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O.R. Applications

Application of Benders' decomposition to power plant preventive maintenance scheduling

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Abstract

Power plant preventive maintenance scheduling consists of knowing which generating units to take off line for regular safety inspection. This problem is extremely important because a failure in a power station may cause a general breakdown in an electric network. The principal danger is that customer electricity demand will not be satisfied in such cases. The problem is approached from the operations research perspective as a question of optimisation. Benders' decomposition technique is used to solve the resulting model. An example of this application is included. The algorithm is put to use in a real power plant setting. The obtained result is a schedule that allows the efficient organisation of preventive maintenance over the time horizon considered.

Keywords: Global optimisation; Benders' decomposition; Maintenance; Power plant; Scheduling

1. Introduction

In this paper, the problem of power plant preventive maintenance scheduling is approached. The issue to be solved is how to determine the period for which generating units of an electric system should be taken off line for planned preventive maintenance over a specific time horizon. Benders' decomposition method is used for the resolution of this optimisation problem. A new model for the problem and the way to solve it are the most outstanding points presented.

Preventive maintenance consists of the periodic inspection of a power plant to detect potential failures. This is planned maintenance of the plant equipment and facilities that is designed to prolong their useful life and avoid any unplanned maintenance activity. Its purpose is to minimize breakdowns and excessive depreciation during normal functioning. Neither equipment nor facilities should be allowed to reach the breaking point. An appropriate preventive maintenance program should include non-destructive testing, periodic inspection,

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pre-planned maintenance activities, and maintenance to correct deficiencies found through testing or inspections. The amount of preventive maintenance needed at a power plant can vary greatly.

The problem of power plant preventive maintenance scheduling is usually dealt with in the framework of the long term exploitation of electric energy production systems (Yamayee, 1982; Kralj and Petrovic, 1988). The importance of the problem under study springs from the real necessity to shut down power plants regularly and review the functioning of the machinery, the complementary equipment, and the facilities. The main aim is to maintain efficiency. Because power plants are integrated into a global electric system, it is possible for an unforeseen failure to affect the rest of the system. Hence, an unexpected shutdown in a power plant may provoke an undesirable interruption in the electric supply. Customer dissatisfaction is the immediate consequence. The information process has a great influence on this problem (Swanson, 2003).

Different authors have focused their attention on power plant preventive maintenance planning with a wide variety of methodologies, namely: heuristic techniques (Christiaanse and Palmer, 1972; De Cuadra et al., 1995; Shimomura et al., 2002), mixed integer programming (Dopazo and Merill, 1975; Escudero et al., 1980; Gurevich et al., 1996; Ben-Daya et al., 2000), dynamic programming (Zürn and Quintana, 1975; Agogino et al., 1999), stochastic programming (Silva et al., 1995; Chattopadhyay, 2004; Doyle, 2004), decomposition methods (Yellen et al., 1992; Marwali and Shahidehpour, 1998), tabu search (El-Amin et al., 2000), expert systems (Lin et al., 1992), and multiobjective optimisation (Kralj and Rajakovic, 1994). They apply different models to the problem. Currently, in most cases the industry solves this problem via heuristic techniques.

2. Problem description

It is possible to distinguish between two aspects of the problem of power plant preventive maintenance scheduling:

- Minimization of the reliability impact (Zürn and Quintana, 1977; Gertsbakh, 2000) and
- Minimization of the economic impact (Ben-Daya et al., 2000; Chattopadhyay, 2004).

The electric energy demand must be supplied under an adequate reliability level. Moreover, the associated cost of shutdown an electric generator set has to be the minimum possible.

The problem being dealt with is combinatorial and non-linear, although the model used is linear. The linearity of the model is an advantage, but its being combinatorial is an inconvenience. The complexity of planning power plant preventive maintenance is due to the enormous size of the system to be modelled. There are a high number of variables, including binary variables. They make the resolution of the problem difficult. This constrains the level of detail represented in the model so that computational efficiency could be reduced. Balancing complexity, problem size, model calculating time, and reality approximation level is essential.

2.1. Time horizon and study periods

The typical time horizon selected is 1 year. This time horizon may be expressed as 13 months or 52 weeks. A month is supposed to have 4 weeks. The study periods to divide the time horizon can be either 13 months or 52 weeks.

2.2. Maintenance duration

Maintenance duration depends on the type of power plant, including:

- *Thermal power plants*: coal (4 weeks), fuel oil, and natural gas (3 weeks).
- *Nuclear power plants*: 6 weeks and
- *Hydroelectric power plants*: variable duration according to each power plant.

The same maintenance duration will be considered for all power plants. Specifically, maintenance duration will be 1 month or 1 period.

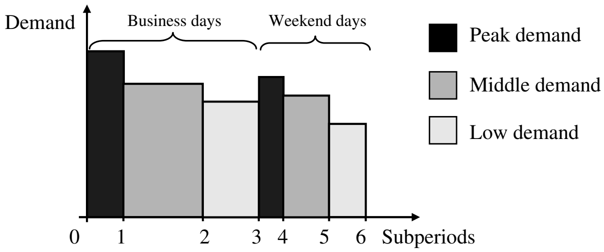

Fig. 1. Qualitative variation of power demand in each subperiod.

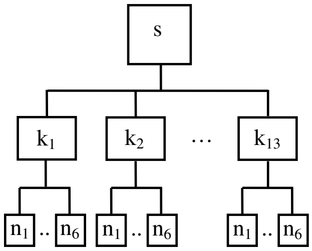

Fig. 2. Structure scenario-period-subperiod.

2.3. Some considerations about electric demand

One period in the time horizon is equal to one month. In each month two parts are distinguished: business days and weekend days, according to the electric energy demand (larger in the first part than in the second one). Each part is divided into three different subparts depending on the electric demand. Those above mentioned subparts, from higher to lower demand are: peak demand, middle demand and low demand. This classification provides six subperiods with distinct duration. Qualitatively, the electric power demand variation in relation to the subperiods included in a period is presented in Fig. 1.

