Memorial University of Newfoundland
Faculty of Engineering and Applied Science

ENGI7824 Intro to Digital Signal Processing
Assignment 2

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Aug 5th 2020

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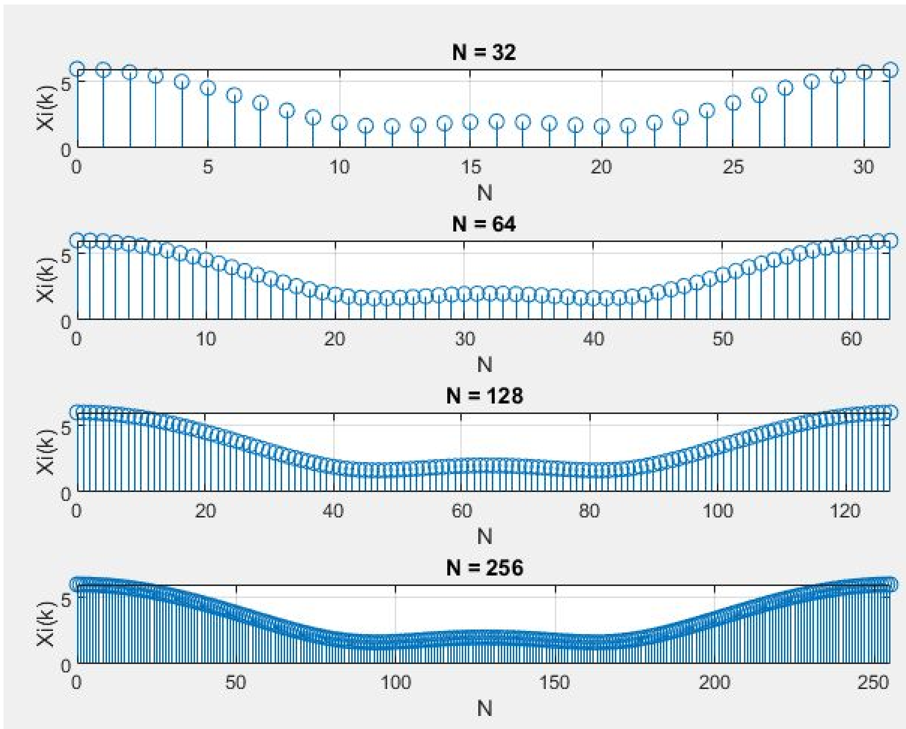
Question 1

1ai)

Computed the magnitude and plotted the DFT of x[n]. See Xn[k] below.



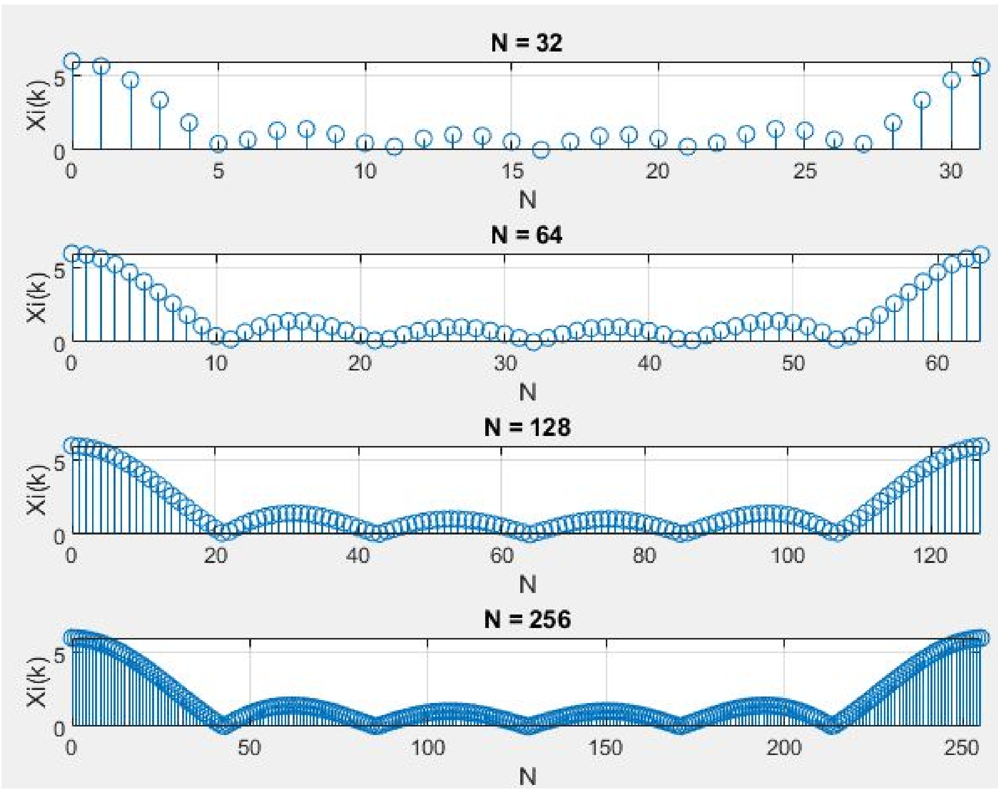
Code

```
% Question 1 ---------------------------------------------------------
% Question 1ai ------
N = [32 64 128 256];
figure('Name','1Ai','NumberTitle','off');
for i = 1:1:4
    xn_k = zeros(N(i), 1);
    xn=zeros(N(i), 1);
    xn(1)=3;
    xn(2)=2;
    xn(3)=1;

    for k = 1:N(i)
        xk = 0;
        for n = 1:N(i)
            xk = xn(n)*exp((-1j)*(2*pi/N(i))*(k-1)*(n-1)) + xk;
        end
        xn_k(k) = xk;
    end
```

```
    %Plotting
    subplot(4,1,i); stem(0:(N(i)-1),abs(xn_k)); grid;
    title('N = ' + string(N(i))); xlabel('N'); ylabel('Xi(k)');
    xlim([0 N(i)-1]);
    %stem(n,hA,'filled','k');
end
```

1aii)

Computed the magnitude and plotted the DFT of x[n]. See Xn[k] below.



