

# SPI - Defining bespoke and archetypal context-dependent Soundscape Perception Indices

Andrew Mitchell<sup>a,\*</sup>, Francesco Aletta<sup>a</sup>, Tin Oberman<sup>a</sup>, Jian Kang

<sup>a</sup>*University College London, Institute for Environmental Design & Engineering,*

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## Abstract

The soundscape approach provides a basis for considering the holistic perception of sound environments, in context. While steady advancements have been made in methods for assessment and analysis, a gap exists for comparing soundscapes and quantifying improvements in the multi-dimensional perception of a soundscape. To this end, there is a need for the creation of single value indices to compare soundscape quality which incorporate context, aural diversity, and specific design goals for a given application. Just as a variety of decibel-based indices have been developed for various purposes (e.g.  $L_{Aeq}$ ,  $L_{Ceq}$ ,  $L_{90}$ ,  $L_{den}$ , etc.), the soundscape approach requires the ability to create novel indices for different uses, but which share a common language and understanding. We therefore propose a unified framework for creating both bespoke and standardised single index measures of soundscape perception based on the soundscape circumplex model, allowing for new metrics to be defined in the future. The implementation of this framework is demonstrated through the creation of a public spaced typology-based index using data collected under the SSID Protocol, which was designed specifically for the purpose of defining soundscape indices. Indices developed under this framework can enable a broader and more efficient application of the soundscape approach.

*Keywords:* keyword1, keyword2

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## 1. Introduction

The EU Green Paper on Future Noise Policy indicates that 80 million EU citizens are suffering from potentially harmful environmental noise levels, according to the World Health Organization (WHO) recommendations (Berglund et al., 1999). The publication of the EU Directive Relating to the Assessment and Management of Environmental Noise (END) (European Union, 2002) more than two decades ago has led to major actions across Europe, with reducing noise levels as their main focus, for which billions of Euros are being spent. However, it is widely recognised that solely reducing sound level in people’s living environments is not always feasible or cost-effective and, more importantly, with only ~30% of environmental noise annoyance depending on facets of parameters such as acoustic energy (Guski), sound level reduction will not necessarily lead to improved quality of life. For this reason, from a public health point of view, it is necessary to explore alternative management and design strategies for acoustic environments that rely on more positive soundscapes, rather than merely environments not affected by noise pollution (Aletta et al., 2018; Kang, 2023; Kang et al., 2023).

Soundscape design, separate from noise control engineering, is about the relationships between human physiology, perception, the sound environment, and its social/cultural context (Kang, 2006). Soundscape

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\*Corresponding author

Email addresses: andrew.mitchell.18@ucl.ac.uk (Andrew Mitchell), f.aletta@ucl.ac.uk (Francesco Aletta), t.oberman@ucl.ac.uk (Tin Oberman), j.kang@ucl.ac.uk (Jian Kang)

research represents a paradigm shift in that it combines physical, social, and psychological approaches and considers environmental sounds as a ‘resource’ rather than ‘waste’ (Kang and Schulte-Fortkamp, 2016) relating to perceptual constructs rather than just physical phenomena. However, the current research is still at the stage of describing and identifying the problems and tends to be fragmented and focussed on only special cases e.g. subjective evaluations of soundscapes for residential areas (Schulte-Fortkamp and Kang, 2013). In the movement from noise control to soundscape creation (Aletta and Kang, 2015), a vital step is the standardisation of methods to assess soundscape quality.

A common aim for implementing soundscape assessment in practice is to compare the quality of different soundscapes. Often (but not always) the goal is to identify a ‘good’ soundscape compared to a ‘bad’ soundscape. However, this presents several challenges:

- What makes a soundscape good or bad is highly contextual; that is, the same acoustic environment may result in different appreciations and perceptual outcomes, depending on where/when it is happening, and what groups of individuals are there to experience it.
- On what metric should the quality rating be based? Previous attempts at defining objective metrics of “soundscape quality” assessment have fallen short of capturing the multidimensionality of people’s perception of surrounding acoustic environments.
- How can we deal with different requirements and definitions of how a soundscape should be perceived? Soundscape constructs are normally seen as highly individualised, while designing the soundscapes of public spaces should look at accommodating the needs of a given community of a space as a whole.

In many cases, the ultimate aim is to be able to rank soundscapes based on their quality. There is pressure from stakeholders and policymakers to move towards such simplified assessment protocols. However, any ranking metric should be flexible and be able to handle a variety of contexts and definitions of what a ‘good’ soundscape is for a given purpose. To address this, we will propose the Soundscape Perception Index (SPI) framework, a flexible method for defining single value indices of soundscape quality based on distributions within the Soundscape Circumplex Model (SCM) (Axelsson et al., 2010, Axelsson et al. (2012), Mitchell et al. (2022)).

As previous suggested, the primary motivation behind the development of the Soundscape Perception Indices (SPI) framework stems from the need to address the existing gap in quantifying and comparing soundscape quality across diverse contexts and applications. By creating a unified framework for defining these indices, the aim is to facilitate a broader and more efficient application of the soundscape approach in various domains, such as urban planning, environmental management, acoustic design, and policy development.

The overarching aim of this framework is to empower stakeholders, decision-makers, and researchers with the ability to create tailored indices that align with their specific objectives and design goals, while simultaneously enabling cross-comparisons and benchmarking against empirically-defined soundscape archetypes. This dual approach not only acknowledges the context-dependent nature of soundscape perception but also fosters a common language and understanding, facilitating knowledge sharing and collaborative efforts within the field. This paper will demonstrate the SPI framework and test whether it is capable of both scoring soundscape quality and generating consistent rankings of soundscapes across different contexts.

