Sampling

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Learning Objectives

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- Distinguish between a population and a sample, and between population parameters and sample statistics

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- Distinguish between a population and a sample, and between population parameters and sample statistics
- Describe the concepts of sampling error and sampling distribution
- Compute the standard error of the mean
- Describe how the Central Limit Theorem determines the nature of the sampling distribution of the mean

These learning objectives focus on fundamental concepts in statistical sampling that are crucial for understanding statistical inference and data analysis.

Key teaching points:

- Focus on practical understanding over mathematical formalism
- Build foundation for later statistical concepts

Learning Objectives

- Emphasize real-world applications in business and research

One of the foundational ideas in statistics is that we can make inferences about an entire population based on a relatively small sample of individuals from that population. This concept forms the basis for all statistical sampling and inference.

The chapter introduces:

- How to properly select samples from populations
- Why sampling works mathematically
- How to quantify sampling uncertainty
- The mathematical principles that make sampling reliable
- Real-world applications and examples

Students should understand that these concepts will be essential for later topics in statistics and data analysis, particularly when we discuss hypothesis testing and confidence intervals.

Introduction

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- Real-World Example: Election Polling

Introduction

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- Real-World Example: Election Polling
  - → Nate Silver's predictions:
    - 2008: Correct for 49/50 states
    - 2012: Correct for all 50 states
  - → Used only ~21,000 people to predict 125 million votes
- Why This Matters
  - → Cost-effective decision making
  - → Time-efficient research
  - → Practical business applications

The example of Nate Silver's election predictions demonstrates the power of statistical sampling. Anyone living in the United States will be familiar with the concept of sampling

Introduction

from the political polls that have become a central part of the electoral process. In some cases, these polls can be incredibly accurate at predicting the outcomes of elections.

Silver combined data from 21 different polls, which vary in the degree to which they tend to lean towards either the Republican or Democratic side. Each poll included about 1000 likely voters – meaning that Silver was able to almost perfectly predict the pattern of votes of more than 125 million voters using data from only about 21,000 people, along with other knowledge (such as how those states have voted in the past).

Key points to emphasize:

- The remarkable accuracy possible with relatively small samples
- How combining multiple samples can improve accuracy
- The importance of proper sampling methods
- Cost and time benefits of sampling
- How different polls may have different biases

Introduction

- The value of combining multiple data sources
- The role of historical data and context

Teaching tips:

- Ask students to think about other situations where we make inferences about large populations from small samples (e.g., quality control in manufacturing, customer satisfaction surveys)
- Discuss how polling has evolved with technology
- Consider why some polls fail to predict accurately
- Explore the role of sample size in prediction accuracy

Population vs Sample: Key Terms

Our goal in sampling is to determine the value of a statistic for an entire population using just a small subset:

- Population

Population vs Sample: Key Terms

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- Population
  - → Entire group of interest
  - → Often too large or impractical to measure completely
  - → Described by parameters (usually unknown)
  - → Example: all registered voters in a region
- Sample

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  - → Described by parameters (usually unknown)
  - → Example: all registered voters in a region
- Sample
  - → Subset used to make inferences
  - → Must be representative
  - → Described by statistics we can calculate
  - → Example: 1000 voters in a poll
- Real-world Examples

Population vs Sample: Key Terms

- → Customer satisfaction surveys
- → Quality control in manufacturing
- → Market research studies
- → Medical trials
- → Environmental monitoring

These fundamental concepts form the basis for statistical inference. Our goal in sampling is to determine the value of a statistic for an entire population of interest, using just a small subset of the population. We do this primarily to save time and effort – why go to the trouble of measuring every individual in the population when just a small sample is sufficient to accurately estimate the statistic of interest?

The distinction between population and sample is crucial for understanding statistical methods:

Population:

Population vs Sample: Key Terms

- The entire group about which we want to make inferences
- Often too large or impractical to measure completely
- Described by parameters that are usually unknown
- Examples: all registered voters, all customers, all products manufactured

Sample:

- A subset of the population used to make inferences
- Must be representative of the population
- Described by statistics we can calculate
- Examples: 1000 voters in a poll, 100 customers surveyed, 50 products tested

Key teaching points:

- Population parameters vs sample statistics
- Why we sample instead of measuring entire populations

Population vs Sample: Key Terms

- Importance of representative samples
- Cost and practicality considerations
- The relationship between sample and population values

Common misconceptions to address:

- Bigger samples aren't always better (diminishing returns)
- A sample can be large but still biased
- Parameters are usually unknown in real situations
- The difference between parameters (population) and statistics (sample)
- Why we can't usually know the true population values

Teaching tip: Use concrete examples from students' experience to illustrate these concepts. For instance, discuss how Netflix might use a sample of users to test new features, or how manufacturers use quality control sampling.

