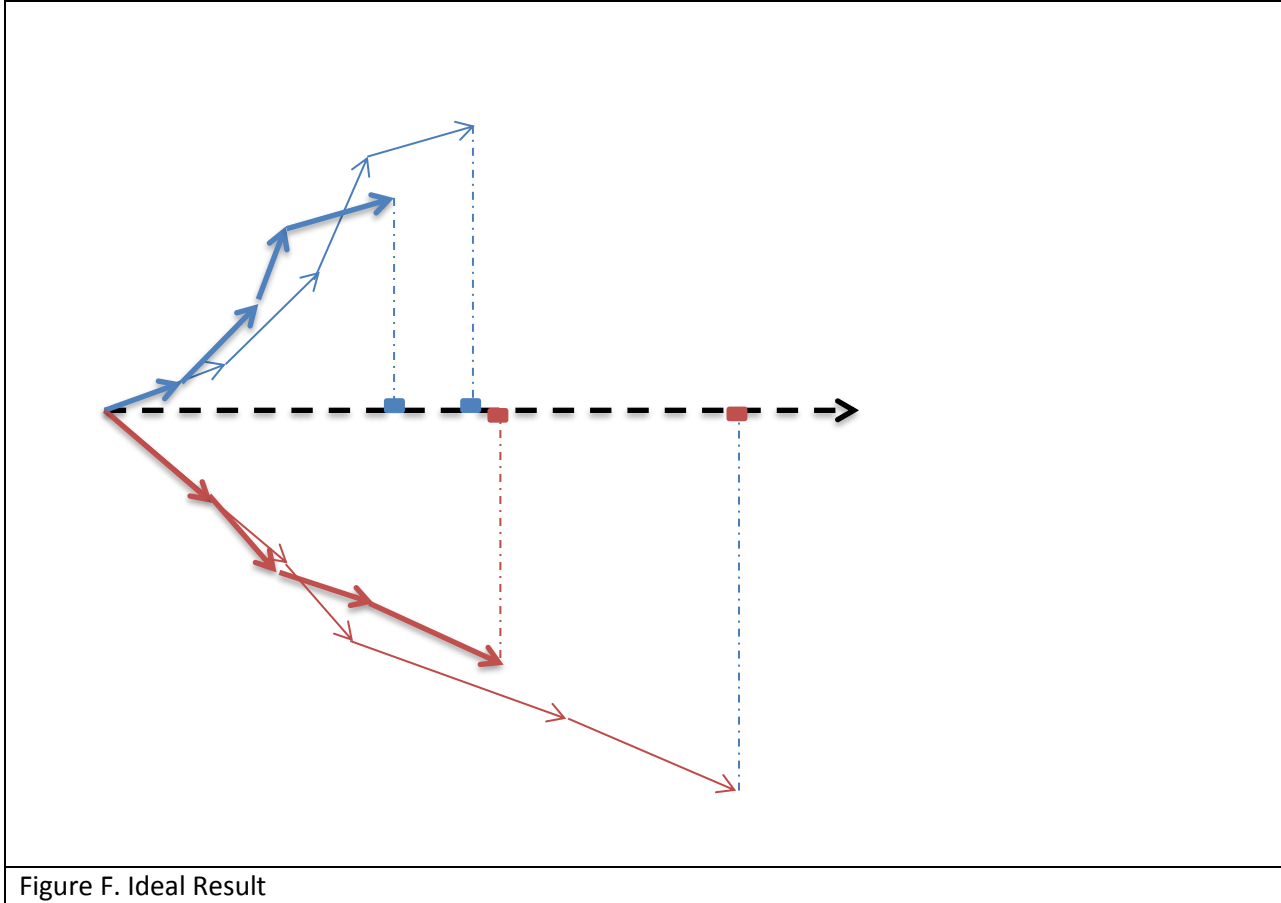


HW 7 Visualization of linear discrimination of two classes on n-D points  
Ideal result of visualization is shown in figure F below.