## 2. Background

In Aletta et al. (2016), the authors defined a framework for categorising the components of a soundscape assessment. They define three aspects: soundscape descriptors, soundscape indicators, and soundscape indices. Soundscape descriptors are defined as ‘measures of how people perceive the acoustic environment’ and soundscape indicators as ‘measures used to predict the value of a soundscape descriptor’. The relationship between soundscape indicator(s) and a soundscape descriptor effectively defines what has been previously referred to as a “predictive soundscape model” (Aletta et al., 2016; Mitchell, 2022). There are primarily

two rationales for modeling the relationship between the physical attributes and the perceived (i.e., soundscape) qualities of the acoustic environment. Firstly, a predictive model can forecast how individuals would perceive the acoustic environment, eliminating the need for labour-intensive surveys (Mitchell et al., 2023). Secondly, a precise predictive model may unveil the root causes of these perceived qualities, thereby serving as a valuable tool for design. Lionello and colleagues (Lionello et al., 2020) provided a review of such models and concluded contextual features play an important role in increasing the quality of the model.

Indices on the other hand, the primary focus of this article, are single numerical values that combine multiple indicators or descriptors to provide a comprehensive representation of the overall soundscape perception and allow for comparison between soundscapes. These indices serve as powerful tools for quantifying and comparing soundscapes, enabling decision-makers and stakeholders to assess the impact of interventions, monitor changes over time, and prioritize areas for improvement (Kang et al., 2019).

The earliest and most commonly used scientific index measuring sound level is the Decibel (dB). To represent the overall level of sound with a single value on one scale, as the Decibel index does, is often desirable. For this purpose, a number of different values representing sounds at various frequencies must be combined. Several frequency weighting networks have been developed since the 1930s, considering typical human responses to sound based on equal-loudness-level contours (Fletcher and Munson) and, among them, the A-weighting network, with resultant decibel values called dBA, has been commonly used in almost all the national/international regulations (Kryter). However, there have been numerous criticisms on its effectiveness (Parmanen) as the correlations between dBA and perceived sound quality (e.g. noise annoyance) are often low (Hellman and Zwicker).

Another set of indices is psychoacoustic magnitudes, including loudness, fluctuation strength or roughness, sharpness, and pitch strength, development with sound quality studies of industrial products since the 1980's (Zwicker and Fastl, 2007). These emerged when it was conceived that acoustic emissions can be characterised beyond just sound level (Blauert and Jekosch). But while psychoacoustic magnitudes have proven to be successful for the assessment of product sound quality, in the field of environmental acoustics, their applicability has been limited (Fastl), since a significant feature of environmental acoustics is that there are multiple/dynamic sound sources. Additionally, while psychoacoustic magnitudes incorporate perceptual aspects, both dB based and psychoacoustic indices are ultimately describing the acoustic signal and not the soundscape perception and may therefore be more accurately described as indicators rather than soundscape indices (Mitchell et al., 2023).

When applied to urban sound and specifically to noise pollution, the soundscape approach introduces three key considerations beyond traditional noise control methods:

1. considering all aspects of the environment which may influence perception, not just the sound level and spectral content;
2. an increased and integrated consideration of the varying impacts which different sound sources and sonic characteristics have on perception; and
3. a consideration of both the positive and negative dimensions of soundscape perception.

This approach can enable better outcomes by identifying positive soundscapes (in line with the END's mandate to 'preserve environmental noise quality where it is good' (European Union, 2002)), better identify specific sources of noise which impact soundscape quality and pinpoint the characteristics which may need to be decreased, and illuminate alternative methods which could be introduced to improve a soundscape where a reduction of noise is impractical (Fiebig, 2018; Kang and Aletta, 2018). These can all lead to more opportunities to truly improve a space by identifying the causes of positive soundscapes, while also potentially decreasing the costs of noise mitigation by offering more targeted techniques and alternative approaches.

Factors such as the presence of natural or human-made sounds, their temporal patterns, and the overall contextual meaning ascribed to these sounds all contribute to the holistic perception of a soundscape.

Consequently, there is a pressing need for the development of robust indices that can encapsulate this multi-dimensional nature of soundscape perception, enabling comparative evaluations and informing targeted interventions to enhance the overall quality of acoustic environments (Chen et al., 2023).

### 2.1. Existing ‘Soundscape Indices’

While the field of soundscape research has witnessed substantial progress, the development of standardized indices for evaluating and comparing soundscapes across diverse contexts has been relatively limited. Existing indices can be broadly seen as arising from two domains: soundscape ecology and soundscape perception.

#### 2.1.1. Soundscape Ecology and Bioacoustics

Within the realm of soundscape ecology, indices such as the Acoustic Diversity Index (ADI) and Frequency-dependency Acoustic Diversity Index (FADI) (Xu et al., 2023) have been developed to quantify the diversity and complexity of acoustic signals within a given soundscape. Similar indices (e.g. ADI, NDSI, ACI) have also been developed to analyse the acoustic signal of complex acoustic environments and indicate the richness and diversity of biophonic (natural) and anthrophonic (human-made) sound sources. However, while these indices contribute valuable insights into the ecological aspects of soundscapes, they do not directly address the perceptual dimensions that are central to the soundscape approach (Schulte-Fortkamp et al., 2023). The multi-dimensional nature of soundscape perception, encompassing factors such as pleasantness, eventfulness, and familiarity, necessitates a more comprehensive and context-sensitive approach.