The previous distribution is repeated three times because three electric demand scenarios are taken into account: high (s_{h}), medium (s_{m}), and low (s_{l}). This is the way to model uncertainty in power demand. Each scenario has a probability associated with it according to the chance of occurrence. Following an intuitive criterion, s_{m} has the highest probability whereas s_{l} has the smallest. The time structure is represented in Fig. 2.

The meaning for every element is:

$s =$ Power demand scenario $(s_{\mathrm{h}}, s_{\mathrm{m}}, s_{\mathrm{l}})$,
$k =$ Period (from 1 to 13) and
$n =$ Subperiod (from 1 to 6).

3. Problem formulation

The problem is an optimisation problem. Objectives such as cost minimization or reliability maximization can be proposed, satisfying a set of constraints.

3.1. Main variables

The most relevant variables to know in this problem are the maintenance variables, whose notation is $x_{i,k}$. They are binary variables (0/1) and indicate:

$x_{i,k} = 0 \Rightarrow$ Generator i is not in maintenance in the period k,
$x_{i,k} = 1 \Rightarrow$ Generator i is in maintenance in the period k.

Other variables 0/1 directly connected are $c_{i,k}$. They denote the maintenance start-up:

$c_{i,k} = 0 \Rightarrow$ Maintenance of generator i does not start at the beginning of the period k,
$c_{i,k} = 1 \Rightarrow$ Maintenance of generator i starts at the beginning of the period k.

3.2. Objective function

Costs are selected as the objective function for this problem (Gurevich et al., 1996; Mukerji et al., 1991). General costs, operation costs or exploitation costs can be divided into three types (Chattopadhyay et al., 1995):

- *Start-up cost*
  This is the cost to put a generator into operation after being disconnected.
- *Production cost*
  This cost refers to the cost of producing 1 MW h in a generator and
- *Maintenance cost*
  This is the cost to put a generator into preventive maintenance.

Fixed costs and shutdown costs are not taken into account. Maintenance costs are insignificant with regard to start-up and production costs, given that the first ones are much smaller than the second ones (on the order of one thousand times) and third ones (on the order of one million times). Hydroelectric generators do not have start-up costs or production costs. The expression is

$$\text{Minimize} \quad \left[\sum_{i\in I}\sum_{s\in S}\sum_{k\in K}\sum_{n\in N} p_s \cdot (h_{i,s,k,n} + w_{i,s,k,n}) + \sum_{i\in I}\sum_{k\in K} z_{\mathrm{i,k}} \cdot x_{i,k}\right],$$

where

$h_{i,s,k,n} =$ Start-up cost of generator i, in subperiod n, of period k, in scenario s (\$),
$w_{i,s,k,n} =$ Production cost of generator i, in subperiod n, of period k, in scenario s (\$),
$z_{i,k} =$ Maintenance cost of generator i, in period k (\$) and
$p_s =$ Probability of demand scenario s.

Start-up and production costs are included in the first term. Maintenance cost is considered in the second term. Substituting each cost for its value results in

$$\text{Minimize} \quad \left[\sum_{i\in I}\sum_{s\in S}\sum_{k\in K}\sum_{n\in N} p_s \cdot (f_i \cdot y_{i,s,k,n} + g_i \cdot t_{i,s,k,n} \cdot \tau_n) + \sum_{i\in I}\sum_{k\in K} z_{i,k} \cdot x_{i,k}\right],$$

where

$f_i =$ Start-up cost of generator i (\$),
$y_{i,s,k,n} =$ Start-up variable of generator i, at the beginning of subperiod n, of period k, in scenario s,
$g_i =$ Electric energy cost produced by generator i (\$/MW h),
$t_{i,s,k,n} =$ Output electric power of generator i, in subperiod n, of period k, in scenario s (MW) and
$\tau_n =$ Duration of subperiod n (h).

$y_{i,s,k,n}$ is a 0/1 variable whose value is given by the next criterion:

$y_{i,s,k,n} = 0 \Rightarrow$ Generator i does not start at the beginning of subperiod n, of period k, in scenario s,
$y_{i,s,k,n} = 1 \Rightarrow$ Generator i starts at the beginning of subperiod n, of period k, in scenario s.

The cost of the electric energy produced by a generator is only applicable to thermal and nuclear power plants. Hydroelectric power plants do not have production costs giving that the resource used is free. The start-up costs are also equal to zero. The final aim is

$$\text{Minimize} \quad \sum_{i\in I}\sum_{s\in S}\sum_{k\in K}\sum_{n\in N} p_s \cdot \left(f_i \cdot y_{i,s,k,n} + g_i \cdot t_{i,s,k,n} \cdot \tau_n\right).$$

3.3. Constraints

There are four set of constraints (Vemuri and Lemonidis, 1990; Ben-Daya et al., 2000): maintenance, economic unit commitment, maintenance and connection, and generating volume constraints. Some of them are newly modelled.

3.3.1. Maintenance constraints

(1) *Maintenance duration constraint*
The maintenance of the generator i lasts a given number of periods, β_i.

$$\sum_{k\in K} x_{i,k} = \beta_i \quad \forall i \in I.$$

(2) *Period constraint*
A maximum number of maintenance ψ_k is imposed in the period k.

$$\sum_{i\in I} x_{i,k} \leqslant \psi_k \quad \forall k \in K.$$

(3) *Non-stop maintenance constraint*
The maintenance of a generator is carried out in consecutive periods.

$$x_{i,k} - x_{i,k-1} \leqslant c_{i,k} \quad \forall i \in I \ \forall k \in K.$$

For $k = 1$ select $x_{i,0} = 0$.

