

# Supplementary Material for: Soundscape Perception Indices (SPI)

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## 1. Supplementary Material

### 1.1. Multi-objective Optimization to Derive an SPI target

To set up the optimisation task, we first need to express the parameter space and any constraints. Since our goal is to identify an optimised soundscape target distribution, the parameters we will search over are:

- $\xi = (\xi_x, \xi_y)$ ,  $-1 \leq \xi \leq 1$
- $\Omega = \begin{pmatrix} \text{var}(x) & \text{cov}(x, y) \\ \text{cov}(y, x) & \text{var}(y) \end{pmatrix}$ 
  - $0 \leq \text{var}() \leq 1$
  - $-1 \leq \text{cov}() \leq 1$
  - $\Omega$  must be symmetric and positive definite
- $\alpha = (\alpha_x, \alpha_y)$ ,  $-5 \leq \alpha \leq 5$
- $-1 \leq x, y \leq 1$  In `pymoo`, each objective function is supposed to be minimized. Therefore, we need to convert both SPI and `r()` to minimize problems.
- $\min -r(\text{ranks}_{\text{quality}}, \text{ranks}_{\text{target}})$

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- $\min -\text{mean}(SPI_{target}(X_i))$

The final objective function is:

- $f_1 = -r(\text{ranks}_{quality}, \text{ranks}_{target})$
- $f_2 = -\text{mean}(SPI_{target}(X_i))$

So our variables to optimize are:

- $-1 \leq \xi_x \leq 1$
- $-1 \leq \xi_y \leq 1$
- $0 \leq \text{var}(x) \leq 1$
- $0 \leq \text{var}(y) \leq 1$
- $-1 \leq \text{cov}(x, y) \leq 1$
- $-5 \leq \alpha_x \leq 5$
- $-5 \leq \alpha_y \leq 5$

Constraint: -  $\Omega$  must be symmetric and positive definite - `np.linalg.eigvals(omega) > 0`

We then define the objective functions based on the two goals given above. For each step in the algorithm with a given trial set of parameters, a target distribution will be produced, the SPI for each test location assessed according to the protocol described in Section~??, and the resulting set of SPI scores and ranking will be scored using the objective functions. Goal (1) is assessed by calculating the Spearman rank correlation between the *a priori* ranking and the SPI ranking:

```
import warnings
from pathlib import Path

import numpy as np
import pandas as pd
import soundscapy as sspy
from soundscapy.surveys.survey_utils import LANGUAGE_ANGLES, PAQ_IDS

import optimize_target as ot
from MultiSkewNorm import MultiSkewNorm

warnings.filterwarnings("ignore")
```

```
# Load latest ISD dataset
# data = sspy.isd.load_zenodo()
# Load latest ISD dataset

data = sspy.isd.load()
data, excl_data = sspy.isd.validate(data)
data = data.query("Language != 'cmn'")

# Exclude RegentsParkJapan outliers
# excl_id = list(data.query(
#     # "LocationID == 'RegentsParkJapan'"
#     # ).query("ISOEventful > 0.72 | ISOEventful < -0.5").index)
```

```
# Excluded RegentsParkFields outliers
# excl_id = excl_id + list(data.query(
#     # "LocationID == 'RegentsParkFields' and ISOPleasant < 0").index) # Helicopters
excl_id = [652, 706, 548, 550, 551, 553, 569, 580, 609, 618, 623, 636, 643]
data.drop(excl_id, inplace=True)
data
```

