A Summary Of An Investigation into Holographic Entanglement Entropy

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In this study, we focused on a proposal by Ryu and Takayanagi [2] that relates to A gravity theory and a way of measuring information in a quantum system called entanglement entropy. Entanglement entropy measures how much two parts of a quantum system are correlated, and it plays a crucial role in understanding the behaviour of quantum systems. Ryu and Takayanagi proposed a formula for calculating entanglement entropy using the Anti-de Sitter(gravity theory)/Conformal Field Theory(quantum theory) (Ads/CFT) correspondence, which is a powerful tool in theoretical physics that relates gravity in a higher-dimensional space (Ads) to a quantum field theory living on its boundary (CFT). It is interesting to see how these two very different theoretical systems are related. Explaining each term in turn.

Entanglement is a unique quantum phenomenon where the properties of two or more particles become correlated so that the state of the entire system cannot be described independently of the individual states of its constituent parts. This means that the properties of one part of the system can instantly affect the properties of another part, even if they are spatially separated. Why is it important to study entanglement entropy? Entanglement entropy plays a crucial role in many areas of physics, including quantum information theory and condensed matter physics. In quantum computing, we would use this entanglement entropy to describe the amount of information that can be encoded due to the usage of entangled particles to do the computation. In condensed matter physics, we use entanglement entropy to help describe the large body systems and how they interact in a complex system that can change states. And these highly coupled systems will play a large part in our future. These quantum systems are equivalent to a theory of gravity in a hypothetical space-time called Ads and these can be described as CFTs, so that is why we call it Ads/CFT.

Ads space, also known as Anti-de Sitter space, is a mathematical construct used in theoretical physics to describe negatively curved spacetime that looks as in 1. It is a higher-dimensional space with distinct geometric properties that make it useful for studying gravity and quantum field theory. The Ads/CFT correspondence is a powerful theoretical framework that relates a gravitational theory in Ads space to a quantum field theory (CFT) living on the boundary of that space. This duality has provided profound insights into the fundamental nature of gravity and quantum field theory

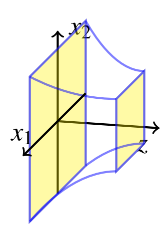

Figure 1. Ads shape of Ads spacetime

and has been a subject of intense research in recent years.

We Used Ryu and Takayanagi's Proposed (and Lewkowycz and Maldacena later proved[1]) that the entanglement entropy of the CFT is proportional to the area of the minimal area in Ads space. This is the easiest way to compute the entanglement entropy for strongly coupled systems. Even an undergraduate can do it.

We derived the case where the dimension of the $d_{cft} = 2$. during this analysis, we found the general form as expected, but we also obtained the results for the finite parts. This solution agrees with the result from the standard CFT computation methods.

We extended Ryu and Takayanagi's proposal to two different shapes in the bulk of Ads, a strip and a sphere, in d-dimensional space. To study a system with temperature, we needed to add a black hole to the Ads space, which required numerical methods. We found the shape of the surfaces to look as in figures 2 and 3. As we increased the size of the strip, it kept going along the black hole, leading it to become the thermal state of the CFT.

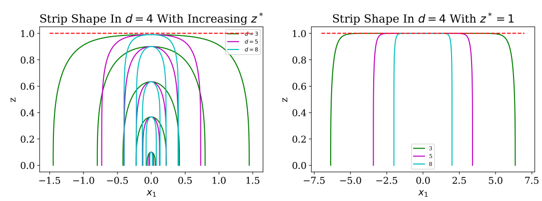

Figure 2. On the left, we have the shape of the strip with increasing strip width and on the right, the largest strip width computed, both with varying dimension

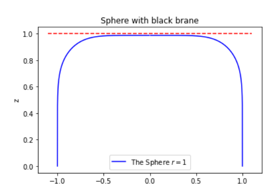

Figure 3. The minimal surface sphere with r from 0 to 1 at d=4

In conclusion, the AdS/CFT correspondence has opened up exciting possibilities for understanding quantum states and their properties, particularly in the realm of entanglement entropy. However, many important questions remain unanswered. Can AdS/CFT shed light on the scaling behaviour of entanglement entropy in different quantum states or states far from equilibrium? Can it offer insights into the role of quantum entanglement in black hole physics and the resolution of the information paradox? What is the relationship between entanglement entropy and quantum phase transitions? Further exploration of these open questions in AdS/CFT may unlock new insights into the quantum nature of the universe, with potential applications in fields ranging from condensed matter physics to black hole thermodynamics.

REFERENCES

[1] Lewkowycz, A. and Maldacena, J. (2013). Generalized gravitational entropy.

[2] Ryu, S. and Takayanagi, T. (2006). Aspects of holographic entanglement entropy. *Journal of High Energy Physics*, 2006:045–045.