Representative Sampling

The way we select our sample is critical:

- Key Principle

Representative Sampling

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- Key Principle
  - → Every member of population should have equal chance of selection
  - → Results should reflect population characteristics
  - → Goal is to avoid systematic differences from population
- Common Biases to Avoid

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  - → Results should reflect population characteristics
  - → Goal is to avoid systematic differences from population
- Common Biases to Avoid
  - → Selection bias (e.g., only Democratic party names)
  - → Voluntary response bias (only those who choose to respond)
  - → Convenience sampling (easily accessible subjects)
  - → Time-of-day bias (e.g., surveys only during business hours)
- Impact of Bias
  - → Systematic errors in estimates
  - → Misleading conclusions

Representative Sampling

- → Poor business decisions
- → Wasted resources
- → Invalid research findings

Representative sampling is crucial for valid statistical inference. The way in which we select the sample is critical to ensuring that the sample is representative of the entire population, which is a main goal of statistical sampling.

A biased sample can lead to incorrect conclusions regardless of its size. For example, if a pollster only called individuals whose names they had received from the local Democratic party, then it would be unlikely that the results of the poll would be representative of the population as a whole.

In general, we would define a representative sample as being one in which every member of the population has an equal chance of being selected. When this fails, then we have to worry about whether the statistic that we compute on the sample is biased - that is, whether its

Representative Sampling

value is systematically different from the population value (which we refer to as a parameter).

Examples of bias:

- Political polls using only one party's contact list
- Customer surveys only during business hours
- Student research using only their friends
- Online surveys that only reach certain demographics
- Quality control testing only during day shift

Practical implications:

- Need for random selection methods
- Importance of considering all population segments
- Methods to minimize bias

Representative Sampling

- Cost of proper sampling procedures
- Impact of bias on business decisions

Keep in mind that we generally don't know the population parameter, because if we did then we wouldn't need to sample! But we will use examples where we have access to the entire population, in order to explain some of the key ideas.

Teaching tips:

- Have students identify potential sources of bias in real-world examples
- Discuss methods to reduce or eliminate bias
- Consider cost-benefit tradeoffs in sampling design
- Explore how technology can help or hinder representative sampling

Sampling Methods

Two fundamental approaches to sampling:

- With Replacement

Sampling Methods

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Two fundamental approaches to sampling:

- With Replacement
  - → Item returned to population after sampling
  - → Can be selected again
  - → Used in bootstrapping and simulation
  - → Maintains independence between selections
  - → Important for theoretical studies
- Without Replacement

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- With Replacement
  - → Item returned to population after sampling
  - → Can be selected again
  - → Used in bootstrapping and simulation
  - → Maintains independence between selections
  - → Important for theoretical studies
- Without Replacement
  - → Item removed after sampling
  - → Cannot be selected again
  - → More common in practice
  - → More efficient for estimation
  - → Reflects real-world constraints

Sampling Methods

- When to Use Each
  - → With replacement: theoretical studies, bootstrapping
  - → Without replacement: most real-world sampling
  - → Choice affects probability calculations
  - → Impacts statistical properties

The choice between sampling with or without replacement depends on the specific context and goals of the study. It's important to distinguish between these two different ways of sampling:

Sampling with replacement:

- After a member of the population has been sampled, they are put back into the pool
- Can be selected again in subsequent draws
- Used in theoretical studies and certain statistical techniques
- Important for bootstrapping and simulation methods

Sampling Methods

- Allows for independence between draws

Sampling without replacement:

- Once a member has been sampled they are not eligible to be sampled again
- More common in practical applications
- Reflects real-world constraints
- More efficient for population estimation
- Leads to dependent samples

Key points:

- Most real-world sampling is without replacement
- With replacement is important for certain statistical techniques
- Each method has specific mathematical properties
- Connection to later topics like bootstrapping

Sampling Methods

- Impact on probability calculations
- Effect on sample independence

Examples to discuss:

- Quality control sampling (typically without replacement)
- Customer surveys (without replacement)
- Population studies (without replacement)
- Computer simulations (often with replacement)
- Bootstrap resampling (with replacement)
- Theoretical probability studies (with replacement)

We will see sampling with replacement again when we discuss bootstrapping in later chapters, where it plays a crucial role in statistical inference.