	LocationID	SessionID	GroupID	RecordID	start_time	end_time	latit
0	CarloV	CarloV2	2CV12	1434	2019-05-16 18:46:00	2019-05-16 18:56:00	37.1
1	CarloV	CarloV2	2CV12	1435	2019-05-16 18:46:00	2019-05-16 18:56:00	37.1
2	CarloV	CarloV2	2CV13	1430	2019-05-16 19:02:00	2019-05-16 19:12:00	37.1
3	CarloV	CarloV2	2CV13	1431	2019-05-16 19:02:00	2019-05-16 19:12:00	37.1
4	CarloV	CarloV2	2CV13	1432	2019-05-16 19:02:00	2019-05-16 19:12:00	37.1
...	...	...	...	...	...	...	...
1693	Noorderplantsoen	Noorderplantsoen1	NP161	61	2020-03-11 12:42:00	2020-03-11 12:55:00	NaN
1694	Noorderplantsoen	Noorderplantsoen1	NP162	63	2020-03-11 12:39:00	2020-03-11 13:00:00	NaN
1695	Noorderplantsoen	Noorderplantsoen1	NP162	62	2020-03-11 12:54:00	2020-03-11 12:58:00	NaN
1696	Noorderplantsoen	Noorderplantsoen1	NP162	64	2020-03-11 12:56:00	2020-03-11 12:59:00	NaN
1697	Noorderplantsoen	Noorderplantsoen1	NP163	70	2020-03-11 23:08:00	2020-03-11 23:18:00	NaN

## 1.2. Calculate ISOPleasant and ISOEventful coordinates

Here we use the adjusted angles from Aletta et al. (2024) for each language included.

```
for i, row in data.iterrows():
    lang = row["Language"]
    angles = LANGUAGE_ANGLES[lang]
    iso_pl, iso_ev = (
        sspsy.surveys.processing._adj_iso_pl(row[PAQ_IDS], angles, scale=4),
        sspsy.surveys.processing._adj_iso_ev(row[PAQ_IDS], angles, scale=4),
    )
    data.loc[i, "ISOPleasant"] = iso_pl
    data.loc[i, "ISOEventful"] = iso_ev
```

```
# Separate out parks and non-parks
```

```
parks = [
    "RegentsParkFields",
    "RegentsParkJapan",
    "Noorderplantsoen",
    "StPaulsCross",
    "MiradorSanNicolas",
    "RussellSq",
    "Noorderplantsoen",
    "MonumentoGaribaldi",
    "CampoPrincipe",
]
```

```
not_parks = [
```

```

    "MarchmontGarden",
    "PancrasLock",
    "TateModern",
    "PlazaBibRambla",
    "SanMarco",
    "StPaulsRow",
    "CarloV",
    "CamdenTown",
    "EustonTap",
    "TorringtonSq",
]

park_data = data.query("LocationID in @parks")
not_park_data = data.query("LocationID in @not_parks")

rank_on = "sss01"

# Creating a somewhat arbitrary ranking of parks
park_quality = pd.DataFrame(
    park_data.groupby("LocationID")[rank_on].mean().sort_values(ascending=False)
)
park_quality["Rank"] = range(1, len(park_quality) + 1)
park_quality

```

LocationID	sss01	Rank
RegentsParkJapan	4.617978	1
RegentsParkFields	4.467290	2
CampoPrincipe	4.345455	3
MonumentoGaribaldi	4.156250	4
RussellSq	4.020548	5
MiradorSanNicolas	3.964286	6
StPaulsCross	3.803030	7
Noorderplantsoen	2.412371	8

### 1.3. *pymoo* Multi-objective Optimization

Defining the optimization problem:

- $\max r(\text{ranks}_{\text{quality}}, \text{ranks}_{\text{target}})$
- $\max \text{mean}(SPI_{\text{target}}(X_i))$

where  $r$  is the rank correlation coefficient,  $\text{ranks}_{\text{quality}}$  and  $\text{ranks}_{\text{target}}$  are the ranks of the quality and target values, and  $SPI_{\text{target}}(X_i)$  is the SPI for a given target on the data for the  $i$ -th location. Therefore we are trying to achieve the best correlation between the desired ranking and the ranking produced by  $SPI_{\text{target}}$  and to achieve the highest mean  $SPI_{\text{target}}$ .

$\text{ranks}_{\text{quality}}$  is pre-defined.  $\text{ranks}_{\text{target}}$  is calculated by sorting the target values and assigning ranks to them.  $SPI_{\text{target}}$  is calculated for each location and target.