Code

```
% Question 1aii ------
N = [32 64 128 256];
figure('Name','1Aii','NumberTitle','off');
for i = 1:1:4
    xn_k = zeros(N(i), 1);
    xn=zeros(N(i), 1);
    xn(1)=1;
    xn(2)=1;
    xn(3)=1;
    xn(4)=1;
    xn(5)=1;
    xn(6)=1;
```

```
    for k = 1:N(i)
        xk = 0;
        for n = 1:N(i)
            xk = xn(n)*exp((-1j)*(2*pi/N(i))*(k-1)*(n-1)) + xk;
        end
        xn_k(k) = xk;
    end

    %Plotting
    subplot(4,1,i); stem(0:(N(i)-1),abs(xn_k)); grid;
    title('N = ' + string(N(i))); xlabel('N'); ylabel('Xi(k)');
    xlim([0 N(i)-1]);
    %stem(n,hA,'filled','k');
end
```

1bi)

Using the table of DTFT the transform for x[n] is as follows.

$$X(e^{jw}) = 3 + 2e^{-jw} + e^{-2jw} \; ; \; w = given\ in\ 1b)$$

Using matlab to compute the magnitude givens the following outputs:

Question 1bi

1bi) Xi with w = 0.19635

ans =

5.9360

1bi) Xi with w = 3.5343

ans =

1.8603

The first magnitude and w corresponds with k = 8

The second magnitude and w corresponds with k = 81

CODE

```
% Question 1b ------
% Question 1bi ------
fprintf('Question 1bi \n');
xn =0;
xn(1)=3;
xn(2)=2;
xn(3)=1;

Xi = 0;
w = [2*pi/32 9*pi/8];
```

```
%Finding Xi
for m=1:2
    for i=1:length(xn)
        Xi = xn(i)*exp((-1j)*(w(m))*(i-1)) + Xi;
    end
    fprintf('1bi) Xi with w = ' + string(w(m)))
    abs(Xi)
    Xi = 0;
end
```

1bii)

Using the table of DTFT the transform for our rectangular pulse is as follows.

$$X(e^{jw}) \;=\; \sum_{n=0}^{5} (e^{-jw})^{n} \;;\; w = given\ in\ 1b)$$

Using matlab to compute the magnitude

1bii) Xi with w= 0.19635

ans =

5.6681

1bii) Xi with w= 3.5343

ans =

0.9420

The first magnitude and w corresponds with k = 8

The second magnitude and w corresponds with k = 37

The values for the DFTs and DTFT align as I expect them too. For higher frequencies the magnitude decays faster for ii) due to the nature of the calculated DTFT.

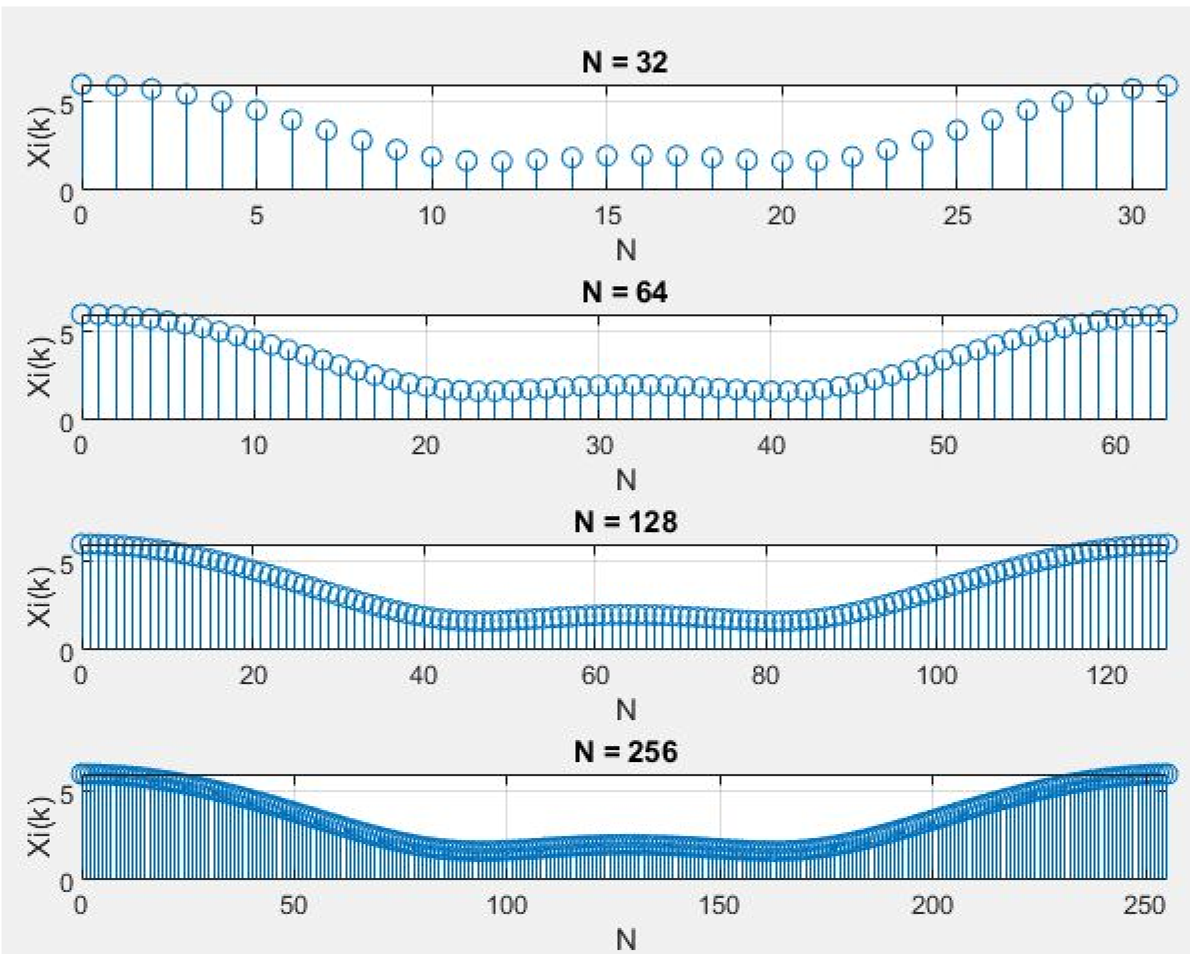
Code

```
% Question 1bii ------
xn=0;
xn(1)=1;
xn(2)=1;
xn(3)=1;
xn(4)=1;
xn(5)=1;
xn(6)=1;

Xi = 0;
```