Teaching tips:

Sampling Methods

- Use physical demonstrations (e.g., drawing cards)
- Compare efficiency of methods
- Discuss when each method is appropriate
- Explore impact on sample size requirements

Sampling Error

Even with perfect sampling methods:

- Definition

Sampling Error

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Sampling Error

Even with perfect sampling methods:

- Definition
  - → Difference between sample statistic and population parameter
  - → Present in all samples
  - → Can be estimated but not eliminated
  - → Natural result of using subset of population
- Why It Matters

Sampling Error

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Even with perfect sampling methods:

- Definition
  - → Difference between sample statistic and population parameter
  - → Present in all samples
  - → Can be estimated but not eliminated
  - → Natural result of using subset of population
- Why It Matters
  - → Affects confidence in results
  - → Influences decision making
  - → Determines required sample size
  - → Impacts research costs
- Real-world Implications
  - → Market research confidence levels

Sampling Error

- → Quality control tolerances
- → Risk assessment accuracy
- → Medical trial reliability
- → Survey result interpretation

Sampling error is inevitable in any sample, but understanding it helps us make better decisions. Regardless of how representative our sample is, it's likely that the statistic we compute from the sample is going to differ at least slightly from the population parameter.

Sampling error is directly related to the quality of our measurement of the population. If we take multiple samples, the value of our statistical estimate will also vary from sample to sample; we refer to this distribution of our statistic across samples as the sampling distribution.

Key concepts:

Sampling Error

- Difference between bias and random error
- How sample size affects sampling error
- Relationship to confidence intervals
- Impact on business decisions
- Nature of sampling variability
- Role of random chance

Clearly we want the estimates obtained from our sample to be as close as possible to the true value of the population parameter. However, even if our statistic is unbiased (that is, we expect it to have the same value as the population parameter), the value for any particular estimate will differ from the population value, and those differences will be greater when the sampling error is greater. Thus, reducing sampling error is an important step towards better measurement.

Examples to discuss:

Sampling Error

- Political polling margins of error
- Quality control tolerances
- Market research uncertainty
- Medical study variation
- Environmental sampling
- Economic indicators

Teaching tips:

- Use simulation to demonstrate sampling variability
- Compare different sample sizes
- Discuss cost-benefit of reducing error
- Connect to real-world decision making
- Explore implications for research design

NHANES Example: Height Data

To demonstrate sampling concepts, we'll use the NHANES dataset:

- NHANES = National Health and Nutrition Examination Survey
- Contains comprehensive health data from US population
- We'll treat this dataset as our "entire population"
- Looking specifically at adult height measurements
- Known population mean: `I(mean(NHANES_adult$Height))` cm
- Known population SD: `I(sd(NHANES_adult$Height))` cm

Important Note: In real life, we rarely know the true population parameters. We're using NHANES as the population here just to demonstrate sampling concepts.

Let's look at multiple samples of 50 individuals each:

```
# create a NHANES dataset without duplicated IDs
NHANES <- NHANES %>%
```

NHANES Example: Height Data

```
  distinct(ID, .keep_all = TRUE)

# create a dataset of only adults
NHANES_adult <- NHANES %>%
  filter(
    !is.na(Height),
    Age >= 18
  )

# sample 50 individuals from NHANES dataset
sample_df <- data.frame(sampnum = seq(5), sampleMean = 0, sampleSD = 0)

for (i in 1:5) {
  exampleSample <- NHANES_adult %>%
    sample_n(50) %>%
```

NHANES Example: Height Data

```
    pull(Height)
  sample_df$sampleMean[i] <- mean(exampleSample)
  sample_df$sampleSD[i] <- sd(exampleSample)
}
sample_df <- sample_df %>%
  dplyr::select(-sampnum)
kable(
  sample_df,
  caption = "Example means and standard deviations for several samples of
Height variable from NHANES."
)
```

sampleMean	sampleSD
170.734	8.715518
166.480	10.079095

NHANES Example: Height Data

sampleMean	sampleSD
170.528	9.252050
168.302	9.822267
168.228	10.397351

This example demonstrates how sample statistics vary from sample to sample using the NHANES dataset as our population. We are going to assume that the NHANES dataset is the entire population of interest, and then draw random samples from this population.

In this example, we know the adult population mean and standard deviation for height because we are assuming that the NHANES dataset is the population. The table shows statistics computed from several samples of 50 individuals from the NHANES population.

Key points to discuss:

- Each sample gives slightly different results

NHANES Example: Height Data

- Variation is expected and natural
- Larger samples tend to be more stable
- Importance of sample size
- Relationship between sample and population values
- Role of random chance in sampling

The sample mean and standard deviation are similar but not exactly equal to the population values. This demonstrates the fundamental nature of sampling - we get close to the true value, but with some variation due to random chance.