(4) *Precedence constraint*
This constraint establishes the order to follow in power plant maintenance. If the maintenance for generator i is prior to the maintenance for generator j, the expression is

$$\sum_{k_n=1}^{k} c_{i,k_n} - c_{j,k} \geqslant 0 \quad \forall k \in K,$$

$$c_{i,k} + c_{j,k} \leqslant 1 \quad \forall k \in K.$$

The index k_n that varies from period 1 until period k.

(5) *Exclusion constraint*
Generators i and j cannot be in maintenance at the same time.

$$x_{i,k} + x_{j,k} \leqslant 1 \quad \forall k \in K.$$

(6) *Time constraint*
Interval constraint
A number of "e" periods are introduced between generators i and j shutdown for maintenance. A sequence is provided. Considering δ as the time horizon, the formula is

$$c_{i,k} = c_{j,k+\beta_i+e} \quad 1 \leqslant k \leqslant \delta - \beta_i - e,$$

$$\sum_{k=1}^{\delta-\beta_i-e} \left(c_{i,k} + c_{j,k+\beta_i+e}\right) = 2.$$

Overlap constraint
The maintenance of generators i and j have an overlap of "u" periods. The generator i is the first one out of service.

$$c_{i,k} = c_{j,k+\beta_i-u} \quad 1 \leqslant k < \delta - \beta_i + u.$$

(7) *One-time maintenance constraint*
Each generator has an outage for maintenance only once along the time horizon considered.

$$\sum_{k \in K} c_{i,k} = 1 \quad \forall i \in I.$$

Workforce and *material resources constraints* could be modelled, since they are limited. However, they may be included in period, precedence, exclusion, or time constraints.

3.3.2. Economic unit commitment constraints

(1) *Generated power limit constraint*
Each generator is designed to work between minimum and maximum power capacity (MW).

$$v_{i,s,k,n} \cdot \underline{t}_i \leqslant t_{i,s,k,n} \leqslant v_{i,s,k,n} \cdot \overline{t}_i \quad \forall i \in I \;\; \forall s \in S \;\; \forall k \in K \;\; \forall n \in N,$$

where
$v_{i,s,k,n}$ = Connecting variable for generator i, in subperiod n, of period k, in scenario s,
$\underline{t}_i$ = Nominal minimum power or technical minimum for generator i (MW) and
$\overline{t}_i$ = Nominal maximum power or technical maximum for generator i (MW).

$v_{i,s,k,n}$ is a 0/1 variable whose value is given according this criterion:
$v_{i,s,k,n} = 0 \Rightarrow$ Generator i is not connected in subperiod n, of period k, in scenario s.
$v_{i,s,k,n} = 1 \Rightarrow$ Generator i is connected in subperiod n, of period k, in scenario s.

(2) *Demand supply constraint*
This constraint establishes a power balance: power production in each subperiod has to be equal to electric demand (constant in the considered subperiod).

$$\sum_{i \in I} t_{i,s,k,n} = d_{s,k,n} \quad \forall s \in S \;\; \forall k \in K \;\; \forall n \in N.$$

(3) *Reserve constraint*
The reserve is the power provided if a generator fails. It is a safety margin. Usually, it is given as a demand proportion or the output maximum power of the biggest power plant on line. If $rr_{s,k,n}$ (MW) is the reserve in subperiod n, of period k, in scenario s, it results:

$$\sum_{i \in I} v_{i,s,k,n} \cdot \overline{t}_i \geqslant d_{s,k,n} + rr_{s,k,n} \quad \forall s \in S \;\; \forall k \in K \;\; \forall n \in N.$$

(4) *Start-up constraint*
It establishes the start-up logic for thermal and nuclear power plants. $y_{i,s,k,n}$ has to satisfy it to model correctly the start-up costs.

$$y_{i,s,k,n} \geqslant v_{i,s,k,n} - v_{i,s,k,n-1} \quad \forall i \in I - I_2 \;\; \forall s \in S \;\; \forall k \in K \;\; \forall n \in N.$$

I_2 is the index set for hydroelectric generators. When $n = 1$ the applied contour condition is the corresponding one to the last subperiod of the previous period (in the case of $k = 1$, 0 is selected for connecting variable).

3.3.3. Maintenance and connection constraints

The correlation between maintenance variables, $x_{i,k}$, and connection variables, $v_{i,s,k,n}$, is

$$x_{i,k} = 0 \Rightarrow v_{i,s,k,n} = 0/1 \quad \text{and} \quad x_{i,k} = 1 \Rightarrow v_{i,s,k,n} = 0.$$

That expression can be expressed in this unique formula

$$x_{i,k} + v_{i,s,k,n} \leqslant 1.$$

For nuclear power plants only equality exists, because they are always connected, except when in maintenance. Therefore:

$$x_{i_1,k} + v_{i_1,s,k,n} \leqslant 1 \quad \forall i_1 \in I_1 \;\; \forall s \in S \;\; \forall k \in K \;\; \forall n \in N$$
$$x_{i_2,k} + v_{i_2,s,k,n} \leqslant 1 \quad \forall i_2 \in I_2 \;\; \forall s \in S \;\; \forall k \in K \;\; \forall n \in N,$$
$$x_{i_3,k} + v_{i_3,s,k,n} = 1 \quad \forall i_3 \in I_3 \;\; \forall s \in S \;\; \forall k \in K \;\; \forall n \in N,$$

where

I_1 = Index set for thermal power plants and
I_3 = Index set for nuclear power plants.

3.3.4. Generating volume constraints

(1) *Minimum volume constraint*
This constraint is related to a particular country. In terms of applicability, the use of this constraint depends on the specific electric system considered. It is used for coal thermal power plants. A minimum production using national coal is required. The reason is maintaining jobs in the sector, although it is not profitable.

$$\sum_{n \in N} t_{i_1,s,k,n} \cdot \tau_n \geqslant \underline{E}_{i_1} \cdot (1 - x_{i_1,k}) \quad \forall i_1 \in I_1 \;\; \forall s \in S \;\; \forall k \in K.$$

$\underline{E}_{i_1}$ is the minimum energy (MW h) to be produced by generator i_1. τ_n is the duration of subperiod n (hour).

