004 \text{ J K}^{-1} \text{ kg}^{-1} \cdot 101,320 \text{ Pa}}{9.81 \text{ m s}^{-2}} \\ &= 1.037 \times 10^7 \text{ J K}^{-1} \text{ m}^{-2}\end{aligned}\tag{1}$$

$$\begin{aligned}\bar{C}_O &= \rho_w c_w d_w = 10^3 \text{ kg m}^{-3} \cdot 4,218 \text{ J kg}^{-1} \text{ K}^{-1} \cdot d_w \\ &= d_w \cdot 4.218 \times 10^6 \text{ J K}^{-1} \text{ m}^{-2}\end{aligned}\tag{2}$$

Equations 1 and 2 model the average specific heat capacity of the entire atmosphere, and the volumetric thermal capacity of the first d_w meters in depth of the ocean.

The summer months (June, July, and August) have (30, 31, 31) days respectively, so the time period consists of $\Delta t = (30 + 31 + 31) \cdot 24 \cdot 60 \cdot 60 = 7,948,800 \text{ s}$.

$$F_A = \frac{\bar{C}_A \Delta T}{\Delta t} = \frac{1.037 \times 10^7 \text{ J K}^{-1} \text{ m}^{-2} \cdot 20 \cdot \text{K}}{7.9488 \times 10^6 \text{ s}} = 26 \text{ W m}^{-2}\tag{3}$$

$$F_O = \frac{\bar{C}_O \Delta T d_w}{\Delta t} = \frac{4.218 \times 10^6 \text{ J K}^{-1} \text{ m}^{-3} \cdot 5 \text{ K} \cdot 100 \text{ m}}{7.9488 \times 10^6 \text{ s}} = 265.32 \text{ W m}^{-2}\tag{4}$$

The mean heating contributed to the atmosphere during the 3-month season is modeled by Equation 3, and Equation 4 shows the mean heating of the ocean surface down to depth $d_w = 100 \text{ m}$. Although the above value $F_O = 265.32 \text{ W m}^{-2}$ doesn't match the textbook's reported value of $F_O = 270 \text{ W m}^{-2}$, assuming that all three months have 30 days returns the expected value of 270 W m^{-2} .

4.4:

The surface blackbody emission can be linearized with respect to reference temperature T_0 , $\sigma T_s^4 \approx \sigma T_0^4 + 4\sigma T_0^3(T_s - T_0) + \dots$. Similarly, the sensible surface cooling can be approximated $SH \approx c_p \rho C_D |u| (T_s - T_a) + \dots$. Calculate and compare longwave emission and sensible heat flux rates of change wrt constant T_s given: $T_0 = 288 \text{ K}$, $\rho = 1.2 \text{ kg m}^{-3}$, $c_p = 1004 \text{ J kg}^{-1} \text{ K}^{-1}$, $C_D = 2 \times 10^{-3}$, $|u| = 5 \text{ m s}^{-1}$.

$$\begin{aligned} \sigma T_s^4 &\approx F^{\uparrow s} := \sigma T_0^4 + 4\sigma T_0^3(T_s - T_0) + \dots \\ \frac{dF^{\uparrow s}}{dT_s} &= 4\sigma T_0^3 = 4\sigma \cdot (288 \text{ K})^3 = 5.4182 \text{ W m}^{-2} \text{ K}^{-1} \end{aligned} \quad (5)$$

$$\begin{aligned} SH &\approx c_p \rho C_D |u| (T_s - T_a) + \dots \\ \frac{dSH}{dT_s} &= c_p \rho C_D |u| = (1005 \text{ J kg}^{-1} \text{ K}^{-1} \cdot 1.2 \text{ kg m}^{-3} \cdot 2 \times 10^{-3} \cdot 5 \text{ m s}^{-1}) \\ &= 12.048 \text{ W m}^{-2} \text{ K}^{-1} \end{aligned} \quad (6)$$

Equations 5 and 6 are linearized approximations of the longwave surface emissivity and the sensible heat flux from the surface, respectively. Since both surface heat sinks are expressed in terms of only one variable, their derivative with respect to that variable is a scalar value that can be solved for directly, as shown in the equations. These results indicate that near the reference temperature $T_0 = 288 \text{ K}$, per Kelvin increase in surface temperature, the amount of heat lost to sensitive heat flux increases more than twice as fast as the amount of heat lost to longwave emission.