`target_success(target, pre_ranks, data)`

`target = MultiSkewNorm( $\xi$ ,  $\Omega$ ,  $\alpha$ )` parameters

- $\xi = (\xi_x, \xi_y)$ ,  $-1 \leq \xi \leq 1$
- $\Omega = \begin{pmatrix} \text{var}(x) & \text{cov}(x, y) \\ \text{cov}(y, x) & \text{var}(y) \end{pmatrix}$ 
  - $0 \leq \text{var}() \leq 1$
  - $-1 \leq \text{cov}() \leq 1$
  - $\Omega$  must be symmetric and positive definite
- $\alpha = (\alpha_x, \alpha_y)$ ,  $-5 \leq \alpha \leq 5$
- $-1 \leq x, y \leq 1$  In `pymoo`, each objective function is supposed to be minimized. Therefore, we need to convert both `SPI` and `r()` to minimize problems.
- $\min -r(\text{ranks}_{\text{quality}}, \text{ranks}_{\text{target}})$
- $\min -\text{mean}(\text{SPI}_{\text{target}}(X_i))$

The final objective function is:

- $f_1 = -r(\text{ranks}_{\text{quality}}, \text{ranks}_{\text{target}})$
- $f_2 = -\text{mean}(\text{SPI}_{\text{target}}(X_i))$

So our variables to optimize are:

- $-1 \leq \xi_x \leq 1$
- $-1 \leq \xi_y \leq 1$
- $0 \leq \text{var}(x) \leq 1$
- $0 \leq \text{var}(y) \leq 1$
- $-1 \leq \text{cov}(x, y) \leq 1$
- $-5 \leq \alpha_x \leq 5$
- $-5 \leq \alpha_y \leq 5$

Constraint: -  $\Omega$  must be symmetric and positive definite - `np.linalg.eigvals(omega) > 0`

### 1.3.1. Problem Definition

```
import pathos
from pymoo.core.callback import Callback
from pymoo.core.problem import ElementwiseProblem, StarmapParallelization
from pymoo.visualization.scatter import Scatter
from pyrecorder.recorder import Recorder
from pyrecorder.writers.streamer import Streamer
from pyrecorder.writers.video import Video
from pymoo.decomposition.asf import ASF
```

```

class MyProblem(ElementwiseProblem):
    def __init__(self, data, ranking, **kwargs):
        super().__init__(
            n_var=7,
            n_obj=2,
            n_constr=0,
            xl=np.array([-1, -1, 0, 0, -1, -50, -50]),
            xu=np.array([1, 1, 0.5, 0.5, 1, 50, 50]),
            n_eq_constr=1,
            elementwise_evaluation=True,
            **kwargs,
        )

        self.data = data
        self.ranking = ranking

    def _evaluate(self, X, out, *args, **kwargs):
        # Check if the matrix is positive definite
        h = 1 - int(
            np.all(np.linalg.eigvals(np.array([[X[2], X[4]], [X[4], X[3]]])) > 0)
        )
        out["H"] = h
        if h != 0:
            out["F"] = np.column_stack([0, 0])
            return
        else:
            tgt = MultiSkewNorm()
            tgt.define_dp(
                np.array([X[0], X[1]]),
                np.array([[X[2], X[4]], [X[4], X[3]]]),
                np.array([X[5], X[6]]),
            )
            tgt.sample()
            r, wspi, spi_ranks, target = ot.target_success(tgt, self.ranking, self.data)

            f1 = -r[0]
            f2 = -wspi / 100

            out["F"] = np.column_stack([f1, f2])

class VideoCallback(Callback):
    def __init__(self) -> None:
        super().__init__()
        self.rec = Recorder(Streamer(sleep=0.1))

    def notify(self, algorithm):
        sc = Scatter(
            title="Gen %s" % algorithm.n_gen,
            labels=["spearman", "WSPI"],
        )