(2) *Maximum value constraint*
It is applied to thermal power plants. Legislation imposes a maximum limit of energy production to reduce the environmental impact.

$$\sum_{n \in N} t_{i_1,s,k,n} \cdot \tau_n \leqslant \overline{E}_{i_1} \cdot (1 - x_{i_1,k}) \quad \forall i_1 \in I_1 \;\; \forall s \in S \;\; \forall k \in K.$$

$\overline{E}_{i_1}$ is the maximum energy (MW h) to be produced by the thermal generator i_1.

(3) *Water volume constraint*
It is related to hydroelectric generators. Basin water reserve cannot only be used to produce electricity (human consumption, irrigation, etc.).

$$\sum_{n \in N} t_{i_2,s,k,n} \cdot \tau_n = E_{i_2,s,k} \cdot (1 - x_{i_2,k}) \quad \forall i_2 \in I_2 \;\; \forall s \in S \;\; \forall k \in K.$$

$E_{i_2,s,k}$ is the energy (MW h) to be produced by the hydroelectric generator i_2 in period k of scenario s (MW).

4. Model and methodology to solve the problem under study

Benders' decomposition method (Benders, 1962) is based on decomposing the original problem into a master problem and a subproblem. The original problem can be divided into periods, which makes the resolution more attainable. In addition, the efficiency is increased because a highly complex problem is tackled by solving smaller and homogeneously structured problems. The procedure is iterative and convergence is required. Thus, this methodology (Al-Khamis et al., 1992; Yellen et al., 1992) is useful.

4.1. Global model

In compliance with the above objective function and constraints, the problem is modelled like a 0/1 mixed integer linear programming problem of the form:

$$
\begin{aligned}
\text{Minimize}\quad & \sum_{i\in I}\sum_{s\in S}\sum_{k\in K}\sum_{n\in N} p_s\cdot(f_i\cdot y_{i,s,k,n}+g_i\cdot t_{i,s,k,n}\cdot\tau_n)\\
\text{subject to}\quad & \sum_{k\in K} x_{i,k}=\beta_i \quad \forall i\in I,\\
& \sum_{i\in I} x_{i,k}\leqslant \psi_k \quad \forall k\in K,\\
& x_{i,k}-x_{i,k-1}\leqslant c_{i,k} \quad \forall i\in I\ \forall k\in K,\\
& \sum_{k_n=1}^{k} c_{i,k_n}-c_{j,k}\geqslant 0 \quad \forall k\in K,\\
& c_{i,k}+c_{j,k}\leqslant 1 \quad \forall k\in K,\\
& x_{i,k}+x_{j,k}\leqslant 1 \quad \forall k\in K,\\
& c_{i,k}=c_{j,k+\beta_i+e} \quad 1\leqslant k\leqslant \delta-\beta_i-e,\\
& \sum_{k=1}^{\delta-\beta_i-e}(c_{i,k}+c_{j,k+\beta_i+e})=2,\\
& c_{i,k}=c_{j,k+\beta_i-u} \quad 1\leqslant k\leqslant \delta-\beta_i+u,\\
& \sum_{k\in K} c_{i,k}=1 \quad \forall i\in I,\\
& v_{i,s,k,n}\cdot \underline{t}_i\leqslant t_{i,s,k,n}\leqslant v_{i,s,k,n}\cdot \overline{t}_i \quad \forall i\in I\ \forall s\in S\ \forall k\in K\ \forall n\in N,\\
& \sum_{i\in I} t_{i,s,k,n}=d_{s,k,n} \quad \forall s\in S\ \forall k\in K\ \forall n\in N,\\
& y_{i,s,k,n}\geqslant v_{i,s,k,n}-v_{i,s,k,n-1} \quad \forall i\in I-I_2\ \forall s\in S\ \forall k\in K\ \forall n\in N,\\
& x_{i_1,k}+v_{i_1,s,k,n}\leqslant 1 \quad \forall i_1\in I_1\ \forall s\in S\ \forall k\in K\ \forall n\in N,\\
& x_{i_2,k}+v_{i_2,s,k,n}\leqslant 1 \quad \forall i_2\in I_2\ \forall s\in S\ \forall k\in K\ \forall n\in N,\\
& x_{i_3,k}+v_{i_3,s,k,n}=1 \quad \forall i_3\in I_3\ \forall s\in S\ \forall k\in K\ \forall n\in N,\\
& \sum_{n\in N} t_{i_1,s,k,n}\cdot\tau_n\geqslant \underline{E}_{i_1}\cdot(1-x_{i_1,k}) \quad \forall i_1\in I_1\ \forall s\in S\ \forall k\in K,\\
& \sum_{n\in N} t_{i_1,s,k,n}\cdot\tau_n\leqslant \overline{E}_{i_1}\cdot(1-x_{i_1,k}) \quad \forall i_1\in I_1\ \forall s\in S\ \forall k\in K,\\
& \sum_{n\in N} t_{i_2,s,k,n}\cdot\tau_n= E_{i_2,s,k}\cdot(1-x_{i_2,k}) \quad \forall i_2\in I_2\ \forall s\in S\ \forall k\in K,\\
& x_{i,k},c_{i,k},v_{i,s,k,n}\in\{0,1\} \quad \forall i\in I\ \forall s\in S\ \forall k\in K\ \forall n\in N,
\end{aligned}
$$

where

I = Index set for power plants,
I_1 = Index set for thermal power plants,
I_2 = Index set for hydroelectric power plants,
I_3 = Index set for nuclear power plants,
S = Index set for demand scenarios,
K = Index set for periods in demand scenario,
N = Index set for subperiods in a period,
p_s = Probability for demand scenario s,
f_i = Start-up cost of generator i ($),
$y_{i,s,k,n}$ = Start-up variable of generator i, at the beginning of subperiod n, of period k, in scenario s,
g_i = Electric energy cost produced by generator i ($/MW h),
$t_{i,s,k,n}$ = Output electric power of generator i, in subperiod n, of period k, in scenario s (MW),
τ_n = Duration of subperiod n (h),
$x_{i,k}$ = Maintenance variable of generator i in period k,
β_i = Maintenance duration for generator i (periods),
ψ_k = Maximum number of maintenances in period k,