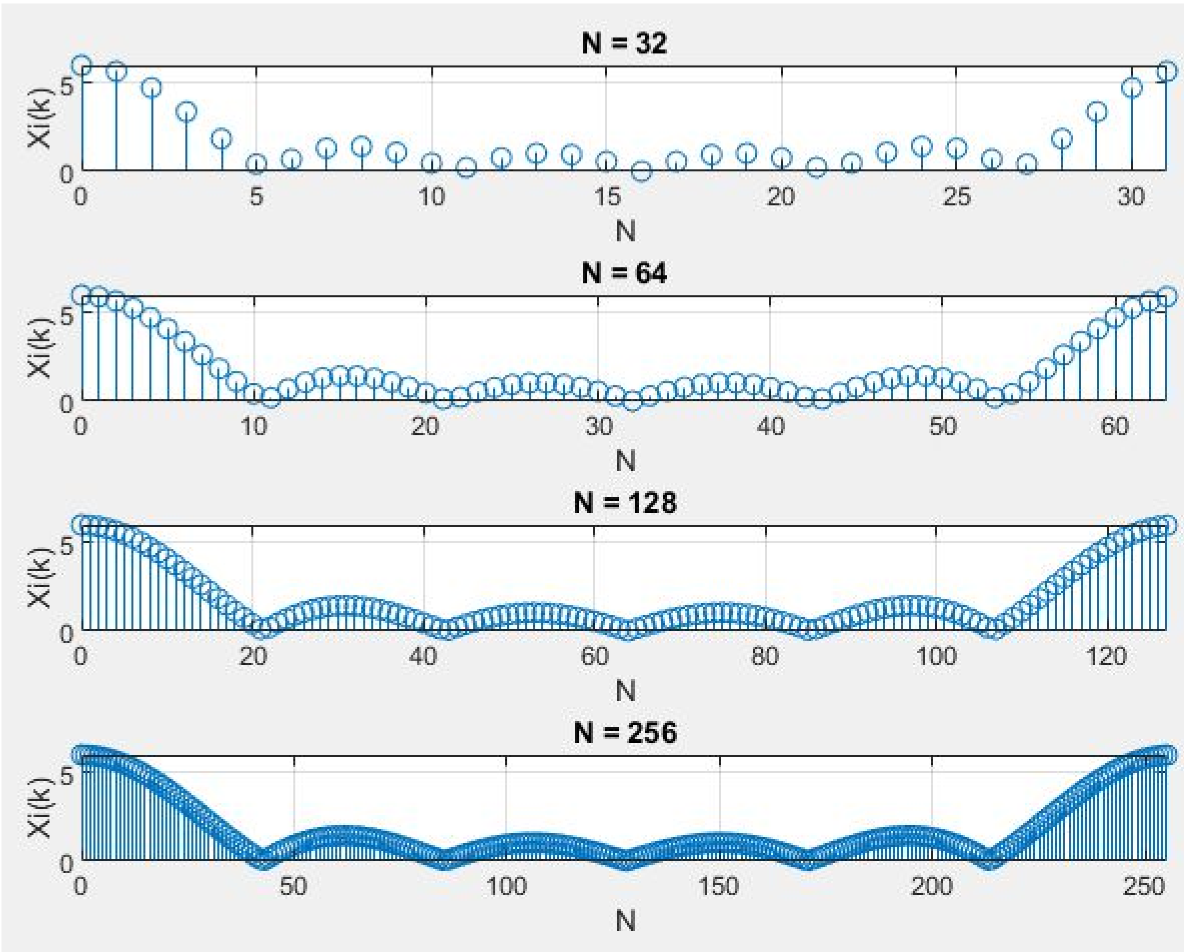
```
%Finding Xi
for m=1:2
    for i=1:length(xn)
        Xi = xn(i)*exp((-1j)*(w(m))*(i-1)) + Xi;
    end
    fprintf('1bii) Xi with w= ' + string(w(m)))
    abs(Xi)
    Xi = 0;
end
```

2ai)

Using the FFT function in MATLAB replicated the results from question 1 with much less work. The following figures for 2ai and 2aii match the previous question.