Teaching tips:

- Ask students to predict how results might change with different sample sizes
- Discuss why we see variation between samples
- Compare sample values to population parameters
- Consider practical implications of sampling variation

NHANES Example: Height Data

- Explore how this relates to real-world sampling situations

Note that we will have more to say in the next chapter about exactly how the generation of "random" samples works in a computer.

Sampling Distribution Visualization

To understand sampling distributions, we'll:

- Take 5000 different samples of 50 individuals each
- Compute mean height for each sample
- Compare distribution of sample means to population

Key Features:

- Blue histogram: Distribution of 5000 sample means
- Gray histogram: Original population distribution
- Vertical line: True population mean (168.3497 cm)
- Sample means cluster around true population mean
- Sampling distribution is narrower than population
- Shape approximates normal distribution (preview of CLT)

Sampling Distribution Visualization

```r
# compute sample means across 5000 samples from NHANES data
sampSize <- 50 # size of sample
nsamps <- 5000 # number of samples we will take

# set up variable to store all of the results
sampMeans <- array(NA, nsamps)

# Loop through and repeatedly sample and compute the mean
for (i in 1:nsamps) {
  NHANES_sample <- sample_n(NHANES_adult, sampSize)
  sampMeans[i] <- mean(NHANES_sample$Height)
}

sampMeans_df <- tibble(sampMeans = sampMeans)
```

Sampling Distribution Visualization

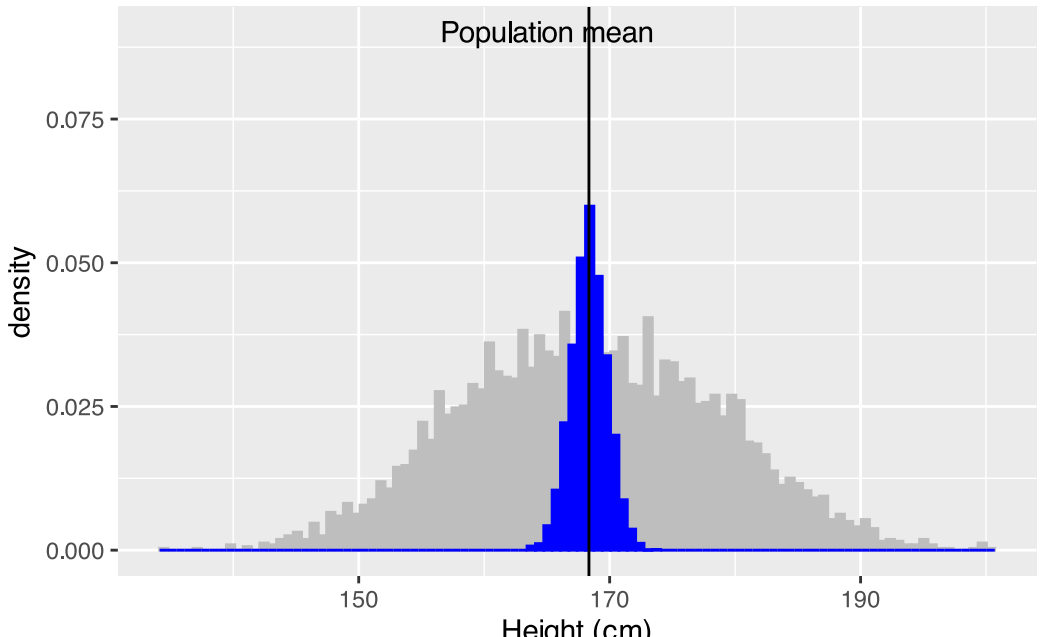
```
sampMeans_df %>%
  ggplot(aes(sampMeans)) +
  geom_histogram(
    data = NHANES_adult,
    aes(Height, ..density..),
    bins = 100,
    col = "gray",
    fill = "gray"
  ) +
  geom_histogram(
    aes(y = ..density.. * 0.2),
    bins = 100,
    col = "blue",
    fill = "blue"
  ) +
```

Sampling Distribution Visualization

```
  geom_vline(xintercept = mean(NHANES_adult$Height)) +
  annotate(
    "text",
    x = 165,
    y = .09,
    label = "Population mean"
  ) +
  labs(
    x = "Height (cm)"
  )
```

```
Warning: The dot-dot notation (`..density..`) was deprecated in ggplot2
3.4.0.
ℹ Please use `after_stat(density)` instead.
```

Sampling Distribution Visualization



This visualization shows the results of taking a large number of samples of 50 individuals, computing the mean for each sample, and looking at the resulting sampling distribution of means.