```

```

sc.add(algorithm.pop.get("F"))
sc.do()
self.rec.record()

```

```

from pymoo.algorithms.moo.nsga2 import NSGA2
from pymoo.operators.crossover.sbx import SBX
from pymoo.operators.mutation.pm import PM
from pymoo.operators.sampling.rnd import FloatRandomSampling
from pymoo.optimize import minimize
from pymoo.termination.default import DefaultMultiObjectiveTermination

algorithm = NSGA2(
    pop_size=150,
    sampling=FloatRandomSampling(),
    crossover=SBX(),
    mutation=PM(),
    eliminate_duplicates=True,
    # callback=VideoCallback()
)

termination = DefaultMultiObjectiveTermination(n_max_gen=100)

```

### 1.3.2. Park Quality optimization

```

# initialize the thread pool and create the runner
mp = pathos.helpers.mp
n_process = 12
pool = mp.Pool(n_process)
runner = StarmapParallelization(pool.starmap)

park_problem = MyProblem(
    data=park_data, ranking=park_quality.sort_index()["Rank"], elementwise_runner=runner
)

park_res = minimize(
    park_problem, algorithm, termination, seed=42, save_history=True, verbose=True
)

pool.close()

park_F = park_res.F
park_X = park_res.X

```

```

=====
n_gen | n_eval | n_nds | cv_min | cv_avg | eps | indicator
=====
1 | 150 | 2 | 0.000000E+00 | 0.7599240000 | - | -
2 | 300 | 2 | 0.000000E+00 | 0.3732960000 | 0.1111111111 | ideal
3 | 450 | 4 | 0.000000E+00 | 0.000000E+00 | 0.2500000000 | ideal

```

4	600	5	0.000000E+00	0.000000E+00	0.0335522234	f
5	750	5	0.000000E+00	0.000000E+00	0.0680302031	ideal
6	900	9	0.000000E+00	0.000000E+00	0.2047687061	ideal
7	1050	11	0.000000E+00	0.000000E+00	0.2929936306	nadir
8	1200	11	0.000000E+00	0.000000E+00	0.0966259235	ideal
9	1350	7	0.000000E+00	0.000000E+00	0.0782387648	ideal
10	1500	8	0.000000E+00	0.000000E+00	0.0611556026	ideal
11	1650	12	0.000000E+00	0.000000E+00	0.0333333333	ideal
12	1800	12	0.000000E+00	0.000000E+00	0.0334725144	f
13	1950	12	0.000000E+00	0.000000E+00	0.0059250141	f
14	2100	13	0.000000E+00	0.000000E+00	0.0304028573	f
15	2250	14	0.000000E+00	0.000000E+00	0.0027672766	f
16	2400	14	0.000000E+00	0.000000E+00	0.0362256249	ideal