2aii)



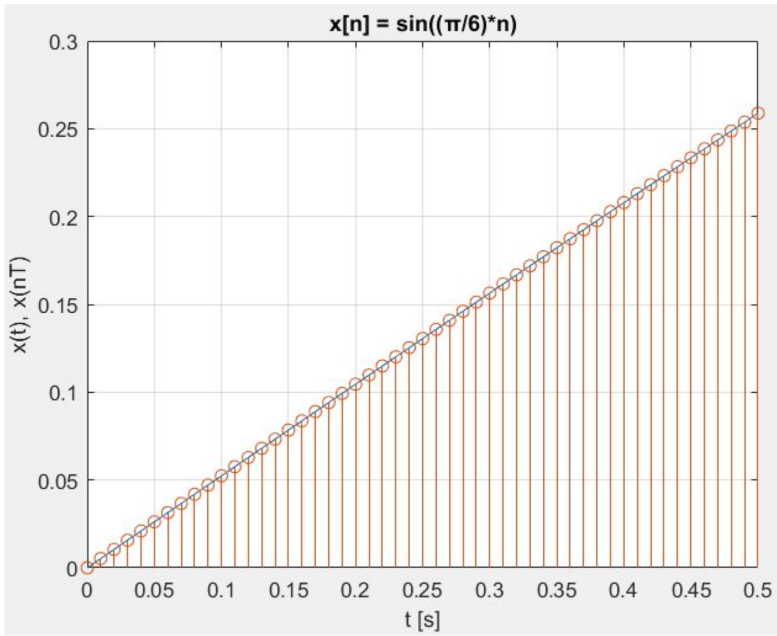
Code

Using the same code from the previous question but without the manual for loop calculation and using the fft function. See appendix for full solution.

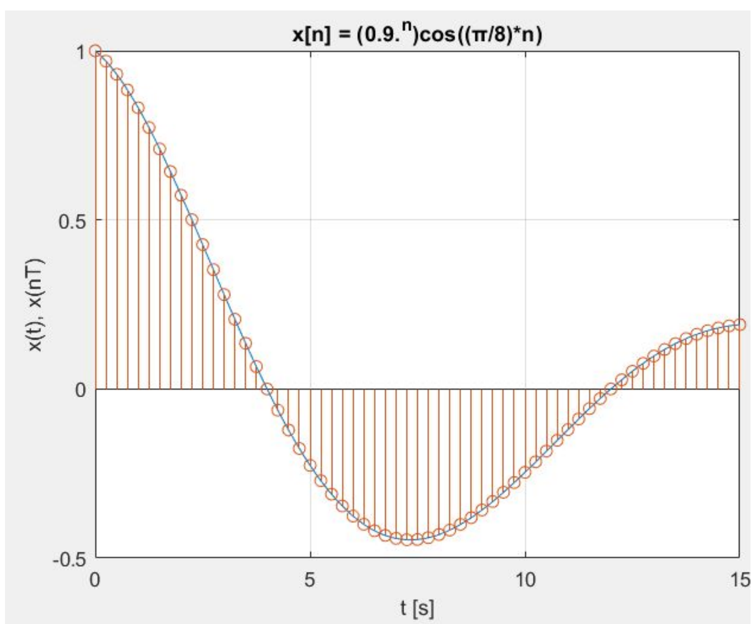
```
xn_k = fft(xn);
```

3a)

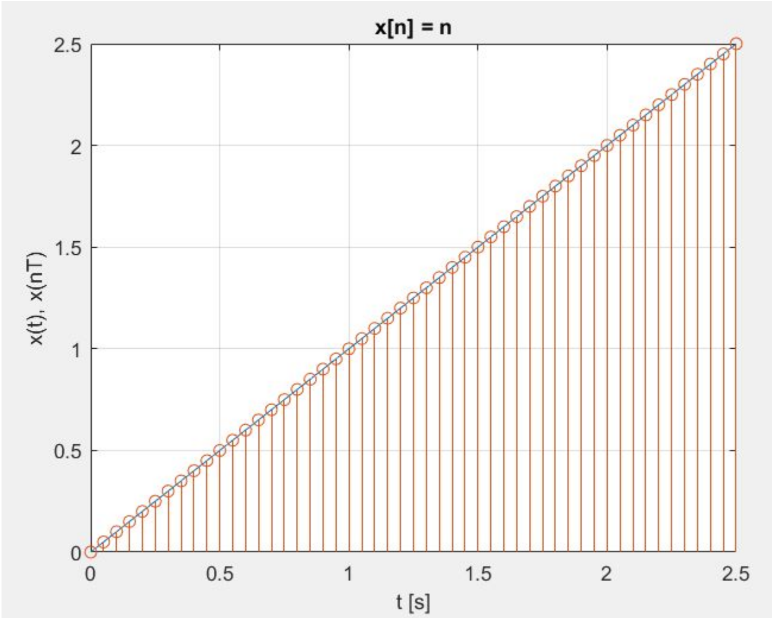
See below plots for questions a, b, and c. The blue continuous line is the original x[t] while the red is the sampled x[n] at the target sampling frequencies.