$c_{i,k}$ = Maintenance start-up variable of generator i in period k,
k_n = Index that varies from period 1 until period k,
e = Number of periods between two given maintenances,
δ = Time horizon,
u = Number of overlap periods between two given maintenances,
$\underline{t}_i$ = Nominal minimum power or technical minimum for generator i (MW),
$\overline{t}_i$ = Nominal maximum power or technical maximum for generator i (MW),
$d_{s,k,n}$ = Power demand in subperiod n, of period k, in scenario s (MW),
$rr_{s,k,n}$ = Power reserve for subperiod n, of period k, in scenario s (MW),
$\underline{E}_{i_1}$ = Minimum level of energy to be produced by thermal generator i_1 (MW h),
$\overline{E}_{i_1}$ = Maximum level of energy to be produced by thermal generator i_1 (MW h) and
$E_{i_2,s,k}$ = Energy to be produced by hydroelectric generator i_2 in period k of scenario s (MW h).

4.2. Problem size

Number of variables

$$\mathrm{card}(I) \times \mathrm{card}(K) \times [3 \times \mathrm{card}(S) \times \mathrm{card}(N) + 2].$$

Number of constraints

$$2 \times \mathrm{card}(I) + [1 + 2 \times r_1 + r_2 + \mathrm{card}(I)] \times \mathrm{card}(K) + r_3 \cdot (1 + \delta) - \sum_{l \in L} \beta_l - \sum_{r_3 \in R_3} e_{r_3} + r_4 \times \delta - \sum_{q \in Q} \beta_q$$
$$+ \sum_{r_4 \in R_4} u_{r_4} + \{[2 + 4 \times \mathrm{card}(I) - \mathrm{card}(I_2)] \times \mathrm{card}(N) + 2 \times \mathrm{card}(I_1) + \mathrm{card}(I_2)\} \times \mathrm{card}(S) \times \mathrm{card}(K),$$

where

r_1 = Number of precedence constraints,
r_2 = Number of exclusion constraints,
R_3 = Index set for interval constraints,
r_3 = card (R_3),
L = Index set for those generators whose maintenance take place first in the interval constraint,
R_4 = Index set for overlap constraints,
r_4 = card (R_4) and
Q = Index set for those generators whose maintenance takes place first in the overlap constraint.

4.3. Solution methodology

If well structured, the modelled problem's features allow the application of Benders' partitioning to solve it (Geoffrion, 1972; Pereira et al., 1985; Holmberg, 1994). There are:

- *Complicating variables*: $x_{i,k}$ and
- *Non-complicating variables*: $v_{i,s,k,n}$, $y_{i,s,k,n}$, $t_{i,s,k,n}$.

The start-up maintenance variables ($c_{i,k}$) are not considered as complicating variables, because they are connected with maintenance variables ($x_{i,k}$).

The resolution problem process via Benders' decomposition method contains the next stages: see Fig. 3.

4.3.1. Subproblems

Structure

Since generation volume constraints are modelled by groups of periods, there is a subproblem per period. The global number of subproblems is equal to the number of demand scenarios multiplied by the number of periods in each one. Each subproblem is

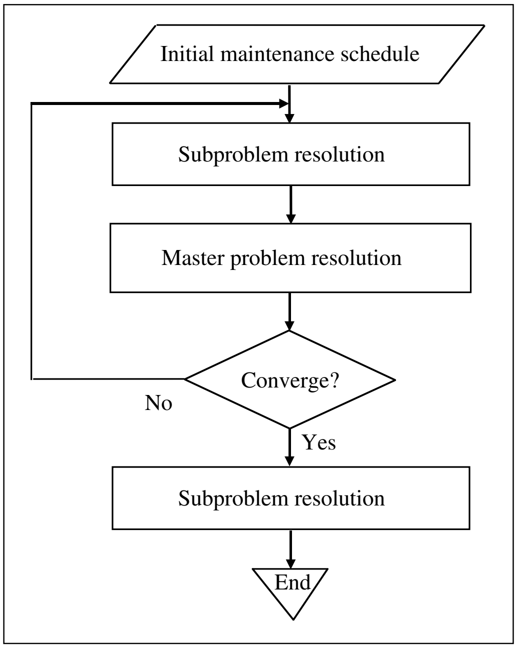

Fig. 3. Benders' decomposition flow chart.

$$
\begin{aligned}
\text{Minimize} \quad & \sum_{i\in I}\sum_{n\in N}(f_i \cdot y_{i,s,k,n} + g_i \cdot t_{i,s,k,n}\cdot \tau_n) \\
\text{subject to} \quad & v_{i,s,k,n}\cdot \underline{t}_i \leqslant t_{i,s,k,n} \leqslant v_{i,s,k,n}\cdot \overline{t}_i \quad \forall i\in I\ \forall n \in N, \\
& \sum_{i\in I} t_{i,s,k,n} = d_{s,k,n} \quad \forall n\in N, \\
& \sum_{i\in I} v_{i,s,k,n}\cdot \overline{t}_i \geqslant d_{s,k,n} + rr_{s,k,n} \quad \forall n \in N, \\
& \sum_{n\in N} t_{i_1,s,k,n}\cdot \tau_n \geqslant \underline{E}_{i_1}\cdot(1-x_{i_1,k}) \quad \forall i_1 \in I_1, \\
& \sum_{n\in N} t_{i_1,s,k,n}\cdot \tau_n \leqslant \overline{E}_{i_1}\cdot(1-x_{i_1,k}) \quad \forall i_1 \in I_1, \\
& \sum_{n\in N} t_{i_2,s,k,n}\cdot \tau_n = E_{i_2,s,k}\cdot(1-x_{i_2,k}) \quad \forall i_2 \in I_2, \\
& x_{i,k} = x^c_{i,k} : \lambda_{i,k}, \\
& x_{i,k}, v_{i,s,k,n} \in \{0,1\} \quad \forall i\in I\ \forall n\in N,
\end{aligned}
$$

where

$x^c_{i,k}$ = Known value for maintenance variable of generator i in period k (obtained from initial maintenance scheduling or from master problem resolution in previous iteration) and
$\lambda_{i,k}$ = Dual variable associated to $x_{i,k} = x^c_{i,k}$ constraint.