#### 2.1.2. Soundscape Perception

In the domain of soundscape perception, the Green Soundscape Index (GSI) (Kogan et al., 2018) has emerged as a notable attempt to quantify the perceived quality of soundscapes, particularly in urban environments. This index incorporates factors such as the presence and levels of natural sounds, human-made sounds, and their respective contributions to the overall soundscape perception.

The GSI is the ratio of the perceived extent of natural sounds (PNS) to the perceived extent of traffic noise (PTN). The GSI is noted to range between 1/5 and 5, with several ranges of values given which correspond to general categories of the perceived dominance of traffic noise. Subsequently, Cao and colleagues (Cao et al., 2020; Yang et al., 2022) argued that for urban soundscape design the GSI would not be suitable for all applications and should be complemented by a Red Soundscape Index (RSI), defined as the ratio of natural sounds (PNS) to human sounds (PHS). Xiang et al. (2023) defined a pool of soundscape indices; namely: the soundscape diversity index (SDI), the soundscape richness index (SRI), the soundscape dominance index (SDO), and soundscape evenness index (SEI), and showed that some of them could be explained by existing acoustic indicators.

While all these indices represent a commendable effort to bridge the gap between objective measurements and subjective perceptions, they remain limited in their ability to capture the full complexity of soundscape perception across diverse contexts.

The Soundscape Perception Index framework presented in this paper differs from these existing indices in two key ways. Firstly, it is not an analysis of an acoustic signal but rather is an index of perception based on soundscape descriptors. Secondly, it does not represent a single target in a particular context, but is a generalisable, extensible, and adaptable framework for scoring soundscapes against any goal defined by the user. The remainder of the paper will introduce and demonstrate this framework, providing a case study of defining an appropriate target.

## 3. Methodology

The index framework, ‘the Soundscape Perception Indices (SPI)’ introduced in this paper is defined here as the agreement between an observed or modelled soundscape perception distribution and a target soundscape perception distribution. Its goal is to determine whether a soundscape - whether it be a real-world location,

a proposed design, or a hypothetical scenario - aligns with the desired perception of that soundscape. This is achieved by first defining the target distribution, which could represent what is considered to be the ‘ideal’ soundscape perception for a given context or application. The test distribution is then compared to the target distribution using a distance metric, which quantifies the deviation between the two distributions. The resulting distance value serves as the basis for calculating the SPI, with smaller distances indicating a closer alignment between the perceived soundscape and the target soundscape perception.

We refer to this as an index framework rather than a single index, as the SPI can be tailored to specific contexts and applications by defining a range of target distributions. A single index is thus created for each target distribution. An SPI value therefore does not represent a ‘good’ or ‘bad’ soundscape, but rather a measure of how closely the perceived soundscape aligns with the desired target soundscape perception. This approach allows for the development of bespoke indices tailored to specific design goals and objectives, while also enabling cross-comparisons and benchmarking against empirically-defined soundscape archetypes.

SPI is grounded in the soundscape circumplex model (SCM) (Axelsson et al., 2010, 2012), a robust theoretical foundation for understanding and representing the multi-dimensional nature of soundscape perception. The reason for grounding the SPI in the soundscape circumplex is that we have observed this model (and its corresponding PAQs) to become the most prevalent assessment model in soundscape literature (Aletta and Torresin, 2023).

The SCM is built on a series of descriptors referred to as the Perceived Affective Quality (PAQ), proposed by (Axelsson et al., 2010). These PAQs are based on the pleasantness-activity paradigm present in research on emotions and environmental psychology, in particular Russell’s circumplex model of affect (Russell, 1980). As summarised by Axelsson: “Russell’s model identifies two dimensions related to the perceived pleasantness of environments and how activating or arousing the environment is.”

One benefit of the circumplex model is that, as a whole, it encapsulates several of the other proposed soundscape descriptors - in particular, annoyance, pleasantness, tranquility, and possibly restorativeness (Aletta et al., 2016). According to Axelsson (2015), the two-dimensional circumplex model of perceived affective quality provides the most comprehensive information for soundscape assessment. It is also possible that the overall soundscape quality could itself be derived from the pleasant-eventful scores derived for a soundscape. The circumplex also lends itself well to questionnaire-based methods of data collection, as proposed in ISO/TS 12913-2:2018 (2018). In contrast to methods such as soundwalks, interviews, and lab experiments, in-situ questionnaires are able to provide the quality and amount of data which is necessary for statistical modelling. Combined, these factors make the circumplex most appropriate for a single index as it provides a comprehensive summary of soundscape perception.

There are four steps involved in calculating the SPI, as shown in Figure 1:

1. Define and parameterise the target circumplex distribution;
2. Sample the target distribution and prepare the test distribution;
3. Compare test and target distributions using the distance metric (2-dimensional Kolmogorov-Smirnov distance);
4. Calculate  $SPI = 100 * (1 - KS)$ .

These steps and their required background are discussed in detail in the following sections. Section 3.5 will then present strategies for defining targets and their applications.

Throughout this paper, we use the data contained in the International Soundscape Database (ISD) (Mitchell et al., 2024), which includes 1300+ individual responses on the PAQ scales collected across 13 locations in London and Venice, according to the SSID Protocol Mitchell et al. (2020).