3b)



3c)



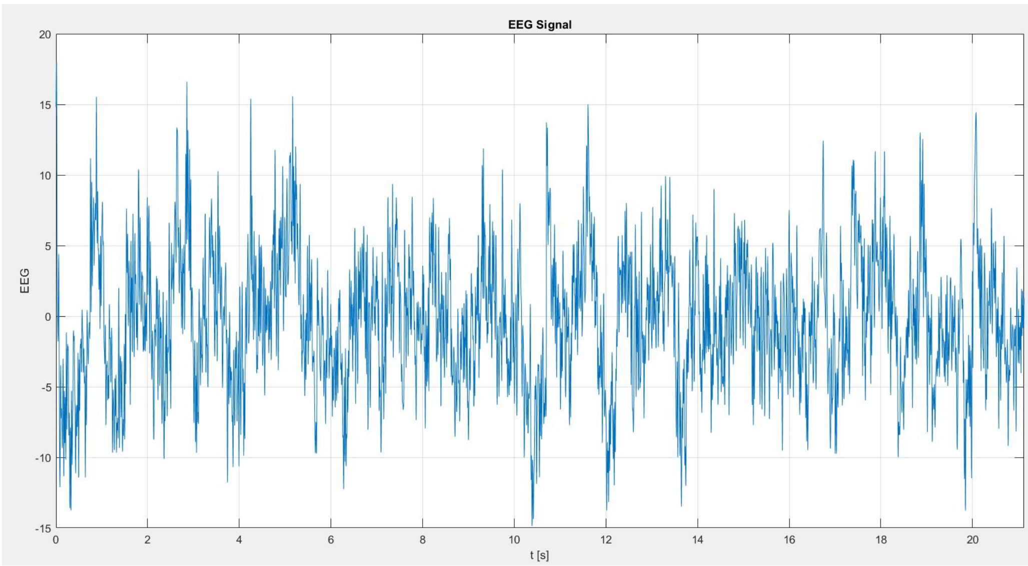
CODE

```
% Question 3 ---------------------------------------------------------------
% Question 3a ------
fs = 100;
Ts = 1/fs;
t = 0:0.0001:0.5;
n = 0:Ts:0.5;

xt = sin(pi/6*t);
xn = sin(pi/6*n);

%Plotting
figure('Name','3A','NumberTitle','off');
plot(t, xt); hold on;
stem(n, xn); hold off;
grid; title('x[n] = sin((π/6)*n)'); xlabel('t [s]');
ylabel('x(t), x(nT)');

% Question 3b ------
fs = 4;
Ts = 1/fs;
t = 0:0.0001:15;
n = 0:Ts:15;

xt = (0.9.^t).*cos(pi/8*t);
xn = (0.9.^n).*cos(pi/8*n);

%Plotting
```

```
figure('Name','3B','NumberTitle','off');
plot(t, xt); hold on;
stem(n, xn); hold off;
grid; title('x[n] = (0.9.^n)cos((π/8)*n)'); xlabel('t [s]');
ylabel('x(t), x(nT)');

% Question 3c ------
fs = 20;
Ts = 1/fs;
t = 0:0.0001:2.5;
n = 0:Ts:2.5;

xt = t;
xn = n;

%Plotting
figure('Name','3C','NumberTitle','off');
plot(t, xt); hold on;
stem(n, xn); hold off;
grid; title('x[n] = n'); xlabel('t [s]');
ylabel('x(t), x(nT)');
```

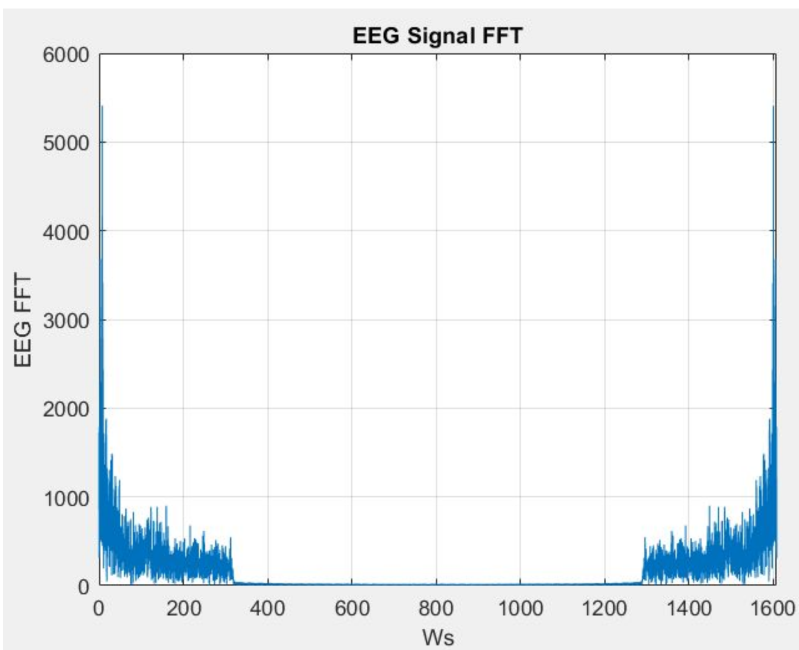
4a)

The number of data points was found by inspection of the data variable after loading it. The duration of the recording was found by dividing that by the sampling frequency. The duration is 21.1289 seconds which is verified on the plot of the EEG



4b)

The cutoff frequency of the lowpass filter is about 500 rad/s which is 500/(2pi) = 79.57 Hz.