17	2550	11	0.000000E+00	0.000000E+00	0.0257801437	f
18	2700	12	0.000000E+00	0.000000E+00	0.0329304891	nadir
19	2850	8	0.000000E+00	0.000000E+00	0.0276813880	ideal
20	3000	8	0.000000E+00	0.000000E+00	0.0111481604	f
21	3150	9	0.000000E+00	0.000000E+00	0.0232629795	ideal
22	3300	10	0.000000E+00	0.000000E+00	0.0026909865	f
23	3450	11	0.000000E+00	0.000000E+00	0.0083226943	f
24	3600	13	0.000000E+00	0.000000E+00	0.0314069149	f
25	3750	13	0.000000E+00	0.000000E+00	0.0128504961	f
26	3900	13	0.000000E+00	0.000000E+00	0.0013253064	f
27	4050	14	0.000000E+00	0.000000E+00	0.0231753198	ideal
28	4200	14	0.000000E+00	0.000000E+00	0.0054205862	f
29	4350	12	0.000000E+00	0.000000E+00	0.0314117047	ideal
30	4500	12	0.000000E+00	0.000000E+00	0.0165317133	f
31	4650	13	0.000000E+00	0.000000E+00	0.0053217968	f
32	4800	14	0.000000E+00	0.000000E+00	0.0045225121	f
33	4950	14	0.000000E+00	0.000000E+00	0.0069662325	f
34	5100	14	0.000000E+00	0.000000E+00	0.0094260974	f
35	5250	14	0.000000E+00	0.000000E+00	0.0003031699	f
36	5400	14	0.000000E+00	0.000000E+00	0.0030972357	f
37	5550	14	0.000000E+00	0.000000E+00	0.000000E+00	f
38	5700	15	0.000000E+00	0.000000E+00	0.0032527831	f
39	5850	14	0.000000E+00	0.000000E+00	0.0004001843	f
40	6000	14	0.000000E+00	0.000000E+00	0.0009926650	f
41	6150	15	0.000000E+00	0.000000E+00	0.0076455607	f
42	6300	15	0.000000E+00	0.000000E+00	0.000000E+00	f
43	6450	15	0.000000E+00	0.000000E+00	0.0000258705	f
44	6600	16	0.000000E+00	0.000000E+00	0.0046279606	f
45	6750	16	0.000000E+00	0.000000E+00	0.0010413888	f
46	6900	16	0.000000E+00	0.000000E+00	0.0010413888	f
47	7050	15	0.000000E+00	0.000000E+00	0.0031917732	f
48	7200	15	0.000000E+00	0.000000E+00	0.0009410395	f
49	7350	16	0.000000E+00	0.000000E+00	0.0154731488	ideal
50	7500	16	0.000000E+00	0.000000E+00	0.0003014637	f
51	7650	16	0.000000E+00	0.000000E+00	0.0020714439	f
52	7800	16	0.000000E+00	0.000000E+00	0.0020714439	f
53	7950	16	0.000000E+00	0.000000E+00	0.0020714439	f
54	8100	17	0.000000E+00	0.000000E+00	0.0040563409	f
55	8250	15	0.000000E+00	0.000000E+00	0.0042581308	f