### 3.1. Define and Parameterise a Soundscape Circumplex Distribution

To move the 8-item PAQ responses into the 2-dimensional circumplex space, we use the projection method first presented in ISO 12913-3:2018. This projection method and its associated formulae were recently

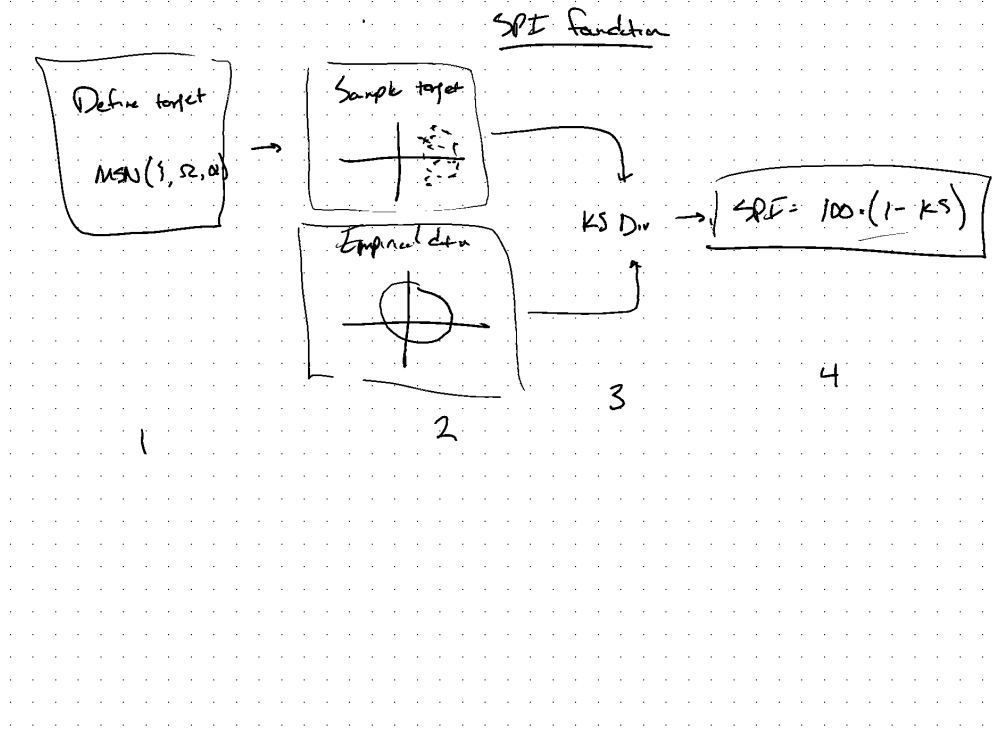


Figure 1: Steps for calculating the SPI.

updated further in [Mitchell and Aletta](#) to include a correction for the language in which the survey was conducted. [Mitchell and Aletta](#) also provides adjusted angles for translations of the circumplex attributes to be used in calculating the  $P_{ISO}$  and  $E_{ISO}$  coordinates.

Once the individual perceptual responses are projected into the circumplex space, the resulting data for each location is treated as a circumplex distribution. There are several advancements in considering circumplex distributions compared to the discussions originally given in [Mitchell et al. \(2022\)](#), which are necessary for SPI. Before exploring the SPI method and target setting more specifically, we will first address these developments.

The circumplex is defined by two axes:  $P_{ISO}$  and  $E_{ISO}$ , which are limited to the range  $[-1, +1]$ . Typically, data in the soundscape circumplex is treated as a combination of two independent normal distributions, one for each axis ([Mitchell et al., 2022](#); [Ooi et al., 2022](#)). In some applications this approach is sufficient for capturing the distribution of soundscape perception, however defining a target distribution for SPI requires a more precise approach. The independent normal distribution approach relies on three key assumptions:

1. The two axes are normally distributed.
2. The two axes are independent of each other.
3. The two axes are symmetrically distributed.

While the first assumption is generally valid, the second and third assumptions are not always met in practice. In particular, the distribution of soundscape perception responses in the circumplex is often characterised by a high degree of skewness, which can lead to inaccuracies in the calculation of the SPI. Soundscape circumplex distributions are most appropriately described as a bivariate skew-normal distribution ([Azzalini, b](#)) which accurately reflects the relationship between the two dimensions of the circumplex and the fact that real-world perceptual distributions have been consistently observed to not be strictly symmetric.

The skew-normal distribution is defined by three parameters: location ( $\mu$ ), scale ( $\sigma$ ), and shape ( $\alpha$ ). The location parameter defines the centre of the distribution, the scale parameter defines the spread of the distribution and the shape parameter defines the skew of the distribution. The one-dimensional skew-normal distribution is defined as (Azzalini and Valle):

$$\phi(z; \alpha) = 2\phi(z)\Phi(\alpha z) \quad \text{for } z \in \mathbb{R}$$

where  $\phi$  and  $\Phi$  are the standard normal probability density function and distribution function, respectively, and  $\alpha$  is a shape variable which regulates the skewness. The distribution reduces to a standard normal density when  $\alpha = 0$ . The bivariate skew-normal distribution extends this concept to two dimensions, allowing for the modelling of asymmetric and skewed distributions in a two-dimensional space such as the soundscape circumplex. The multivariate skew-normal (MSN) distribution including scale and location parameters is given by combining the normal density and distribution functions (Azzalini and Capitanio):

$$Y = 2\phi_k(y - \xi; \Omega)\Phi\{\alpha^T \omega^{-1}(y - \xi)\}$$

where  $\phi_k$  is the  $k$ -dimensional normal density with location  $\xi$ , shape  $\alpha$ , and covariance matrix  $\Omega$ .  $\Phi\{\cdot\}$  is the normal distribution function and  $\alpha$  is a  $k$ -dimensional shape vector. When  $\alpha = 0$ ,  $Y$  reduces to the standard multivariate normal  $N_k(\xi, \Omega)$  density. A circumplex distribution can therefore be parameterised<sup>1</sup> with a 2x2 covariance matrix  $\Omega$ , a 2x1 location vector  $\xi$ , and a 2x1 shape vector  $\alpha$ , written as:

$$Y \sim MSN(\xi, \Omega, \alpha)$$

By fitting an MSN distribution to empirical soundscape perception responses, it becomes possible to accurately capture the asymmetry and skewness of the distribution. A bivariate skew-normal distribution can be summarised as a set of these three parameters. Once parameterised, the distribution can then be sampled from to generate a synthetic distribution of soundscape perception responses.