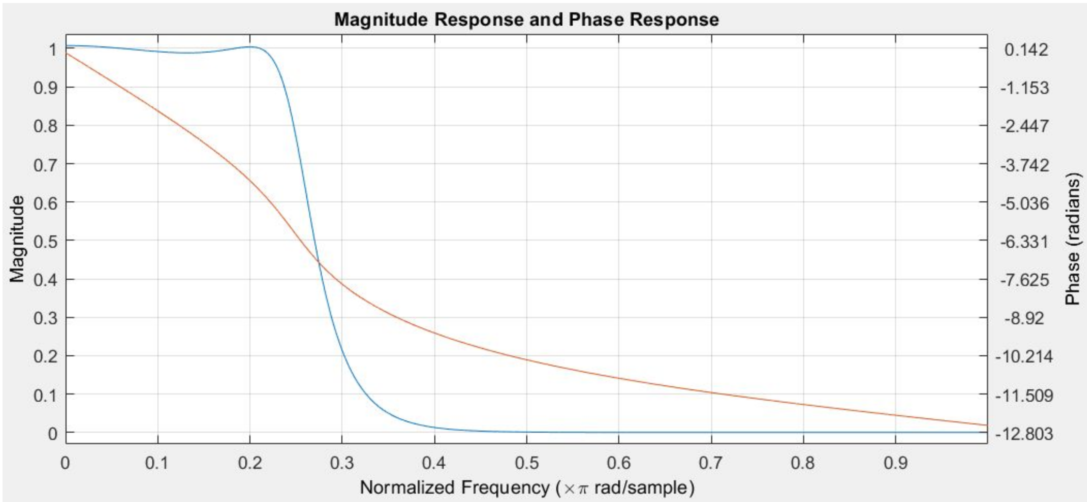


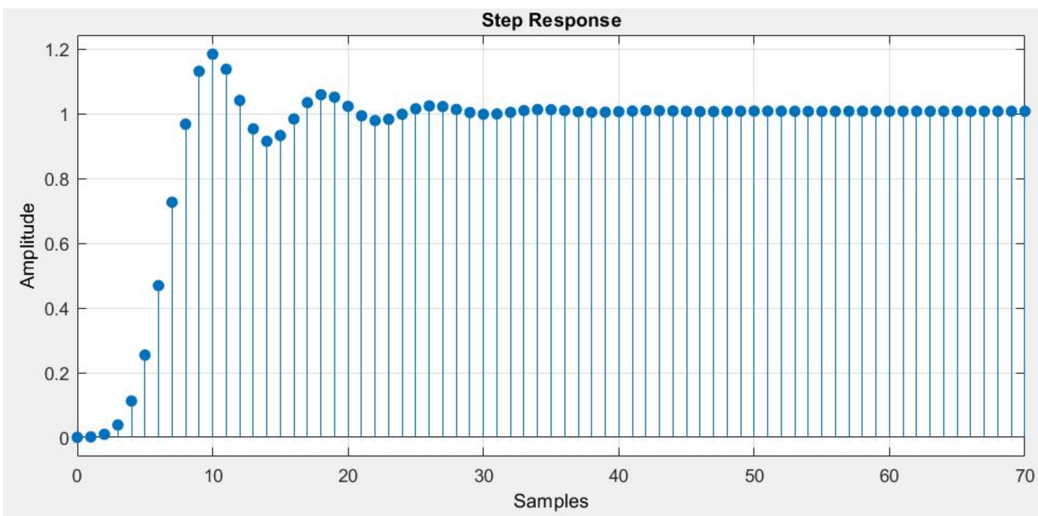
CODE

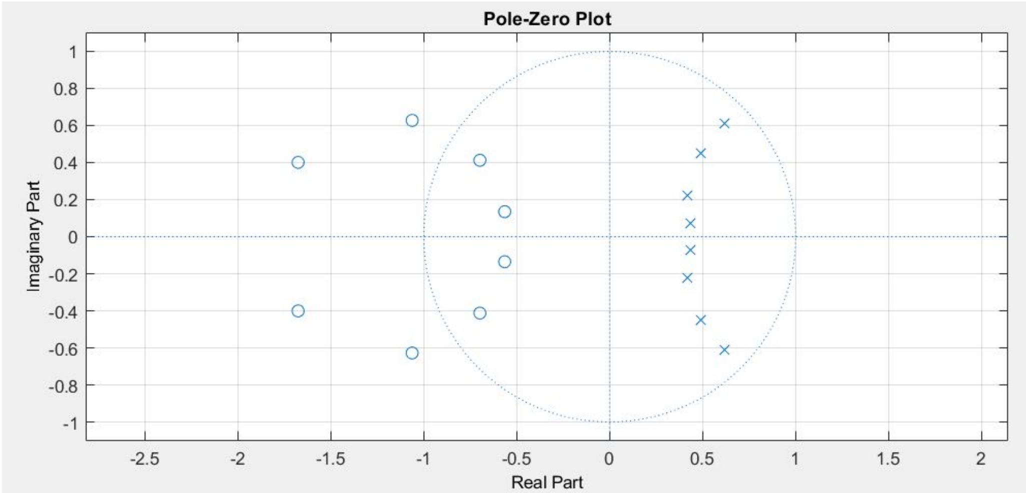
```
% Question 4 ---------------------------------------------------------------
% Question 4a ------
EEG = load('Assign2_eeg.mat');
fs = 256;
Ts = 1/fs;
Ns = 5409; %From inspection after loading file
time = Ts*(Ns-1);
t = 0:Ts:time;

figure('Name','4A','NumberTitle','off');
plot(t,EEG.data);  grid;
title('EEG Signal'); xlabel('t [s]'); ylabel('EEG');
xlim([0 time]);

% Question 4b ------
eeg_fft = abs(fft(EEG.data));
ws = fs*2*pi;
Ws = 0:ws/(Ns-1):ws;

figure('Name','4B','NumberTitle','off');
plot(Ws,eeg_fft);  grid;
title('EEG Signal FFT'); xlabel('Ws'); ylabel('EEG FFT');
xlim([0 ws]);
```

5a)

See below plots from fvtool.