### Actual HW

You are given an equation  $y = -3x_1 + 2x_2 + 6x_3 + 5x_4 + 10$  that separates 4-D points of two classes  
Consider 4-D point  $A = (1, 1, 1, 1)$  with  $y = -3 \cdot 1 + 2 \cdot 1 + 6 \cdot 1 + 5 \cdot 1 + 10 = -3 + 2 + 6 + 5 + 10 = 20$  from class 1 and  
4-D point  $B = (1, 2, 1, 2)$  with  $y = -3 \cdot 1 + 2 \cdot 2 + 6 \cdot 1 + 5 \cdot 2 + 10 = -3 + 4 + 6 + 10 + 10 = 27$  from class 2.  
Formally here the classification rule is:

if  $y < 25$  then class 1 else class 2,

where  $T = 25$  is a threshold.

- 1) Form a set of coefficients  $C = (c_1, c_2, c_3, c_4, c_5) = (-3, 2, 6, 5, 10)$  and compute the absolute value of its max value, which is 10,

$$c_{\max} = \text{abs}(\max_{i=1:4}(c_i)) = 10$$

- 2) Normalize coefficients  $C$  by creating as set of normalized coefficients  $k_i$ :

$$k_i = c_i / c_{\max}$$

$$K = (k_1, k_2, k_3, k_4, k_5) = (-0.3, 0.2, 0.6, 0.5, 1),$$

Normalized equation  $y = -0.3x_1 + 0.2x_2 + 0.6x_3 + 0.5x_4 + 1.0$   
 Normalized rule: if  $y < 2.5$  then class 1 else class 2.

**Statement.** If classes are linearly separated with coefficients C then these classes are linearly separated with coefficients K.

**Proof.** The statement follows from the fact that all  $c_i$  and a threshold T are normalized by the same number.

- 3) Compute all angles  $Q_i = \arccos(\text{abs}(k_i))$  of absolute values of  $k_i$ .

$\arccos(Q)$	angle Q in radians	angle Q in degree
0.3	$Q_1 = 1.27$	72.8
0.2	$Q_2 = 1.37$	78.5
0.6	$Q_3 = 0.93$	53.3
0.5	$Q_4 = 1.05$	60.2

- 4) Draw figure 1

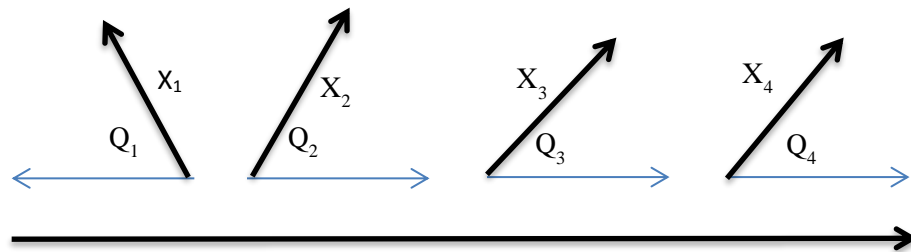


Figure 1. GLC coordinates  $X_1$ - $X_4$  with angles  $(Q_1, Q_2, Q_3, Q_4)$ .  $X_1$  is directed to the left due to negative  $k_1 = -0.3$ . Always draw coordinates for negative  $k_i$  directed to the left.

- 5) Draw Figure 2

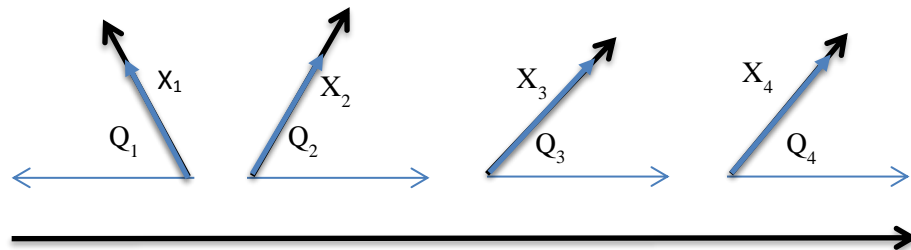


Figure 2. 4-D point  $A = (1, 1, 1, 1)$  in GLC coordinates  $X_1$ - $X_4$  with angles  $(Q_1, Q_2, Q_3, Q_4)$ . The length of all  $x_i$  is the same,  $x_i = 1$ .