Soundscape targets can thus be set by defining the desired MSN distribution. To demonstrate this, we will construct three arbitrary targets which will be used later to score three SPIs. The parameters chosen for the example targets are given in Table 1.

Table 1: The MSN direct parameterizations for three arbitrary example target distributions.  $\text{tgt}_1$  is located in the pleasant half, with a wide variance, and a positive skew along the pleasantness axis.

Target	Location $\xi$	Covariance Matrix $\Omega$	Shape $\alpha$
$\text{tgt}_1$	[0.5, 0.0]	$\begin{bmatrix} 0.2 & 0.0 \\ 0.0 & 0.2 \end{bmatrix}$	[1, 0]
$\text{tgt}_2$	[1.0, -0.4]	$\begin{bmatrix} 0.18 & -0.04 \\ -0.04 & 0.09 \end{bmatrix}$	[-8, 1]
$\text{tgt}_3$	[0.5, 0.7]	$\begin{bmatrix} 0.1 & 0.05 \\ 0.05 & 0.1 \end{bmatrix}$	[0, -5]

<sup>1</sup>It is important to note that the parameters which appear in the density expression ( $\xi, \Omega, \alpha$ ) are what are called ‘direct parameters’ (DP). They directly parameterise an MSN density and are typically only estimated by fitting an MSN to a sample. The more familiar and interpretable components (mean, standard deviation, and skewness) are termed the centred parameters (CP). It is possible to move from one parameterization to another, however “while any choice of the DP components is admissible, the same is not true for CP”; i.e. we can always move DP  $\rightarrow$  CP but not always CP  $\rightarrow$  DP. In this context, it is most important for readers not to confuse the location parameter  $\xi$  with the sample mean  $\mu$ . A more complete explanation of these parameterizations can be found in Azzalini (a)



### 3.2. Sample a Target Distribution

Once the parameters for an MSN are defined (i.e. the target), the MSN is then sampled using the `sn` package (Azzalini, 2021) in R (R Core Team, 2018). This is to prepare the target distribution to be compared with the empirical test distribution. Several restrictions to the possible parameter values apply, most importantly the covariance matrix  $\Omega$  must be a positive-definite matrix. In depth discussions of how these parameterizations should be defined and their restrictions can be found in Azzalini (a). Figure 2 shows the result of sampling (n=1000) the three example distributions given in Table 1 and plotting them as soundscape distributions.

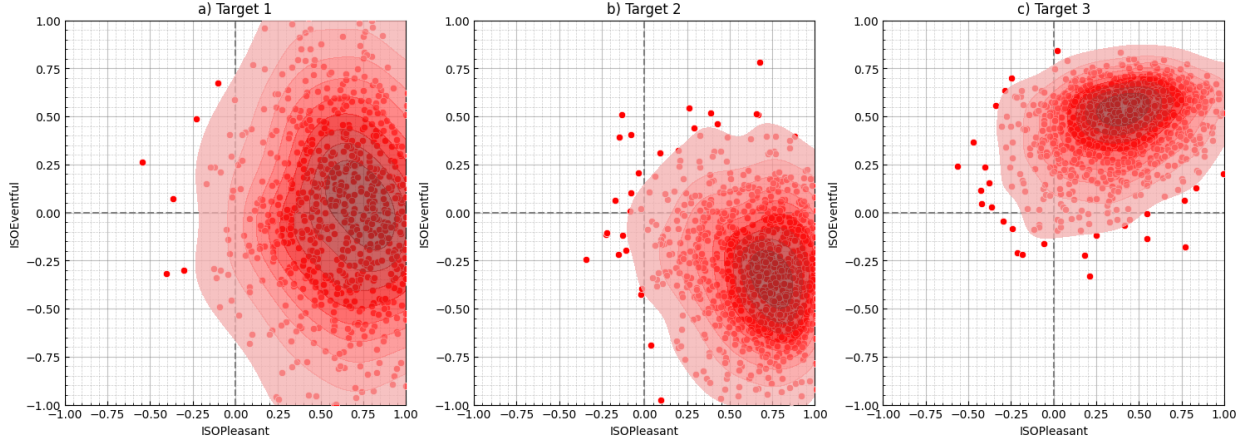


Figure 2: Example of defining and sampling from three arbitrary bespoke targets.

Source: SPI - Defining bespoke and archetypal context-dependent Soundscape Perception Indices

### 3.3. Compare the target and test distributions

Central to the SPI framework is the concept of a distance metric, which quantifies the deviation of a given soundscape from a desired target soundscape. This distance metric serves as the basis for calculating the SPI value, with smaller distances indicating a closer alignment between the perceived soundscape and the target soundscape perception. The distance between the test and target soundscape distributions is calculated using a two-dimensional Kolmogorov-Smirnov test (Fasano and Franceschini). The KS test is a non-parametric test of the equality of continuous distributions which is sensitive to both the location and shape of the distributions (Chakravati et al.).

Various other distance metrics were considered when developing the SPI method. The simplest method is to define a single point target, rather than a target distribution, and calculate a normalized mean Euclidean distance between points in the test distribution and the target point. While this is conceptually simple and requires defining only a single coordinate point as a target, rather than the MSN parameters described in Section 3.1, the shape and spread of a soundscape distribution is itself an important factor in describing the collective perception of a soundscape and would not be captured by this method (Mitchell et al., 2022).