Sampling Distribution Visualization

Components of the visualization:

- Blue histogram: sampling distribution of means
- Gray histogram: original population distribution
- Vertical line: true population mean

We have to decide how many samples to take in order to do a good job of estimating the sampling distribution – in this case we take 5000 samples so that we are very confident in the answer. Note that simulations like this one can sometimes take a few minutes to run, and might make your computer huff and puff.

Key points:

- Sampling distribution centers on population mean
- Shape is approximately normal (preview of CLT)
- Spread is smaller than population distribution

Sampling Distribution Visualization

- Demonstrates law of large numbers
- Shows convergence to true population value

The histogram shows that the means estimated for each of the samples of 50 individuals vary somewhat, but that overall they are centered around the population mean. The average of the 5000 sample means is very close to the true population mean, demonstrating how sampling works in practice.

Teaching tips:

- Explain why we need so many samples
- Discuss the relationship between sample and population distributions
- Connect to real-world sampling situations
- Consider what happens with different sample sizes
- Explore why the sampling distribution is narrower than the population distribution

Standard Error of the Mean

The standard error of the mean (SEM) measures the precision of our sample mean. For our NHANES height example:

- Population SEM = 1.44
- Observed SD of sample means = 1.42
- These values are very close, demonstrating how SEM works

The formula and its implications:

- Formula: SEM = frac{hat{sigma}}{sqrt{n}}

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The formula and its implications:

- Formula: SEM = frac{hat{sigma}}{sqrt{n}}
- Components
  - → $\hat{\sigma}$ = estimated standard deviation (population variability)
  - → n = sample size (under our control)
  - → Square root relationship is crucial
  - → Smaller SEM = better precision
  - → Larger n = smaller SEM

Standard Error of the Mean

- Key Points
  - → Measures precision of sample mean
  - → Critical for sample size planning
  - → Shows diminishing returns with larger samples
  - → Helps balance precision vs cost
  - → Essential for statistical inference

The standard error of the mean (SEM) is a fundamental concept in statistical inference. Later in the course it will become essential to be able to characterize how variable our samples are, in order to make inferences about the sample statistics.

The SEM can be thought of as the standard deviation of the sampling distribution of the mean. To compute it, we divide the estimated standard deviation by the square root of the sample size:

$$\text{SEM} = \hat{\sigma}/\sqrt{n}$$

Standard Error of the Mean

Important aspects:

- Relationship between sample size and precision
- Why we use estimated standard deviation
- Connection to confidence intervals
- Role in hypothesis testing
- Mathematical basis for inference
- Limitations and assumptions

Note that we have to be careful about computing SEM using the estimated standard deviation if our sample is small (less than about 30).

The formula for the standard error of the mean implies that the quality of our measurement involves two quantities:

1. The population variability ($\hat{\sigma}$)

Standard Error of the Mean

2. The size of our sample (n)

Because the sample size is the denominator in the formula for SEM, a larger sample size will yield a smaller SEM when holding the population variability constant. We have no control over the population variability, but we do have control over the sample size.

Practical applications:

- Determining required sample sizes
- Assessing measurement precision
- Planning research studies
- Quality control limits
- Survey design
- Clinical trials

Teaching tips:

Standard Error of the Mean

- Work through the formula components
- Demonstrate effect of changing n
- Connect to real-world applications
- Discuss practical limitations
- Consider cost-benefit tradeoffs

Small Sample Considerations

Special attention needed when $n < 30$:

- Statistical Issues

Small Sample Considerations

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Small Sample Considerations

Special attention needed when $n < 30$:

- Statistical Issues
  - $\longrightarrow$ Less reliable estimates
  - $\longrightarrow$ Normal approximations may not hold
  - $\longrightarrow$ Standard errors less trustworthy
  - $\longrightarrow$ Greater chance of misleading results
- Practical Solutions

Small Sample Considerations

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Small Sample Considerations

Special attention needed when $n < 30$:

- Statistical Issues
  - → Less reliable estimates
  - → Normal approximations may not hold
  - → Standard errors less trustworthy
  - → Greater chance of misleading results
- Practical Solutions
  - → Increase sample size if possible
  - → Use appropriate statistical methods
  - → Report limitations clearly
  - → Consider alternative approaches
  - → Be more conservative in conclusions
- Impact on Decision Making

Small Sample Considerations

- → Greater uncertainty in results
- → Need for larger margins of error
- → More conservative conclusions
- → Higher risk of incorrect decisions
- → May need additional validation

Small samples require special consideration in statistical analysis. The cutoff of n=30 is commonly used because it relates to the Central Limit Theorem and the reliability of our statistical estimates.