5b)

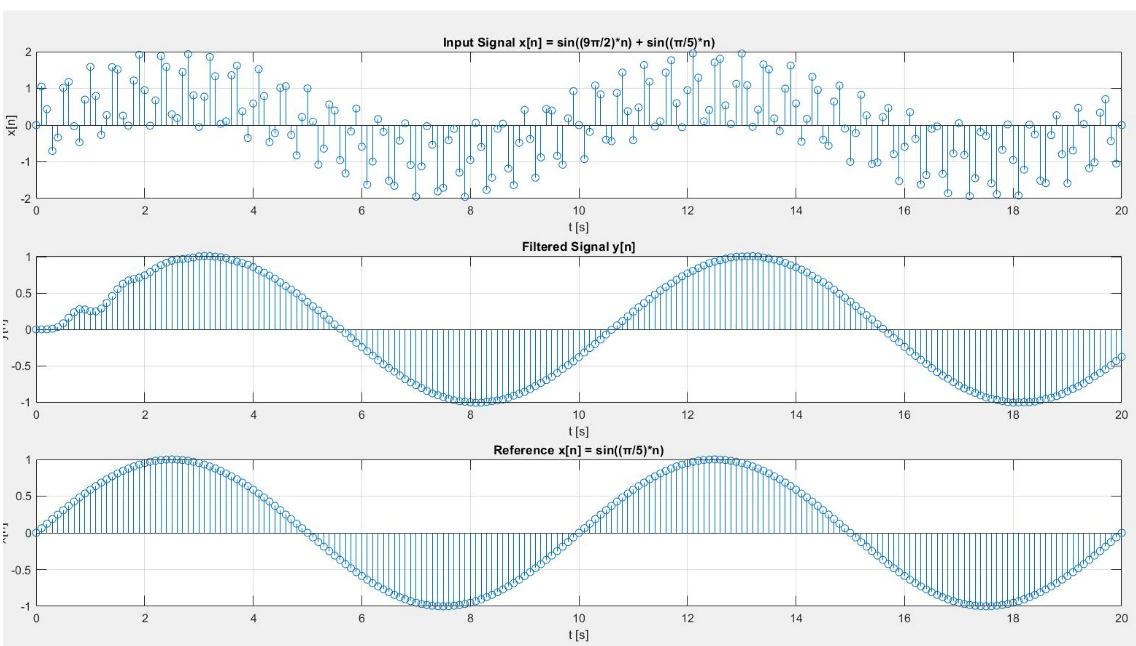
i) The system is a FIR filter

ii) Low pass filter

iii) The approximate cutoff is 0.21 rad/s

5c)

By plotting the signal in the first subplot and running it through the filter H(z) the second plot y[n] is shown. You can see this matches the 3rd reference subplot of one of the original component signals. The higher frequency component is filtered out besides some minor setting for t < 2s