56		8400		13		0.000000E+00		0.000000E+00		0.0019451644		f
57		8550		14		0.000000E+00		0.000000E+00		0.0040756506		f
58		8700		14		0.000000E+00		0.000000E+00		0.0016595430		f
59		8850		14		0.000000E+00		0.000000E+00		0.0040902713		ideal
60		9000		14		0.000000E+00		0.000000E+00		0.000000E+00		f
61		9150		14		0.000000E+00		0.000000E+00		0.000000E+00		f
62		9300		14		0.000000E+00		0.000000E+00		0.000000E+00		f
63		9450		14		0.000000E+00		0.000000E+00		0.000000E+00		f
64		9600		13		0.000000E+00		0.000000E+00		0.0012311058		f
65		9750		13		0.000000E+00		0.000000E+00		0.0012311058		f
66		9900		14		0.000000E+00		0.000000E+00		0.0059885720		f
67		10050		14		0.000000E+00		0.000000E+00		0.000000E+00		f
68		10200		14		0.000000E+00		0.000000E+00		0.000000E+00		f
69		10350		14		0.000000E+00		0.000000E+00		0.000000E+00		f
70		10500		14		0.000000E+00		0.000000E+00		0.000000E+00		f
71		10650		14		0.000000E+00		0.000000E+00		0.0000983841		f
72		10800		14		0.000000E+00		0.000000E+00		0.0009176864		f
73		10950		15		0.000000E+00		0.000000E+00		0.0317293086		nadir
74		11100		14		0.000000E+00		0.000000E+00		0.0004672766		f
75		11250		14		0.000000E+00		0.000000E+00		0.0004672766		f
76		11400		14		0.000000E+00		0.000000E+00		0.0004672766		f
77		11550		14		0.000000E+00		0.000000E+00		0.0004672766		f
78		11700		14		0.000000E+00		0.000000E+00		0.0004672766		f
79		11850		14		0.000000E+00		0.000000E+00		0.0004672766		f
80		12000		14		0.000000E+00		0.000000E+00		0.0004672766		f
81		12150		14		0.000000E+00		0.000000E+00		0.0004672766		f
82		12300		15		0.000000E+00		0.000000E+00		0.0032732863		f
83		12450		15		0.000000E+00		0.000000E+00		0.0009604547		f
84		12600		15		0.000000E+00		0.000000E+00		0.0009604547		f
85		12750		15		0.000000E+00		0.000000E+00		0.0009604547		f
86		12900		14		0.000000E+00		0.000000E+00		0.0017378491		f
87		13050		14		0.000000E+00		0.000000E+00		0.0017378491		f
88		13200		14		0.000000E+00		0.000000E+00		0.0017378491		f
89		13350		14		0.000000E+00		0.000000E+00		0.0017378491		f
90		13500		12		0.000000E+00		0.000000E+00		0.0058632315		f
91		13650		12		0.000000E+00		0.000000E+00		0.000000E+00		f
92		13800		12		0.000000E+00		0.000000E+00		0.000000E+00		f
93		13950		12		0.000000E+00		0.000000E+00		0.000000E+00		f
94		14100		12		0.000000E+00		0.000000E+00		0.000000E+00		f
95		14250		12		0.000000E+00		0.000000E+00		0.000000E+00		f
96		14400		12		0.000000E+00		0.000000E+00		0.000000E+00		f
97		14550		12		0.000000E+00		0.000000E+00		0.000000E+00		f
98		14700		13		0.000000E+00		0.000000E+00		0.0080206105		ideal
99		14850		13		0.000000E+00		0.000000E+00		0.000000E+00		f
100		15000		14		0.000000E+00		0.000000E+00		0.0046174059		f

```
weights = np.array([0.5, 0.5])
decomp = ASF()
```

```
with Recorder(Video("park_nsga2.mp4")) as rec:
    for entry in park_res.history:
```

```

# Get the approximated ideal and nadir points
approx_ideal = park_res.F.min(axis=0)
approx_nadir = park_res.F.max(axis=0)

# Normalize the obtained front
nF = (entry.pop.get("F") - approx_ideal) / (approx_nadir - approx_ideal)
park_I = decomp(nF, weights).argmin()
sc = Scatter(title="Generation: %s" % entry.n_gen)
sc.add(entry.pop.get("F"))
sc.add(entry.pop.get("F")[park_I], color="red", s=30)
sc.do()
rec.record()

with Recorder(Video("park_nsga2_sspy.mp4")) as rec:
    for entry in park_res.history:
        # Get the approximated ideal and nadir points
        approx_ideal = park_res.F.min(axis=0)
        approx_nadir = park_res.F.max(axis=0)

        # Normalize the obtained front
        nF = (entry.pop.get("F") - approx_ideal) / (approx_nadir - approx_ideal)
        park_I = decomp(nF, weights).argmin()
        park_X = entry.pop.get("X")[park_I]
        park_tgt = MultiSkewNorm()
        park_tgt.define_dp(
            np.array([park_X[0], park_X[1]]),
            np.array([[park_X[2], park_X[4]], [park_X[4], park_X[3]]]),
            np.array([park_X[5], park_X[6]]),
        )
        park_tgt.sample()
        ss = sspy.plotting.density_plot(
            data=pd.DataFrame({
                "ISOPleasant": park_tgt.sample_data[:,0],
                "ISOEventful": park_tgt.sample_data[:,1],
            }),
            # x=park_tgt.sample_data[:,0],
            # y=park_tgt.sample_data[:,1],
            title="Generation: %s" % entry.n_gen,
        )
        rec.record()