Essentially, we approach this as a problem of (dis)similarity between soundscapes. The distance metric is then proposed to assess how similar any two given soundscapes distributions are within the circumplex. Taken to the extreme, two perfectly matching distributions in the soundscape circumplex would return a 100% SPI value, while two completely dissimilar distributions would return a 0% SPI value. In practical terms, for the former, this will never be achieved in real world scenarios; for the latter, it is also difficult to estimate how low the SPI value could actually go, and it should be considered that the distance may happen in different directions within the circumplex space. For instance, if a distribution for a vibrant soundscape was taken as a reference, a compared soundscape distribution may exhibit low SPI values for being located in the calm, OR monotonous, OR chaotic regions of the model.



Using the data from one location in the ISD (Piazza San Marco) as the test distribution, the KS statistic and p-value is calculated for each of the target distributions defined above, shown in Table 2.

Table 2: Kolmogorov-Smirnov test comparing the empirical test distribution (Piazza San Marco) against three soundscape target distributions.

Target	D	p
tgt_1	0.64	2.48761e-23
tgt_2	0.82	5.87603e-38
tgt_3	0.28	5.87603e-38

Source: [SPI - Defining bespoke and archetypal context-dependent Soundscape Perception Indices](#)

For the 2D KS test, a p-value less than 0.05 indicates that the empirical distributions are not drawn from the same distribution function. In this use case, where we never expect the distributions to be identical and instead only wish to characterize their degree of (dis)similarity, we discard the p-value and focus only on the test statistic.

### 3.4. Calculate the SPI score

The final step is to convert the KS test statistic into a more interpretable form to use as a comparison across soundscapes. Since the KS test statistic is a measure of dissimilarity, we first subtract it from one to give a measure of similarity between the test distribution and the target distribution. This is then scaled to produce a score which ranges from 0 to 100, giving the final SPI formula:

$$SPI = 100 * (1 - KS\{MSN_{test}, MSN_{tgt}\})$$

The three SPIs can now be calculated for all of the locations in the ISD, shown in Table 3. This produces three separate rankings of soundscape quality for these locations, depending on which target is considered the goal.

Table 3: SPI scores and rankings for the soundscapes of locations included in the International Soundscape Database (ISD).

Ranking	$SPI_1$ (pleasant)	$SPI_2$ (calm)	$SPI_3$ (vibrant)
1	72 CarloV	64 CampoPrincipe	72 SanMarco
2	70 RegentsParkFields	54 CarloV	62 TateModern
3	64 CampoPrincipe	51 RegentsParkFields	60 StPaulsCross
4	64 RegentsParkJapan	50 PlazaBibRambla	59 Noorderplantsoen
5	63 PlazaBibRambla	46 MarchmontGarden	54 PancrasLock
6	62 RussellSq	44 MonumentoGaribaldi	54 TorringtonSq
7	62 MarchmontGarden	41 RussellSq	47 StPaulsRow
8	61 PancrasLock	40 PancrasLock	47 RussellSq
9	61 MonumentoGaribaldi	39 RegentsParkJapan	46 MiradorSanNicolas
10	55 StPaulsCross	32 StPaulsCross	42 CamdenTown
11	50 TateModern	32 MiradorSanNicolas	40 CarloV
12	50 StPaulsRow	31 TateModern	36 MonumentoGaribaldi
13	46 MiradorSanNicolas	30 TorringtonSq	34 MarchmontGarden
14	41 Noorderplantsoen	29 StPaulsRow	33 PlazaBibRambla
15	39 TorringtonSq	17 SanMarco	32 CampoPrincipe
16	35 SanMarco	16 Noorderplantsoen	31 EustonTap
17	22 CamdenTown	14 EustonTap	27 RegentsParkFields

Table 3: SPI scores and rankings for the soundscapes of locations included in the International Soundscape Database (ISD).

Ranking	$SPI_1$ (pleasant)	$SPI_2$ (calm)	$SPI_3$ (vibrant)
18	17 EustonTap	14 CamdenTown	27 RegentsParkJapan

Source: [SPI - Defining bespoke and archetypal context-dependent Soundscape Perception Indices](#)

### 3.5. Deriving a target based on soundscape ranking

#### DRAFT SECTION

Absent from the above methodology has been an exploration of how to actually arrive at a target based on empirical evidence. While arbitrary targets make the SPI framework incredibly flexible, able to score against an effectively infinite set of design goals, often targets should have some sort of systematic foundation. To enable this approach, we therefore present one method of systematically deriving a target distribution based on a given ranking of soundscape quality. Just as one primary goal of the SPI framework is to enable soundscape rankings to be produced from SPI scores, this method allows for rankings which were arrived at separately to produce an optimized SPI target.

In this case study, we will examine a ranking derived from the ISD locations (shown in Table 4). The goal is then to derive an SPI target distribution which, when applied to the SCM data from each of these locations, (1) results in the same ranking order and (2) results in high SPI scores for the top ranked locations and lower scores for the bottom ranked locations.

Table 4: A pre-defined ranking of soundscape quality of the park locations included in the International Soundscape Database (ISD). An SPI target will be derived which aims to reproduce this same ranking when applied to circumplex data from these locations.