Key points: - Why n=30 is often used as a cutoff - Relationship to Central Limit Theorem - When small samples are unavoidable - Methods for handling small samples - Impact on statistical inference - Increased uncertainty in estimates

Small Sample Considerations

When working with small samples: - Standard errors are larger - Normal approximations may not hold - Need for alternative statistical methods - Greater chance of misleading results - More conservative conclusions needed - Careful interpretation required

Examples to discuss: - Rare event studies - Expensive measurements - Pilot studies - Clinical trials with rare conditions - Specialized equipment testing - Historical artifact analysis

Teaching tips: - Compare results from different sample sizes - Discuss when small samples are unavoidable - Explore methods for small sample analysis - Consider cost-benefit tradeoffs - Examine real-world examples

Sample Size Effects

Understanding the square root relationship:

- Larger n = Smaller SEM

Sample Size Effects

Sample Size Effects

Sample Size Effects

Sample Size Effects

Sample Size Effects

Understanding the square root relationship:

- Larger n = Smaller SEM
  - → But diminishing returns
  - → Square root relationship means:
    - Double n → $1/\sqrt{2}$ times smaller SEM
    - 4x n → 1/2 times smaller SEM
    - 9x n → 1/3 times smaller SEM
  - → Cost/benefit tradeoff
- Practical Implications
  - → Doubling n improves precision by $\sqrt{2}$
  - → Need 4x sample size for 2x precision
  - → Each increment costs more
  - → Must balance precision vs resources

Sample Size Effects

→ Important for research planning

Understanding the relationship between sample size and precision is crucial for research planning. The formula for standard error of the mean tells us something very fundamental about statistical sampling – namely, that the utility of larger samples diminishes with the square root of the sample size.

This means that doubling the sample size will not double the quality of the statistics; rather, it will improve it by a factor of $\sqrt{2}$. This principle has important implications for research design and resource allocation.

Key points:

- Diminishing returns principle
- Cost considerations
- Practical limitations

Sample Size Effects

- Connection to statistical power
- Resource allocation decisions
- Optimization strategies

Mathematical relationship:

- To halve the standard error, need 4x sample size
- To reduce SE by 1/3, need 9x sample size
- Each increment of precision costs more
- Must balance precision vs resources

Examples:

- Research budget allocation
- Quality control sampling
- Survey design decisions

Sample Size Effects

- Clinical trial planning
- Market research studies
- Environmental monitoring

In a later section, we will discuss statistical power, which is intimately tied to this idea. The relationship between sample size and precision is crucial for:

- Determining minimum sample sizes
- Budgeting research projects
- Planning data collection
- Justifying research costs
- Optimizing resource allocation

Teaching tips:

- Use numerical examples

Sample Size Effects

- Demonstrate diminishing returns
- Discuss real-world constraints
- Consider cost-benefit analysis
- Explore practical limitations

Central Limit Theorem: Basics

A fundamental principle of statistics:

- Definition

Central Limit Theorem: Basics

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Central Limit Theorem: Basics

A fundamental principle of statistics:

- Definition
  - → Sampling distribution of mean becomes normal
  - → Regardless of population distribution
  - → As sample size increases
  - → Key to statistical inference
- Key Implications

Central Limit Theorem: Basics

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A fundamental principle of statistics:

- Definition
  - → Sampling distribution of mean becomes normal
  - → Regardless of population distribution
  - → As sample size increases
  - → Key to statistical inference
- Key Implications
  - → Works for any distribution shape
  - → Requires sufficient sample size
  - → Foundation for statistical inference
  - → Explains many natural phenomena
- Historical Context
  - → Named after Gaussian distribution

Central Limit Theorem: Basics

- → Developed over centuries
- → Fundamental to modern statistics
- → Enables many statistical methods

The Central Limit Theorem (CLT) is one of the most important concepts in statistics. It tells us that as sample sizes get larger, the sampling distribution of the mean will become normally distributed, even if the data within each sample are not normally distributed.

First, let's discuss the normal distribution. It's also known as the Gaussian distribution, after Carl Friedrich Gauss, a mathematician who didn't invent it but played a role in its development. The normal distribution is described in terms of two parameters: - Mean (location of the peak) - Standard deviation (width of the distribution)

The bell-like shape of the distribution never changes, only its location and width. The normal distribution is commonly observed in data collected in the real world, and the central limit theorem gives us some insight into why that occurs.