Appendix: Matlab Code

```
% -------------------------------------------------------------------
% DSP A2
% Mitchell Stride, 201517901
% ------------------------------------------------------------------
clear;
close all;
clc;

% Question 1 ---------------------------------------------------------------
% Question 1ai ------
N = [32 64 128 256];
figure('Name','1Ai','NumberTitle','off');
for i = 1:1:4
    xn_k = zeros(N(i), 1);
    xn=zeros(N(i), 1);
    xn(1)=3;
    xn(2)=2;
    xn(3)=1;

    for k = 1:N(i)
        xk = 0;
        for n = 1:N(i)
            xk = xn(n)*exp((-1j)*(2*pi/N(i))*(k-1)*(n-1)) + xk;
        end
        xn_k(k) = xk;
    end

    %Plotting
    subplot(4,1,i); stem(0:(N(i)-1),abs(xn_k)); grid;
    title('N = ' + string(N(i))); xlabel('N'); ylabel('Xi(k)');
    xlim([0 N(i)-1]);
    %stem(n,hA,'filled','k');
end

% Question 1aii ------
N = [32 64 128 256];
figure('Name','1Aii','NumberTitle','off');
for i = 1:1:4
    xn_k = zeros(N(i), 1);
    xn=zeros(N(i), 1);
    xn(1)=1;
    xn(2)=1;
    xn(3)=1;
```

```
    xn(4)=1;
    xn(5)=1;
    xn(6)=1;

    for k = 1:N(i)
        xk = 0;
        for n = 1:N(i)
            xk = xn(n)*exp((-1j)*(2*pi/N(i))*(k-1)*(n-1)) + xk;
        end
        xn_k(k) = xk;
    end

    %Plotting
    subplot(4,1,i); stem(0:(N(i)-1),abs(xn_k)); grid;
    title('N = ' + string(N(i))); xlabel('N'); ylabel('Xi(k)');
    xlim([0 N(i)-1]);
    %stem(n,hA,'filled','k');
end

% Question 1b ------
% Question 1bi ------
fprintf('Question 1bi \n');
xn =0;
xn(1)=3;
xn(2)=2;
xn(3)=1;

Xi = 0;
w = [2*pi/32 9*pi/8];

%Finding Xi
for m=1:2
    for i=1:length(xn)
        Xi = xn(i)*exp((-1j)*(w(m))*(i-1)) + Xi;
    end
    fprintf('1bi) Xi with w = ' + string(w(m)))
    abs(Xi)
    Xi = 0;
end

% Question 1bii ------
xn=0;
xn(1)=1;
xn(2)=1;
xn(3)=1;
xn(4)=1;
xn(5)=1;
```

```
xn(6)=1;

Xi = 0;

%Finding Xi
for m=1:2
    for i=1:length(xn)
        Xi = xn(i)*exp((-1j)*(w(m))*(i-1)) + Xi;
    end
    fprintf('1bii) Xi with w= ' + string(w(m)))
    abs(Xi)
    Xi = 0;
end

% Question 2 -------------------------------------------------------------
% Question 2ai ------
figure('Name','2Ai','NumberTitle','off');
for i = 1:1:4
    xn_k = zeros(N(i), 1);
    xn=zeros(N(i), 1);
    xn(1)=3;
    xn(2)=2;
    xn(3)=1;

xn_k = fft(xn);

    %Plotting
    subplot(4,1,i); stem(0:(N(i)-1),abs(xn_k)); grid;
    title('N = ' + string(N(i))); xlabel('N'); ylabel('Xi(k)');
    xlim([0 N(i)-1]);
    %stem(n,hA,'filled','k');
end

% Question 2aii ------
figure('Name','2Aii','NumberTitle','off');
for i = 1:1:4
    xn_k = zeros(N(i), 1);
    xn=zeros(N(i), 1);
    xn(1)=1;
    xn(2)=1;
    xn(3)=1;
    xn(4)=1;
    xn(5)=1;
    xn(6)=1;

xn_k = fft(xn);
```

```
    %Plotting
    subplot(4,1,i); stem(0:(N(i)-1),abs(xn_k)); grid;
    title('N = ' + string(N(i))); xlabel('N'); ylabel('Xi(k)');
    xlim([0 N(i)-1]);
    %stem(n,hA,'filled','k');
end

% Question 3 -------------------------------------------------------------
% Question 3a ------
fs = 100;
Ts = 1/fs;
t = 0:0.0001:0.5;
n = 0:Ts:0.5;

xt = sin(pi/6*t);
xn = sin(pi/6*n);

%Plotting
figure('Name','3A','NumberTitle','off');
plot(t, xt); hold on;
stem(n, xn); hold off;
grid; title('x[n] = sin((π/6)*n)'); xlabel('t [s]');
ylabel('x(t), x(nT)');

% Question 3b ------
fs = 4;
Ts = 1/fs;
t = 0:0.0001:15;
n = 0:Ts:15;

xt = (0.9.^t).*cos(pi/8*t);
xn = (0.9.^n).*cos(pi/8*n);

%Plotting
figure('Name','3B','NumberTitle','off');
plot(t, xt); hold on;
stem(n, xn); hold off;
grid; title('x[n] = (0.9.^n)cos((π/8)*n)'); xlabel('t [s]');
ylabel('x(t), x(nT)');

% Question 3c ------
fs = 20;
Ts = 1/fs;
t = 0:0.0001:2.5;
n = 0:Ts:2.5;

xt = t;
```

```
xn = n;

%Plotting
figure('Name','3C','NumberTitle','off');
plot(t, xt); hold on;
stem(n, xn); hold off;
grid; title('x[n] = n'); xlabel('t [s]');
ylabel('x(t), x(nT)');

% Question 4 --------------------------------------------------------------
% Question 4a ------
EEG = load('Assign2_eeg.mat');
fs = 256;
Ts = 1/fs;
Ns = 5409; %From inspection after loading file
time = Ts*(Ns-1);
t = 0:Ts:time;

figure('Name','4A','NumberTitle','off');
plot(t,EEG.data);  grid;
title('EEG Signal'); xlabel('t [s]'); ylabel('EEG');
xlim([0 time]);

% Question 4b ------
eeg_fft = abs(fft(EEG.data));
ws = fs*2*pi;
Ws = 0:ws/(Ns-1):ws;

figure('Name','4B','NumberTitle','off');
plot(Ws,eeg_fft);  grid;
title('EEG Signal FFT'); xlabel('Ws'); ylabel('EEG FFT');
xlim([0 ws]);

% Question 5 --------------------------------------------------------------
% Question 5a ------
fprintf('Question 5a')
b = [0.0001201 0.0009608 0.003363 0.006725 0.008407 0.006725 0.003363 0.0009608
0.0001201];
a = [1 -3.919 7.325 -8.275 6.106 -2.989 0.9423 -0.1742 0.01442];
Hz=tf(b,a)
%Select option to open by default.
fvtool(b,a,'freq'); %'freq', 'impulse', 'polezero'

% Question 5c ------
%Create Xn
fs = 10;
Ts = 1/fs;
```

```
time = 20; %Sampling 25Hz for 20s
% t = 0:0.0001:time;
n = 0:Ts:time;

% xt = sin((pi/5)*t);
xn = sin((9*pi/2)*n) + sin((pi/5)*n);
yn = filter(b,a,xn);
xn_filt = sin((pi/5)*n);

figure('Name','5C','NumberTitle','off');
subplot(3,1,1); stem(n,xn); grid; title('Input Signal x[n] = sin((9π/2)*n) +
sin((π/5)*n)');
xlabel('t [s]'); ylabel('x[n]'); xlim([0 time]);

subplot(3,1,2); stem(n,yn); grid; title('Filtered Signal y[n]');
xlabel('t [s]'); ylabel('y[n]'); xlim([0 time]);

subplot(3,1,3); stem(n,xn_filt); grid; title('Reference x[n] = sin((π/5)*n)');
xlabel('t [s]'); ylabel('x[n]'); xlim([0 time]);
```