6) Draw figure 3

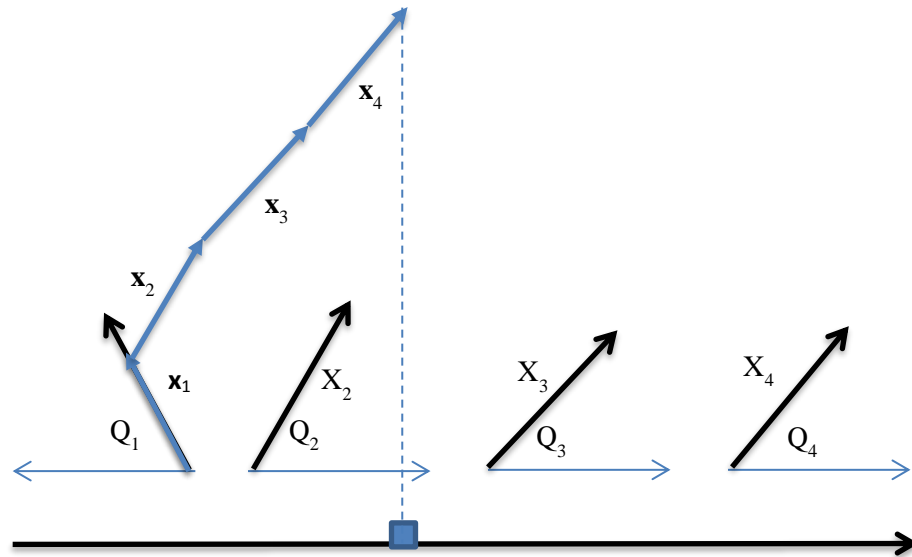


Figure 3. 4-D point  $A=(1,1,1,1)$  in GLC coordinates  $X_1-X_4$  with angles  $(Q_1, Q_2, Q_3, Q_4)$  with vectors  $x_i$  shifted to be connected one after another and the end of last vector projected to the black line.

7) Draw figure 4

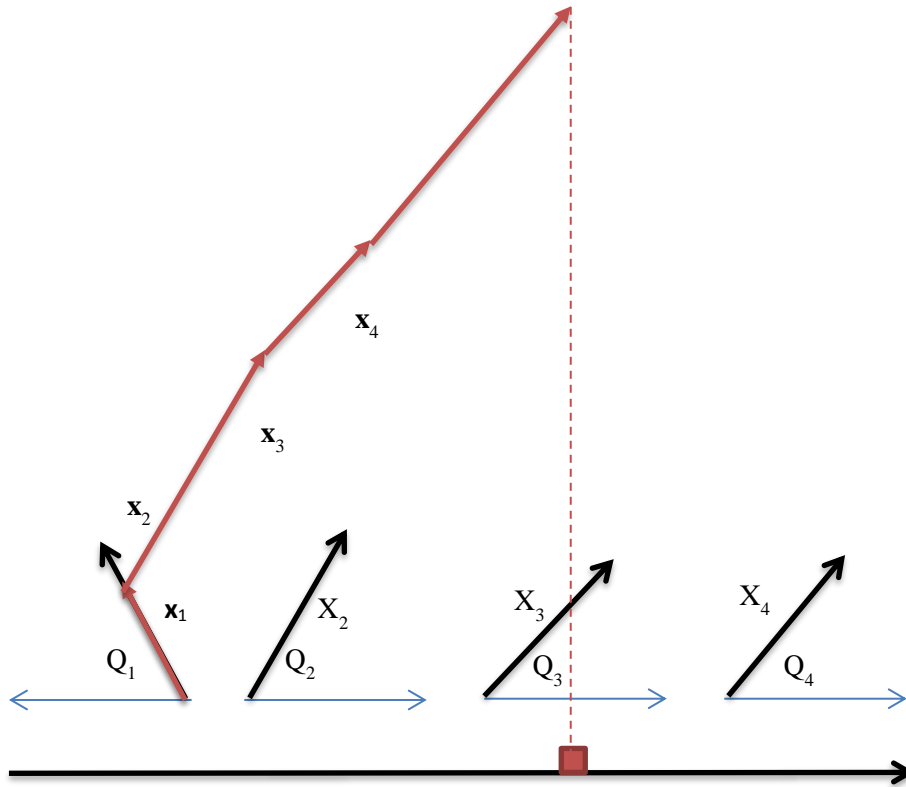


Figure 4. 4-D point  $B=(1,2,1,2)$  in GLC coordinates  $X_1-X_4$  with angles  $(Q_1, Q_2, Q_3, Q_4)$  with vectors  $x_i$  shifted to be connected one after another and the end of last vector projected to the black line.