All subproblems are 0/1 mixed integer linear programming problem.
Size
Number of variables: $3 \times \text{card}(I) \times \text{card}(N)$.
Number of constraints: $[2 + 4 \times \text{card}(I) - \text{card}(I_2)] \times \text{card}(N) + 2 \times \text{card}(I_1) + \text{card}(I_2)$.

4.3.2. Master problem

Structure
Due to original problem characteristic, the master problem is formulated as follows:

$$\begin{aligned}
\text{Minimize}\quad & \alpha \\
\text{subject to}\quad & \sum_{k\in K} x_{i,k} = \beta_i \quad \forall i\in I, \\
& \sum_{i\in I} x_{i,k} \leqslant \psi_k \quad \forall k\in K, \\
& x_{i,k} - x_{i,k-1} \leqslant c_{i,k} \quad \forall i\in I\ \forall k\in K, \\
& \sum_{k_n=1}^{k} c_{i,k_n} - c_{j,k} \geqslant 0 \quad \forall k\in K, \\
& c_{i,k} + c_{j,k} \leqslant 1 \quad \forall k\in K, \\
& x_{i,k} + x_{j,k} \leqslant 1 \quad \forall k\in K, \\
& c_{i,k} = c_{j,k+\beta_i+e} \quad 1\leqslant k\leqslant \delta-\beta_i-e, \\
& \sum_{k=1}^{\delta-\beta_i-e} (c_{i,k} + c_{j,k+\beta_i+e}) = 2, \\
& c_{i,k} = c_{j,k+\beta_i-u} \quad 1\leqslant k\leqslant \delta-\beta_i+u, \\
& \sum_{k\in K} c_{i,k} = 1 \quad \forall i\in I, \\
& \alpha \geqslant F^{(1)} + \sum_{i\in I}\sum_{k\in K} \lambda_{i,k}^{(1)}\cdot(x_{i,k} - x_{i,k}^{c(1)}), \\
& \alpha \geqslant F^{(2)} + \sum_{i\in I}\sum_{k\in K} \lambda_{i,k}^{(2)}\cdot(x_{i,k} - x_{i,k}^{c(2)}), \\
& \vdots \\
& \alpha \geqslant F^{(v)} + \sum_{i\in I}\sum_{k\in K} \lambda_{i,k}^{(v)}\cdot(x_{i,k} - x_{i,k}^{c(v)}), \\
& x_{i,k}, c_{i,k} \in \{0,1\} \quad \forall i\in I\ \forall k\in K.
\end{aligned}$$

$F^{(v)}$ is the sum of the objective function values calculated in the subproblems of the iteration v. One Benders' cut is added per iteration, increasing the size. The result is a 0/1 mixed integer linear programming problem. The larger the number of iterations, the better Benders' cuts reproduce the $\alpha(x)$ function. For one dimension, see Fig. 4.

The meaning for each symbol is:

α_j = Costs,
x_j = Complicating variables,

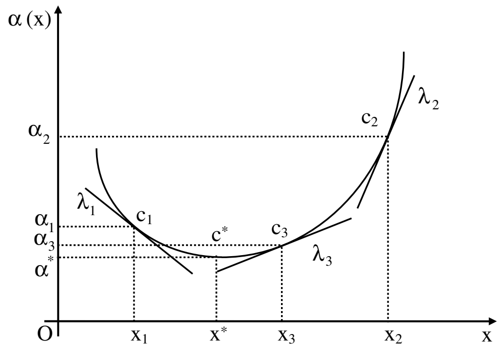

Fig. 4. Approximation of $\alpha(x)$ function for the one dimensional case.

c_j = Points that are defined by the complicating variables and the costs and
λ_j = Dual variables.

By solving the subproblem c_j is obtained. There is a point for each subproblem. The optimum is c^*. The pitch λ_j of each straight line is given by differentiating the cost with respect to the complicating variables. Each straight line is a Benders' cut.

Size

Number of variables: $2 \times \mathrm{card}(I) \times \mathrm{card}(K) + 1$.

Number of constraints:

$$2 \times \mathrm{card}(I) + [1 + 2 \times r_1 + r_2 + \mathrm{card}(I)] \times \mathrm{card}(K) + r_3 \cdot (1 + \delta) - \sum_{l \in L} \beta_l - \sum_{r_3 \in R_3} e_{r_3} + r_4 \times \delta - \sum_{q \in Q} \beta_q + \sum_{r_4 \in R_4} u_{r_4} + v.$$

5. Application example

In order to check the efficiency of the proposed analysis, it is applied to a high dimensioned realistic power system, similar to the Spanish one (75 power plants grid and 13 periods). GAMS is the selected optimisation software for the resolution (Brooke et al., 1998).

5.1. General data

In this section, typical example data are provided for the problem.

Power demand scenario probabilities

Low power demand scenario: 0.1. Medium power demand scenario: 0.6. High power demand scenario: 0.3.

Power demand order for periods

$$\text{Month 1} > \text{Month 8} > \text{Month 13} > \text{Month 7} > \text{Month 12} > \text{Month 2} > \text{Month 11} > \text{Month 3} > \text{Month 10} > \text{Month 4} > \text{Month 9} > \text{Month 6} > \text{Month 5}$$

Peak, middle, and low duration in a period

Peak duration: 10%. Middle duration: 60%. Low duration: 30%.