Central Limit Theorem: Basics

Key points:

- Why normality is important
- Sample size requirements
- Applications in inference
- Historical development
- Mathematical foundations
- Practical implications

The CLT is important because:

- Allows use of normal-theory statistics
- Justifies many statistical procedures
- Explains common patterns in nature
- Enables reliable inference
- Supports sampling theory

Central Limit Theorem: Basics

Examples:

- Quality control measurements
- Financial returns
- Natural phenomena
- Biological measurements
- Social science data
- Industrial processes

Teaching tips:

- Use simulations to demonstrate
- Connect to real-world patterns
- Explain historical context
- Show applications in different fields
- Discuss requirements and limitations

CLT Example: AlcoholYear

To see the Central Limit Theorem in action:

- Using AlcoholYear from NHANES dataset
  - → Measures days of alcohol consumption per year
  - → Highly skewed distribution (not normal)
  - → Real-world example of non-normal data
  - → Perfect for demonstrating CLT

Left panel:

- Original distribution of AlcoholYear
- Shows clear non-normal pattern
- "Funky" shape typical of real data
- Demonstrates real-world complexity

Right panel:

CLT Example: AlcoholYear

- Sampling distribution of means
- Based on samples of size 50
- Shows transformation to normality
- Red line: Theoretical normal distribution
- Demonstrates CLT in action

```
# create sampling distribution function

get_sampling_dist <- function(sampSize, nsamps = 2500) {
  sampMeansFull <- array(NA, nsamps)
  NHANES_clean <- NHANES %>%
    drop_na(AlcoholYear)

  for (i in 1:nsamps) {
    NHANES_sample <- sample_n(NHANES_clean, sampSize)
```

CLT Example: AlcoholYear

```
    sampMeansFull[i] <- mean(NHANES_sample$AlcoholYear)
  }
  sampMeansFullDf <- data.frame(sampMeans = sampMeansFull)

  p2 <- ggplot(sampMeansFullDf, aes(sampMeans)) +
    geom_freqpoly(aes(y = ..density..), bins = 100, color = "blue", size =
0.75) +
    stat_function(
      fun = dnorm, n = 100,
      args = list(
        mean = mean(sampMeansFull),
        sd = sd(sampMeansFull)
      ), size = 1.5, color = "red"
    ) +
    xlab("mean AlcoholYear")
```

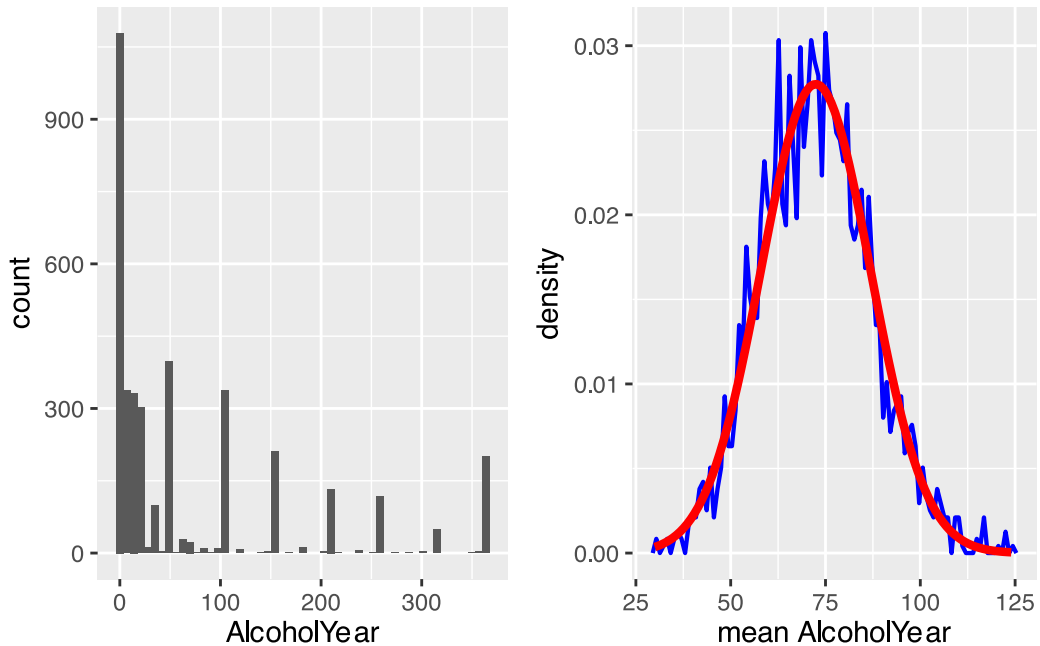
CLT Example: AlcoholYear

```
  return(p2)
}

NHANES_cleanAlc <- NHANES %>%
  drop_na(AlcoholYear)
p1 <- ggplot(NHANES_cleanAlc, aes(AlcoholYear)) +
  geom_histogram(binwidth = 7)

p2 <- get_sampling_dist(50)
```

```
Warning: Using `size` aesthetic for lines was deprecated in ggplot2 3.4.0.
ℹ Please use `linewidth` instead.
```

```
plot_grid(p1,p2)
```