4.5:

Air with temperature $T_a = 27^\circ \text{C}$ moves across a dry parking lot with $|u| = 5 \text{ m s}^{-1}$. Surface solar insolation is $S^\downarrow = 600 \text{ W m}^{-2}$, and the surface downward longwave radiation is $F^\downarrow(0) = 300 \text{ W m}^{-2}$. The longwave emissivity of the surface is $\epsilon_L = 0.85$, and the shortwave albedo is $\alpha_S = 0.1$. **(1)** What is the surface temperature at radiative equilibrium? **(2)** What is the surface temperature if the asphalt is instead concrete with $\alpha_S = 0.3$ (same ϵ_L)? The air density and drag coefficient are as in problem 3. *Hint:* Linearize the blackbody emission around the air temperature and use sfc energy to show (eq in text).

$$\epsilon \sigma T_s^4 \approx \epsilon \sigma T_a^4 + 4\sigma \epsilon T_a^3 (T_s - T_a) \quad (7)$$

$$\begin{aligned} -\epsilon \sigma T_s^4 &= S_s^\downarrow (1 - \alpha_s) + (1 - \epsilon) F_s^\downarrow - c_p \rho C_D u (T_s - T_a) \\ -\epsilon \sigma T_s^4 - 4\sigma \epsilon T_a^3 (T_s - T_a) &= S_s^\downarrow (1 - \alpha_s) + (1 - \epsilon) F_s^\downarrow - c_p \rho C_D u (T_s - T_a) \\ (T_s - T_a) [c_p \rho C_D u - 4\sigma T_a^3 \epsilon] &= S_s^\downarrow (1 - \alpha_s) + \epsilon (F_s^\downarrow - \sigma T_a^4) \\ T_s &= \frac{S_s^\downarrow (1 - \alpha_s) + \epsilon (F_s^\downarrow - \sigma T_a^4)}{[c_p \rho C_D u - 4\sigma T_a^3 \epsilon]} + T_a \end{aligned} \quad (8)$$

Equation 7 shows the linearization approximation of surface emissivity in terms of surface temperature, and Equation 8 uses this approximation to solve the surface energy balance equation in terms of surface temperature T_s .

$$T_s = \frac{600 \text{ W m}^{-2}(1 - .1) + 0.85 (300 \text{ K} - \sigma(300 \text{ K})^4)}{1005 \text{ J kg}^{-1} \text{ K}^{-1} \cdot 1.2 \text{ kg m}^{-3} \cdot 2 \times 10^{-3} \cdot 5 \text{ m s}^{-1} + 4 \cdot .85\sigma \cdot (300 \text{ K})} + 300 \text{ K} \quad (9)$$

$$T_s = 323.43 \text{ K or } 50.434^\circ\text{C}$$

$$T_s = \frac{600 \text{ W m}^{-2}(1 - .3) + 0.85 (300 \text{ K} - \sigma(300 \text{ K})^4)}{1005 \text{ J kg}^{-1} \text{ K}^{-1} \cdot 1.2 \text{ kg m}^{-3} \cdot 2 \times 10^{-3} \cdot 5 \text{ m s}^{-1} + 4 \cdot .85\sigma \cdot (300 \text{ K})} + 300 \text{ K} \quad (10)$$

$$T_s = 316.48 \text{ K or } 43.48^\circ\text{C}$$

Solving for the only unknown, surface temperature, at both $\alpha_s = 0.1$ and $\alpha_s = 0.3$, we recieve the results reported in Equations 9 and 10. The surface temperature is lower when the albedo is higher, which makes sense because the surface layer is absorbing a smaller fraction of incident insolation.

