

Physical Climatology (AES 630) Homework 5

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5.2: Use the bulk aerodynamic formula to calculate the evaporation rate from the ocean with $C_{DE} = 10^{-3}$, $U = 5 \text{ m s}^{-1}$ and reference temperature such that $T_r = T_s - 2^\circ\text{C}$ for reference temperature T_r and surface temperature T_s . Assume fixed air density $\rho_a = 1.2 \text{ kg m}^{-3}$. How would you evaluate the importance of relative humidity versus surface temperature in determining the evaporation rate?

$$LE = \rho L C_{DE} U_r \left((1 - RH) q^*(T_s) + RH \frac{c_p}{L} \frac{T_s - T_a}{B_e} \right) \quad (1)$$

Equation ?? expresses the expanded form of the latent energy equation using the aerodynamic drag approximation over a saturated surface, in watts. In order to determine an evaporation rate per unit area, we must determine the mass of water per unit time removed by evaporation by dividing by the latent energy of vaporization, and the water depth per square meter corresponding to this mass.

Since the density of water $\rho_w = 1,000 \text{ kg m}^{-3}$, each kilogram of evaporation over a unit area corresponds to a 1 mm depth. Thus the evaporation rate can be expressed as follows in Equation ??.

$$E = LE \cdot L_v^{-1} \cdot 86,400 \text{ s day}^{-1} \cdot 1 \text{ mm kg}^{-1} \quad (2)$$

T_s (K)	q_s^* (g kg^{-1})	RH (%)	LE (W)	E (mm day^{-1})
0	3.75	50	32.223	1.113
0	3.75	100	8.175	0.282
30	27	50	226.47	7.823
30	27	100	47.784	1.651

The evaporation rate is proportional to the latent energy release by a constant factor. Equation ?? shows that when the reference-level relative humidity is high, the magnitude of latent energy depends more on the magnitude of the gradient in saturation mixing ratio between the surface and the reference level. In contrast, when the relative humidity is low, the absolute magnitude of the saturation mixing ratio at the surface $q^*(T_s)$ dominates.

Therefore, the evaporation rate's dependence on surface temperature is modulated by the relative humidity at the reference height, such that when relative humidity is low the absolute magnitude of surface temperature contributes more, and when relative humidity is high the difference between the surface and reference level temperatures contributes more.

5.3: Calculate the Bowen ratio using the bulk aerodynamic formula, for surface temperatures 0, 15, and 30°C, relative humidity 70%, and air-water temperature difference $\Delta T = 2^\circ\text{C}$. Assume heat and moisture transfer coefficients are the same.

$$B_e := \frac{c_p}{L} \left(\frac{\partial q^*}{\partial T} \right)^{-1} \Big|_{T=T_s} \approx \frac{c_p}{L} \left(q_s^* \left(\frac{L}{R_v T_s^2} \right) \right)^{-1} \quad (3)$$

$$B_o = B_e \left(1 - \frac{q_a^* - q_a}{q_s^* - q_a} \right) \quad (4)$$

Equation ?? provides an approximation for the equilibrium Bowen ratio B_e over a saturated surface, which can be used to approximate the actual Bowen ratio B_o given an approximate reference-level mixing ratio q_a by substituting it into Equation ??.

$$e_s(T) \approx 611.2 \text{ Pa} \cdot \exp \left[\frac{17.67(T - 273.15 \text{ K})}{T - 29.65 \text{ K}} \right] \quad (5)$$

$$q^*(T, P) \approx \frac{e_s(T) \cdot .622}{P - e_s(T)} \quad (6)$$

Equation ?? is the empirical formula for saturation vapor pressure derived in (Bolton, 1980), which is used to estimate a saturation mixing ratio with Equation ??, assuming standard pressure $P = 1013.2$.

$$q_a^* \approx q^*(T_s) + \frac{\partial q^*}{\partial T} \quad (7)$$

$$q_a \approx RH \cdot q_a^* \quad (8)$$

Equations ?? and ?? are approximations for the saturation and actual mixing ratios at the reference level given a surface temperature, which are used to. By calculating B_e and estimating the mixing ratio values with Equations ??-??, I used Equation ?? to estimate the Bowen ratio at each of the temperatures, and obtained the results below.

RH	T_s	T_a	B_e	q_a^*	B_o
0.7	273.15	271.15	1.466	0.003	0.530
0.7	288.15	286.15	0.579	0.009	0.193
0.7	303.15	301.15	0.250	0.024	0.077