```

```

import matplotlib.pyplot as plt

park_X = park_res.X[park_I]
park_tgt = MultiSkewNorm()
park_tgt.define_dp(
    np.array([park_X[0], park_X[1]]),
    np.array([[park_X[2], park_X[4]], [park_X[4], park_X[3]]]),
    np.array([park_X[5], park_X[6]]),
)

```

```

from pymoo.decomposition.asf import ASF

# Get the approximated ideal and nadir points
approx_ideal = park_res.F.min(axis=0)
approx_nadir = park_res.F.max(axis=0)

# Normalize the obtained front
nF = (park_res.F - approx_ideal) / (approx_nadir - approx_ideal)

weights = np.array([0.48, 0.52])
decomp = ASF()

park_I = decomp(nF, weights).argmin()
print("Best regarding decomposition: Point %s - %s" % (park_I, park_res.F[park_I]))

Best regarding decomposition: Point 2 - [-0.71428571 -1.89225   ]

```

Figure 1

```

park_tgt.sample()

# print(park_tgt.summary())

plot = Scatter()
plot.add(park_res.F, color="blue", alpha=0.2, s=10)
plot.add(park_res.F[park_I], color="red", s=30)
plot.do()
# plot.apply(lambda ax: ax.arrow(0, 0, 0.5, 0.5, color='black',
#                                head_width=0.01, head_length=0.01, alpha=0.4))
plot.show()
plt.show()

# park_tgt.sspy_plot()
df = pd.DataFrame(park_tgt.sample_data, columns=["ISOPleasant", "ISOEventful"])
sspy.plotting.density_plot(
    df, color='red', title=None
)
plt.show()

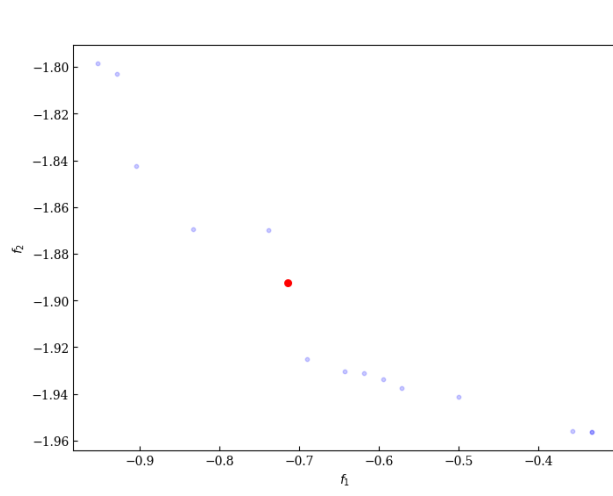
print(park_tgt.summary())

```

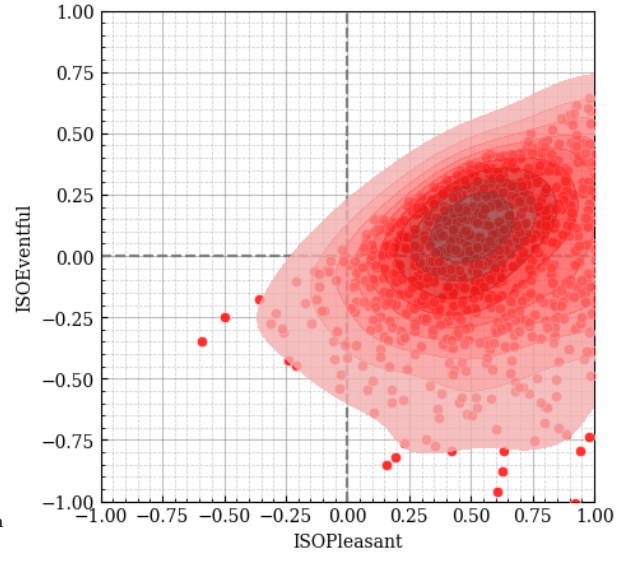
```

Fitted from direct parameters.
Direct Parameters:
xi:      [0.418 0.284]
omega: [[ 0.164 -0.044]
        [-0.044  0.152]]
alpha: [ 16.628 -30.187]

```



(a) Multi-objective optimization Pareto front. The selected solution is indicated in red.



(b) SCM distribution of the derived target distribution.

Figure 2: NSGA-II optimization to learn the MSN parameters which produce the Park ranking.

None

None

## References