4.6: (1) Do problem 4, but with a parking lot with the surrounding air at saturation, and including the effect of latent heat on the surface. Ignore the effects of surface water on the albedo. (2) Compare the surface temperature for wet and dry surfaces. (3) How would the results differ if air wasn't saturated? *Hint:* Use Equation (4.34)

$$q^*(T) := \frac{.622e_s}{P - e_s} \quad (11)$$

$$e_s(T) \approx 6.112 \exp\left(\frac{17.67(T - 273.15)}{T - 29.65}\right) \quad (12)$$

$$B_e = \frac{c_p}{L} \left(\frac{\partial q^*}{\partial T} \right) \Big|_{T=T_s} \quad (13)$$

$$SH = c_p \rho C_{DL} u (T_s - T_a) \quad (14)$$

$$\frac{\partial q^*}{\partial T} \approx q^*(T) \left(\frac{L}{R_v T^2} \right) \quad (15)$$

$$\begin{aligned} LE &= L \rho C_{DL} u (q_s - q_a) \\ &= L \rho C_{DL} u (q_s^*(T_s)(1 - \text{RH}) + \frac{c_p \text{RH}}{L B_e} (T_s - T_a)) \\ (\text{as RH} \rightarrow 1) \quad LE &= L \rho C_{DL} u \frac{c_p (T_s - T_a)}{L B_e} \end{aligned} \quad (16)$$

Equation 11 shows the saturation mixing ratio in terms of the saturation vapor pressure, which is estimated empirically in Equation 12, which was adapted from (Bolton, 1980). Equation 13 shows the equation for the equilibrium Bowen ratio.

Equations 14 and 16 show the energy equations for sensible and latent heat fluxes from the surface. Since we're considering a saturated atmosphere, the surface energy balance is: $0 = S_0^\downarrow(1 - \alpha_s) + (1 - \epsilon_s)F_s^\downarrow + \epsilon F_s^\uparrow - SH - LH$

$$T_s = \frac{S_s^\downarrow(1 - \alpha_s) + \epsilon(F_s^\downarrow - \sigma T_a^4)}{(c_p + \frac{dq^*}{dT}|_{T=T_s})\rho C_D u - 4\sigma T_a^3 \epsilon} + T_a \quad (17)$$

Substituting this to the energy balance in Equation 8 to include the additional term for sensible heat, and making the assumption that the sensible and latent drag coefficients are equal yields Equation 17

$$\begin{aligned} T_s &= \frac{600(1 - .1) + .85(300 - 300^4\sigma)}{(1005 + 0) \cdot 1.2 \cdot .002 \cdot 5 + 4\sigma \cdot .85 \cdot 300^3} + 300 \text{ K} \\ &= 323.4 \text{ K} = 50.434^\circ\text{C} \end{aligned} \quad (18)$$

Using Equation 17 and the mixing ratio approximation 15 for $T_s = 300\text{K}$ and $P_s = 1013.2\text{mb}$, and assuming a dry surface (zero mixing ratio), Equation 18 shows that the equilibrium dry-surface temperature is about 50.434°C , which is the same as the answer received in the previous question since there is no increase in the rate of of heat loss from the latent energy of evaporation.

$$\frac{\partial q^*}{\partial T} \approx q^*(T) \left(\frac{L}{R_v T^2} \right) = \frac{e_s \cdot .622 \cdot 2.5 \times 10^6}{(1013.2 - e_s)(461.5 \cdot 300^2)} = 1.3656 \text{ K}^{-1} \quad (19)$$

The rate of change in saturation mixing ratio is calculated with Equations 11, 12, and 15, which enable us to use the energy balance represented by Equation 17 to calculate the surface temperature, as shown in Equation 20.

$$\begin{aligned} T_s &= \frac{600(1 - .1) + .85(300 - 300^4\sigma)}{(1005 + 5.914 \times 10^3) \cdot 1.2 \cdot .002 \cdot 5 + 4\sigma \cdot .85 \cdot 300^3} + 300 \text{ K} \\ &= 306.95 \text{ K} = 33.948^\circ\text{C} \end{aligned} \quad (20)$$

If the air was unsaturated, the amount of evaporative cooling would increase proportionally in order to re-establish equilibrium, increasing the Bowen ratio. This has the effect of increasing the amount of latent energy, thus further decreasing the surface temperature.