Fuel cost

Thermal power plants: 0.005863 \$/Te. Nuclear power plants: 0.00156 \$/Te. These are average values and come from real power plants in Spain.

Power reserve

A 10% of demand is chosen for the power reserve value (reserve constraint).

Minimum volume constraint

It is applied to all thermal power plants.

Energy to be produced by hydroelectric generators in water volume constraint

$$E_{i_2,s,k} = 0.15 \cdot \frac{\bar{t}_{i_2}}{\sum_{i_2 \in I_2} \bar{t}_{i_2}} \cdot \sum_{n \in N} d_{s,k,n} \cdot \tau_n \quad \forall i_2 \in I_2 \;\; \forall s \in S \;\; \forall k \in K.$$

This is a possible expression. A similar one could also be chosen.

Initial situation for power plants

The different generators are supposed to be disconnected when the time horizon starts. So, they do not produce any electric energy.

5.2. Problem description

The example for the application of this methodology is a system with the next features:

- 75 power plants (50 thermal, 20 hydroelectric, and 5 nuclear) and
- 3 power demand scenarios, 13 periods, and 6 subperiods.

Entry data

The information related to entry data, such as costs, power demand, specific constraints, etc., has not been detailed in this document because of its complexity. However, it will be available to any person interested in it.

Size of the problem

(a) *Global problem*
  - Number of variables: 54,600.
  - Number of constraints: 72,316.

(b) *Subproblem*
  - Number of variables: 1350.
  - Number of constraints: 1812.

(c) *Master problem*
  - Number of variables: 1951.
  - Number of constraints in the iteration number v: $1739 + v$.

5.3. Findings

The convergence was reached after 41 iterations. The chosen accuracy for the difference between the upper bound and the lower bound is 2×10^{-3}. The results are described below.

Objective function: $949.486 million

Final maintenance schedule

The time-plan is given by Table 1.

Power to be produced by three representative generators for the medium demand scenario

Three graphics are provided. They represent the variation of produced electric power (MW) in the medium demand scenario with respect to subperiods (78). A thermal power plant, a hydroelectric power plant and a nuclear power plant are shown.

(a) *Thermal power plant*
  The chosen thermal power plant is the number 10. The graph is in Fig. 5.
(b) *Hydroelectric power plant*
  The selected hydroelectric power plant is number 65. The graphic is given by Fig. 6.
(c) *Nuclear power plant*
  The power plant represented is the number 71. The representation is shown in Fig. 7.
  Cost associated to three representative generators for the medium demand scenario

Table 1
Maintenance scheduling

G/P	1	2	3	4	5	6	7	8	9	10	11	12	13
1				■									
2										■			
3	■												
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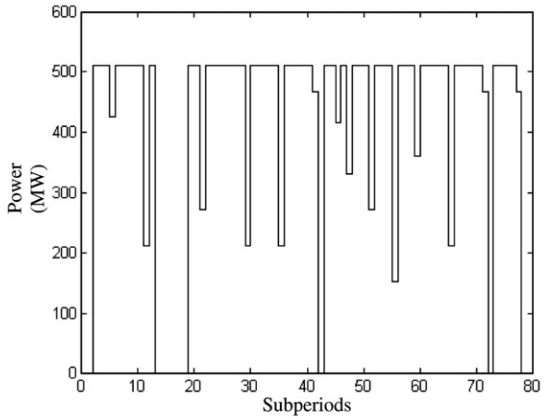

Fig. 5. Power by generator 10 in each subperiod of the medium demand scenario.

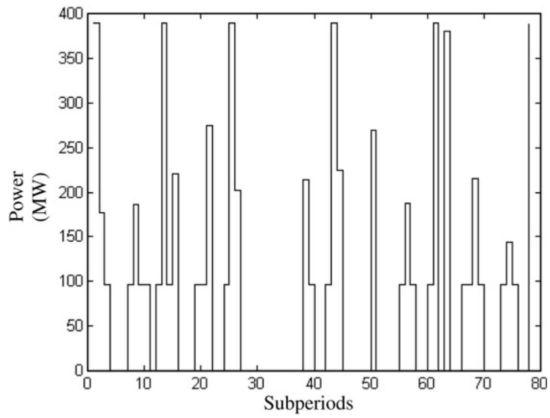

Fig. 6. Power by generator 65 in each subperiod of the medium demand scenario.

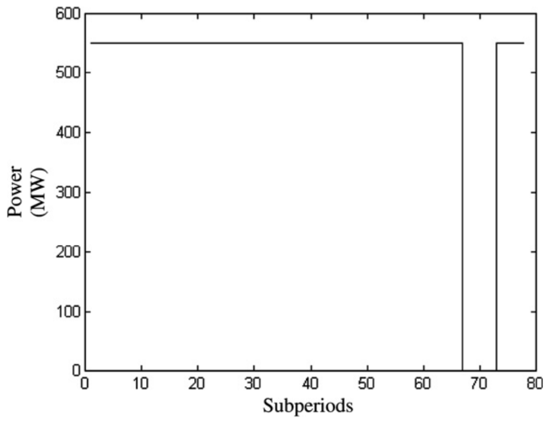

Fig. 7. Power by generator 71 in each subperiod of the medium demand scenario.

Two graphics are shown. They represent the variation of cost (million \$) for a thermal and a nuclear power plants in the medium demand scenario with respect to subperiods (78). The global cost for hydro-electric power plants is equal to zero.

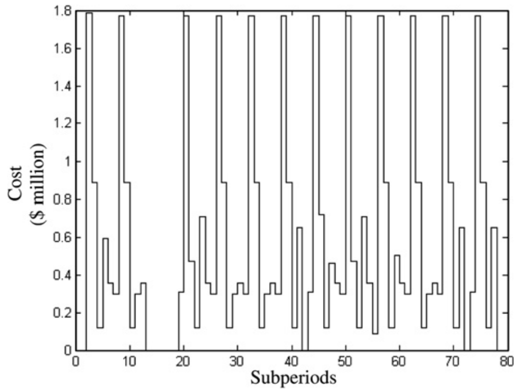

Fig. 8. Cost for generator 10 in each subperiod of the medium demand scenario.