- 8) Draw figures 3 and 4 jointly with two 4-D points A and B in different colors
- 9) Generalize code for 8) to 10 dimensions and 100 10-D points of two classes visualized for the given linear equation
 
$$y = c_1x_1 + c_2x_2 + c_3x_3 + \dots + c_nx_n + c_{n+1}$$
 with reading input data and coefficients  $C=(c_1, c_2, \dots, c_{10})$  from the file.
- 10) Run experiments on 3 datasets with 100 data in each with up to 10 dimensions.
- 11) **Bonus task.** Draw figure 5 and combine it with Figure 3 for the general case of 10-D and 100 10-D points to get visualization similar to the figure F (see page 1) and conduct experiments 10) for these “ideal” visualization. If you do this bonus task you can run experiments 10) only for this visualization. In this visualization all lines of the blue class are above the black line and all lines of red class are below the black line.

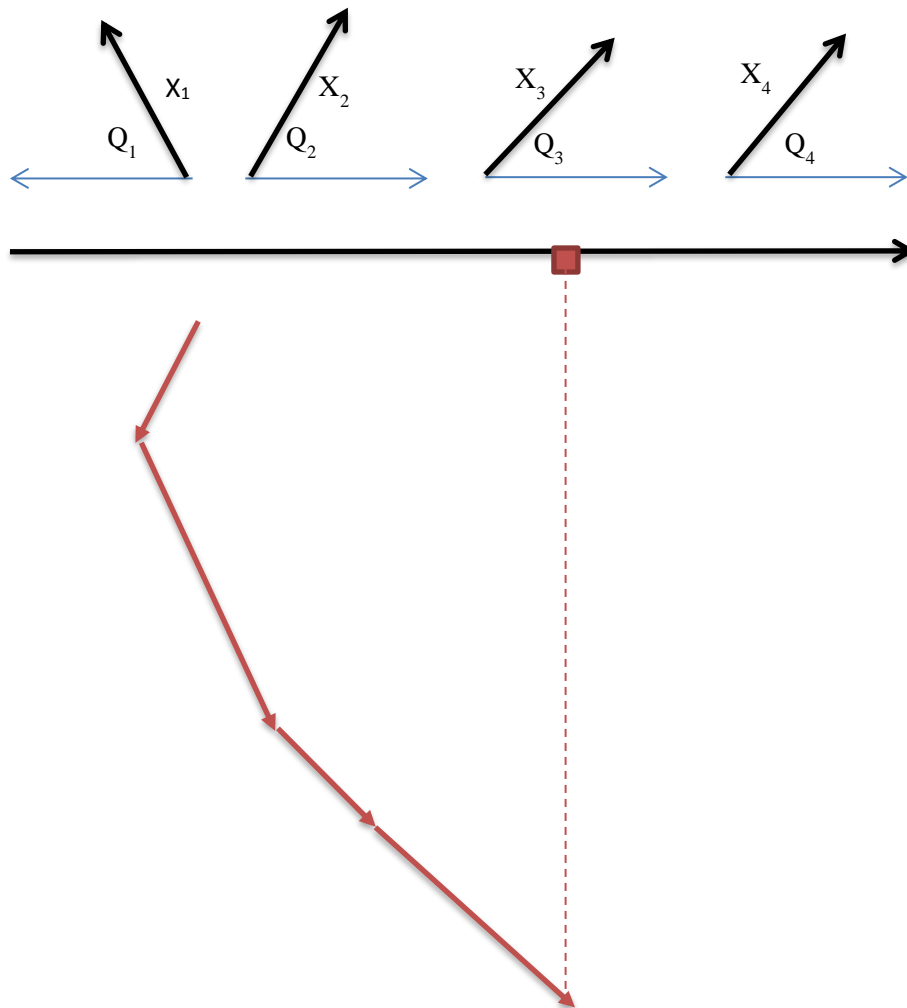


Figure 5. Mirror of 4-D point  $B=(1,2,1,2)$  of the red class relative to the black line from Figure 4.