Rank	Location
1	RegentsParkJapan
2	RegentsParkFields
3	CampoPrincipe
4	MonumentoGaribaldi
5	RussellSq
6	MiradorSanNicolas
7	StPaulsCross
8	Noorderplantsoen

Effectively, this is an optimization task to determine the MSN parameters which best achieve the above goals. Goal (1) is assessed by calculating the Spearman rank correlation between the provided ranking and the SPI ranking. Goal (2) is scored by calculating a weighted sum of the produced SPIs:

$$score_w = \sum_{i=1}^m \frac{1}{rank_i} \cdot SPI_i$$

Through our testing, only optimizing on the rank correlation regularly produced targets which, while they did result in the desired ranking, were in no way representative of the soundscapes in question. We therefore aim to optimize for both a consistent soundscape ranking and for a high SPI score for the top-ranked soundscapes.

We apply an evolutionary multiobjective optimization named NSGA-II ([Deb and Jain](#)).

Defining the optimization problem:

- $\max r(\text{ranks}_{\text{quality}}, \text{ranks}_{\text{target}})$
- $\max \text{mean}(SPI_{\text{target}}(X_i))$

where  $r$  is the rank correlation coefficient,  $\text{ranks}_{\text{quality}}$  and  $\text{ranks}_{\text{target}}$  are the ranks of the quality and target values, and  $SPI_{\text{target}}(X_i)$  is the SPI for a given target on the data for the  $i$ -th location. Therefore we are trying to achieve the best correlation between the desired ranking and the ranking produced by  $SPI_{\text{target}}$  and to achieve the highest mean  $SPI_{\text{target}}$ .

$\text{ranks}_{\text{quality}}$  is pre-defined.  $\text{ranks}_{\text{target}}$  is calculated by sorting the target values and assigning ranks to them.  $SPI_{\text{target}}$  is calculated for each location and target.

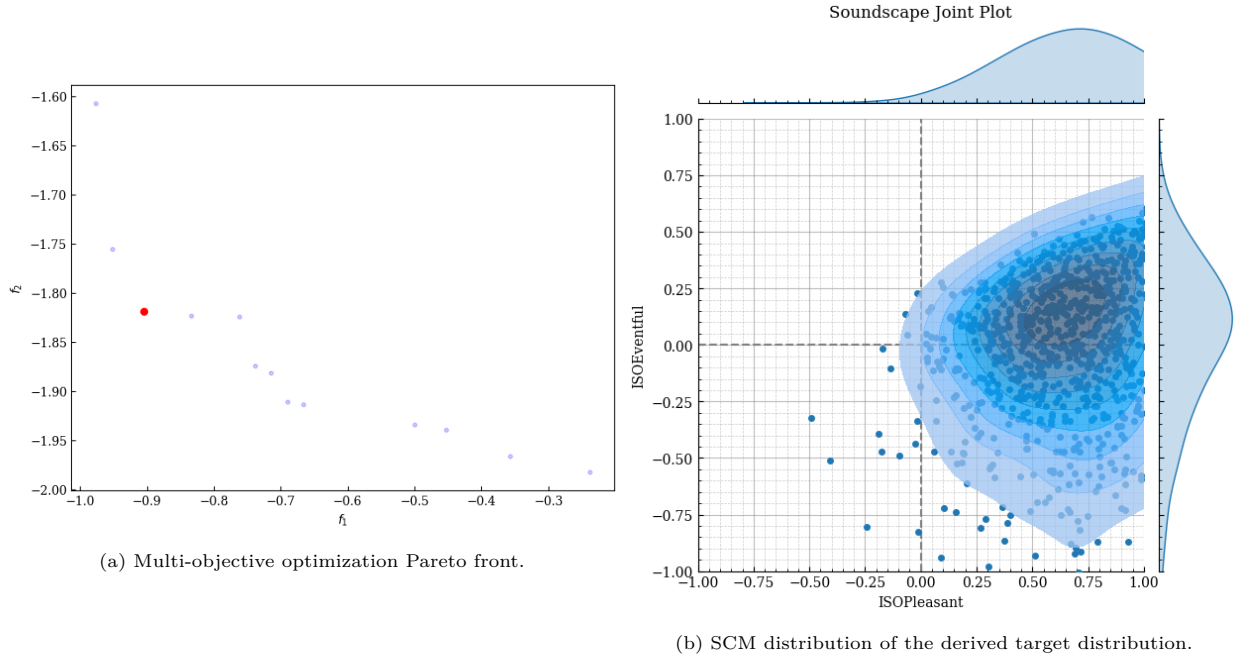


Figure 3: NSGA-II optimization to learn the MSN parameters which produce the Park ranking.

Source: [Load latest ISD dataset](#)

## 4. Expanding the SPI framework

Section 3 has defined and demonstrated the foundational methodology for calculating an SPI score. This included how to: define and sample a target distribution; prepare the test and target distributions for comparison using the KS distance metric; and convert this into an SPI score. To expand this methodology into an applicable framework, we define two distinct types of targets: bespoke targets and archetypal targets, each serving a unique purpose in the index development process.

### 4.0.1. Bespoke Targets

Bespoke targets are essentially a direct application of the foundational method described above. Bespoke targets are tailor-made for specific projects, reflecting the desired soundscape perception for a particular application. These targets can be defined by stakeholders, designers, policymakers, or decision-makers based on their unique requirements, objectives, and constraints. This flexibility allows the SPI for a specific project to be tailored to the desire of the stakeholders for how that specific soundscape should function. It can also provide a consistent and quantifiable baseline for scenarios like a soundscape design contest wherein a target is specified and provided to all participants in the contest and the winning proposal is the

design with the highest SPI score when assessed against that target. Stakeholders could use various methods to decide on a target, subject to the requirements of their project or use case. For example, it could be co-created with other stakeholders or space users, based on trying to match the soundscape of a previous project, or entirely arbitrary.