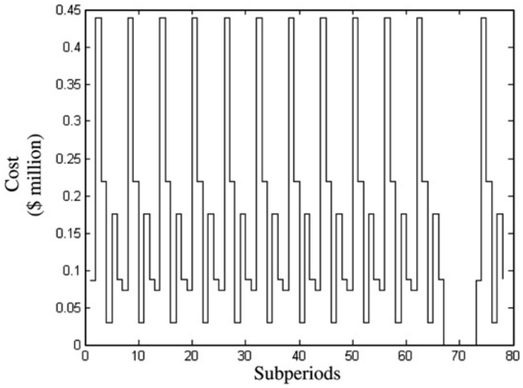

Fig. 9. Cost for generator 71 in each subperiod of the medium demand scenario.

Table 2
Total cost for each electric demand scenario

Electric demand scenario	Cost (million $)
Low	793.170
Medium	922.660
High	1055.250

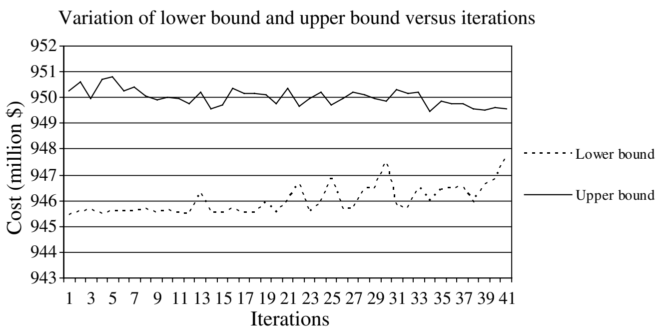

Fig. 10. Lower and upper bound variation versus iterations.

(a) *Thermal power plant*
The chosen thermal power plant is the number 10. The graph is in Fig. 8.
(b) *Nuclear power plant*
The power plant represented is the number 71. The graphic is given by Fig. 9.

Total cost for each electric demand scenario (Table 2).
Total cost of an average electric demand scenario

The cost of each scenario is multiplied by its probability. After adding the three cases, the cost becomes $949.486 million.

The final total cost for the upper bound is 949.568 million $. Regarding the lower bound, its value is 947.764 million $.

The results prove that the analysed methodology works correctly. The graphics for power and costs also show a conventional way of functioning by the thermal, hydroelectric (power only) and nuclear power plants (Figs. 5–9). The convergence represented in Fig. 10 indicates a relatively slow but acceptable progression. Quick or slow convergence is greatly dependant upon the electric system.

During the convergence process, the improvement of the objective function is not very significant. The reason for this behaviour is the initial solution for the maintenance schedule utilised to start that process. This solution is obtained after solving the master problem and satisfies all maintenance constraints, where only complicating variables participate. The value for the objective function that this solution provides is near the optimum.

6. Conclusions

In this article, the problem of power plant preventive maintenance scheduling is approached. The difficulty of solving this highly complex and multi-faceted problem can be overcome using Benders' decomposition technique, which allows its analysis in a complete and realistic way. The inclusion of topics related to the maintenance, economic unit commitment, and generating volume of power plants makes this study quite detailed. The problem under study requires many variables and constraints. However, Benders' partitioning involves dividing it into a set of problems of a smaller order and easier to solve. This method has interesting convergence characteristics. The number of iterations needed is typically under 15, although it can be larger depending on the problem.

The results arrived at after having applied Benders' decomposition technique can be very useful if they are brought to bear on realistic electric system cases. In addition to specific maintenance scheduling, the electric power to be produced by power plants and the associated costs are obtained. Benders' partitioning application to a large-sized electric production system was used to check that the proposed procedure works efficiently.

The application example provides relevant conclusions. The results are consistent and reflect the expected typical behaviour for electric energy production generators. The obtained preventive maintenance scheduling is adequate for the modelled requirements, due to the imposed constraints, and optimises the costs with the objective function.

When electric generators are in preventive maintenance or disconnected, they do not produce any electricity, and their cost can be considered equal to zero. It should be noted that maintenance, start-up, and shutdown costs are negligible. There are other costs such as the capital cost and the cost of residual staff for security and monitoring, but they are also not very significant. Hence, it is necessary to emphasise that this assumption implies a simplification. It can be made without affecting significantly the generality of the model. Hydroelectric power plants have neither production nor start-up costs, so their global cost is zero. Nuclear generators are always connected, except when they are in overhaul. These power plants produce the maximum possible power if the minimum and the maximum technical powers are the same, as in the proposed example. If these technical limits are not equal – although typically they are very similar – the produced power does not have to be the same value in all subperiods.

The most relevant contribution of this work is the use of Benders' decomposition method to solve the problem of power plant preventive maintenance scheduling. Other important contributions can be described as follows:

- *Consideration of precedence, non-stop, interval, and overlap constraints*
  No references to these constraints have been found before. They have been completely described and modelled.
- *Simultaneous use of maintenance variables and maintenance start-up variable*
  This formulation gives a new perspective on the problem.
- *Integration in the same model of maintenance, economic unit commitment, maintenance and connection, and generating volume constraints*
  The inclusion of these constraints has been considered before, but differently from the way they are presented here. Moreover, the above expressions for constraints are not the same.

This study could be of interest to electric companies with production activity, due to its applicability, and particularly to those companies with power plants included in an electric system. The implementation of the work developed in this paper could be beneficial for these companies. In addition, given that reliability is included in the model presented by means of certain constraints, such as electric demand supply, it is possible to maintain a specific quality service level, increasing customer satisfaction.

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