CLT Example: AlcoholYear



This example demonstrates the CLT in action using the AlcoholYear variable from the NHANES dataset, which is highly skewed. The visualization shows:

- Left: Original skewed distribution

CLT Example: AlcoholYear

- Right: Normal sampling distribution

The original distribution is, for lack of a better word, funky – and definitely not normally distributed. Yet when we look at the sampling distribution of the mean for this variable, obtained by repeatedly drawing samples of size 50 from the NHANES dataset and taking the mean, we see something remarkable. Despite the clear non-normality of the original data, the sampling distribution is remarkably close to the normal.

Key points:

- Transformation to normality
- Effect of sample size
- Practical implications
- Why this matters
- Role of sample size
- Universality of the theorem

CLT Example: AlcoholYear

The CLT is important for statistics because:

- Allows us to safely assume normal sampling distributions
- Enables use of normal-theory statistical techniques
- Explains why normal distributions are common
- Supports statistical inference methods

Real-world example: The height of any adult depends on a complex mixture of their genetics and experience; even if those individual contributions may not be normally distributed, when we combine them the result is a normal distribution.

Teaching tips:

- Ask students to predict what would happen with different sample sizes
- Discuss why the transformation occurs
- Connect to real-world examples

CLT Example: AlcoholYear

- Explore implications for inference
- Consider practical applications

Summary

- Key Takeaways

Summary

Summary

Summary

Summary

- Key Takeaways
  - → Population vs sample distinction
  - → Importance of sample size
  - → CLT implications
- Practical Applications

Summary

Summary

Summary

Summary

- Key Takeaways
  - → Population vs sample distinction
  - → Importance of sample size
  - → CLT implications
- Practical Applications
  - → Business decision making
  - → Research design
  - → Quality control
- Common Pitfalls to Avoid
  - → Small sample assumptions
  - → Biased sampling
  - → Overgeneralization

Summary

Reinforce main concepts and their practical applications. This lecture has covered fundamental concepts in statistical sampling that form the basis for statistical inference.

Key points:

- Importance of proper sampling
- Role of sample size
- Real-world applications
- Common mistakes to avoid
- Mathematical foundations
- Practical implications

Core concepts reviewed:

1. Population vs Sample
    - Distinction between parameters and statistics

Summary

- Importance of representative sampling
- Role of random selection

2. Sampling Error
   - Natural variation in samples
   - Relationship to sample size
   - Impact on conclusions
3. Standard Error
   - Mathematical foundation
   - Relationship to sample size
   - Practical implications
4. Central Limit Theorem
   - Fundamental importance
   - Practical applications
   - Connection to normal distribution

Summary

Encourage students to think about:

- Applications in their field
- Future coursework connections
- Research applications
- Real-world sampling situations
- Statistical inference foundations
- Decision-making under uncertainty

Common pitfalls to avoid:

- Biased sampling methods
- Insufficient sample sizes
- Overgeneralization of results
- Ignoring sampling error
- Misinterpreting variation

Suggested Readings

"The Signal and the Noise" by Nate Silver

- Practical sampling applications
- Real-world prediction challenges
- Data-driven decision making

"The Signal and the Noise" by Nate Silver provides excellent real-world examples of sampling and prediction. The book explores:

- How predictions succeed or fail
- Role of probability in forecasting
- Importance of proper sampling
- Impact of bias in predictions
- Bayesian thinking
- Uncertainty quantification

Suggested Readings

Additional suggestions:

- Statistical methods texts
- Online resources
- Research papers
- Case studies
- Professional journals
- Industry applications

Key topics for further study:

- Advanced sampling methods
- Experimental design
- Survey methodology
- Big data sampling
- Bootstrapping techniques

Suggested Readings

- Modern prediction methods

Questions?

Thank you for your attention!

Be prepared to address:

- Common misconceptions about sampling
- Practical applications in various fields
- Connection to future topics in the course
- Real-world examples from business and research
- Questions about sampling methods
- Issues of bias and representation
- Sample size considerations
- Statistical inference foundations
- Relationship to hypothesis testing
- Role in research design

Remember to emphasize:

Questions?

- Importance of proper sampling methods
- Role of randomization
- Impact of sample size
- Practical constraints
- Cost-benefit considerations
- Real-world limitations