#### 4.0.2. Archetypal Targets

##### *DRAFT SECTION*

In contrast to bespoke targets, archetypal targets represent generalized, widely recognized soundscape archetypes which transcend specific applications or projects. These archetypes serve as reference points and enable comparisons across different domains and use cases. Essentially an archetypal target is a target that has been empirically defined to encapsulate the ideal of a particular type of soundscape.

The best methods for empirically determining the ideal soundscape distribution for a given context will no doubt remain a topic of debate and development in the coming years.

**Add more here**

## 5. Discussion and Conclusion

*Probably need to expand and rework subheaders*

The development of bespoke and archetypal context-dependent Soundscape Perception Indices (SPIs) represents a significant step towards enabling more comprehensive and effective applications of the soundscape approach. By providing a unified framework for defining these indices, the potential for quantifying and comparing soundscape quality across diverse contexts and applications is unlocked, while still ensuring that the multi-dimensional and context-driven aspects of soundscape quality are considered.

The proposed framework offers several key advantages. First, it acknowledges the inherent context-dependent nature of soundscape perception, allowing for the creation of indices tailored to specific use cases or design goals through the use of bespoke targets. This flexibility ensures that the resulting SPIs accurately capture the desired soundscape perception for the given application, enabling targeted interventions and optimisations.

Second, the inclusion of archetypal targets facilitates cross-comparisons and benchmarking, enabling a common language and understanding of soundscape quality across different domains. By calculating the distance between a given soundscape and these widely recognized archetypes, stakeholders can identify areas for improvement and prioritize interventions accordingly, aligning their efforts with collectively recognized standards of desirable or undesirable soundscapes.

We expect that this would then expand into collections of SPI targets. As an example, imagine trying to define a soundscape perception index that could be applied across an entire city. A single index is insufficient, because each type of place within the city (e.g. parks, plazas, residential areas) has different requirements for its soundscape. Therefore, each place type would need its own soundscape target.

In this example, these sets of targets would correspond to different types of places within the city (e.g. a single target for parks, a target for plazas etc.). When applying this “urban typology” set of targets, the soundscape of each location being assessed would be scored against its relevant target (i.e how well does a specific park perform in comparison to an archetypal park target). This results in a single score for each location that can be compared against all other locations, regardless of whether or not they are the same type of place, allowing for different soundscapes to be compared on a common scale. This system ensures that context (in this case, the typology of a space) is brought into the assessment, allowing soundscapes to be scored against the most appropriate target. Enabling these context dependent assessments to be expressed on a common scale can facilitate additional use cases such as soundscape mapping, which requires a single scale to be applied across an entire city.

This set of targets made up of e.g. parks, plazas etc. is just one example of an application of archetypal SPIs. Other examples could include a demographics SPI, where different targets are set for respondents from different demographic groups, or a “use case” SPI with different targets set for different intended purposes of spaces (e.g. recreation, restoration, socialising). We encourage users of the SPI to define both their own single archetype targets that can be added these suites of targets for use by others, and their own new sets of archetypes.

(Kogan et al., 2018, Fig.6), in fact displays a startlingly similar concept, showing the locations of the three categories of traffic noise dominance (‘traffic noise’, ‘balanced’, and ‘natural’) plotted in the circumplex perceptual model. It can be clearly seen in this plot that the GSI categories create their own clusters within the circumplex.

Although it is expected that the target distribution would usually represent the ideal or goal soundscape perception, it is also possible to define target distributions that represent undesirable or suboptimal soundscape perceptions. For instance, in a soundscape mapping context, it may be beneficial to map and identify chaotic soundscapes across a city in order to better target areas for soundscape interventions. In this case, the target distribution would be set in the chaotic quadrant and a higher SPI would indicate a closer alignment with the target distribution. This flexibility allows the SPI to be applied to a wide range of contexts and applications, enabling the quantification and comparison of soundscape quality across diverse scenarios.

#### 5.0.1. Data Source

The SPI framework is designed to accommodate a wide range of data sources, including both objective measurements and subjective evaluations. This flexibility enables the framework to be applied to diverse contexts and applications, ranging from urban soundscapes to natural environments, public spaces, and indoor settings.

#### 5.1. Conclusion

The proposed framework addresses the existing gap in quantifying multi-dimensional soundscape perception, facilitating a broader application of the soundscape approach in areas such as urban planning, environmental management, acoustic design, and policy development. Through the creation of bespoke indices tailored to specific design goals and the utilization of archetypal targets for benchmarking, this framework empowers stakeholders and decision-makers to make informed choices and prioritize soundscape improvements aligned with their unique objectives and constraints.

Furthermore, the grounding of the SPI framework in the soundscape circumplex model ensures a robust theoretical foundation, capturing the multi-dimensional nature of soundscape perception. The use of a distance metric enables quantitative assessments and comparisons, fostering a common language and understanding of soundscape quality across different domains. This shared understanding facilitates knowledge exchange, collaborative efforts, and the development of best practices within the field.

As the SPI framework continues to be explored and refined, future research should focus on validating and expanding the range of archetypal targets, as well as investigating the potential for incorporating additional dimensions and factors that influence soundscape perception. The integration of emerging technologies, such as virtual and augmented reality, may also provide new avenues for immersive soundscape evaluation and index development.

Additionally, the application of the framework in diverse real-world scenarios, ranging from urban planning and environmental management to acoustic design and policy development, will provide valuable insights and contribute to the ongoing refinement and adaptation of the SPI framework. Collaboration with stakeholders, end-users, and experts from various domains will be crucial in ensuring the framework’s relevance and applicability across a wide range of contexts.

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