Series RC, RL, and RLC Circuits
Parallel RC, RL, and RLC Circuits

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If you want a good description of the analysis of these circuits, go to the Wikipedia web site, for example http://en.wikipedia.org/wiki/RL_circuit.

Analyses for series RC, parallel RL, and series RLC circuits were taken from class notes for Berkeley's EE 40, Introduction to Microelectronic Circuits. The lecture notes for this course are very well written. http://laser.eecs.berkeley.edu/ee40/lectures/cch-Lec09-021505-2-6p.pdf.

General Information

$$i = \frac{dQ}{dt} \qquad (1a) \qquad\qquad Q = \int_{-\infty}^{t} i(\tau)\,d\tau \qquad (1b)$$

$$v_R = Ri_R \qquad (2a) \qquad\qquad i_R = \frac{1}{R}v_R \qquad (2b)$$

$$v_C = \frac{Q}{C} = \frac{1}{C}\int_{-\infty}^{t} i_C(\tau)\,d\tau \qquad (3a) \qquad\qquad i_C = C\frac{dv_C}{dt} \qquad (3b)$$

$$v_L = L\frac{di_L}{dt} \qquad (4a) \qquad\qquad i_L = \frac{1}{L}\int_{-\infty}^{t} v_L(\tau)\,d\tau \qquad (4b)$$

Time Constants

$$\tau_{RL} = \frac{L}{R} \qquad (5a) \qquad\qquad \tau_{RC} = RC \qquad (5b)$$

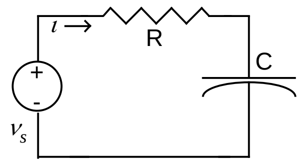
Natural frequency

$$\omega_0^2 = \frac{1}{LC} \qquad (6)$$

Kirchhoff's voltage law (**KVL**): the net voltage (potential) change around a circuit is 0. You end up where you started.

Kirchhoff's current law (**KCL**): the total current into a node = the total current leaving the node. The electrons have to go somewhere.

First Order Series RC circuit



KVL: $$v_R + v_C = v_s \tag{7}$$

Write in terms of v_C:

From equation (2a) $$v_R = Ri_R \tag{8}$$

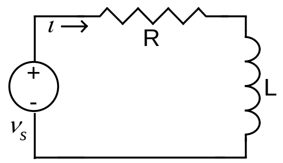
There is only one current in the loop so $$i_R = i_C = i \tag{9}$$

From equation (3b) $$v_R = Ri_C = R\left(C\frac{dv_C}{dt}\right) \tag{10}$$

Plug into equation (7) $$RC\frac{dv_C}{dt} + v_C = v_s \tag{11a}$$

Rewrite using equation (5a) $$\tau_{RC}\frac{dv_C}{dt} + v_C = v_s \tag{11b}$$

First Order Series RL circuit



KVL: $$v_R + v_L = v_s \tag{12}$$

Write in terms of v_C:

From equation (2a) $$v_R = Ri_R \tag{13}$$

From equation (4a) $$v_L = L\frac{di_L}{dt} \tag{14}$$

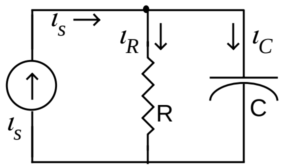
There is only one current in the loop so $$i_R = i_L = i \tag{15}$$

Plug into equation (12) $$L\frac{di}{dt} + Ri = v_s \tag{16}$$

Divide by L $$\frac{di}{dt} + \frac{R}{L}i = \frac{1}{L}v_s \tag{17a}$$

Rewrite using equation (5a) $$\frac{di}{dt} + \frac{1}{\tau_{RL}}i = \frac{1}{L}v_s \tag{17b}$$

First Order Parallel RC circuit



KCL: $$i_C + i_R = i_s \quad (18)$$

Write in terms of v:

There is only one voltage drop so $$v_R = v_C = v \quad (19)$$

From equation (2b) $$i_R = \frac{1}{R}v \quad (20)$$

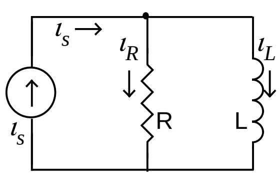
From equation (3b) $$i_C = C\frac{dv}{dt} \quad (21)$$

Plug into equation (18) $$C\frac{dv}{dt} + \frac{1}{R}v = i_s \quad (22)$$

Divide by C $$\frac{dv}{dt} + \frac{1}{RC}v = \frac{1}{C}i_s \quad (23a)$$

Rewrite using equation (5b) $$\frac{dv}{dt} + \frac{1}{\tau_{RC}}v = \frac{1}{C}i_s \quad (23b)$$

First Order Parallel RL circuit



KCL: $$i_L + i_R = i_s \quad (24)$$

Write in terms of i_L:

From equation (2b) $$i_R = \frac{1}{R}v_R\,. \quad (25)$$

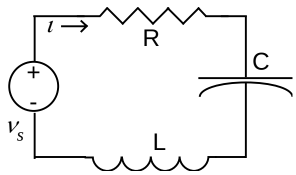
There is only one voltage drop so $$v_R = v_L = v \quad (26)$$

From equation (4a) $$i_R = \frac{1}{R}v_L = \frac{1}{R}\left(L\frac{di_L}{dt}\right) \quad (27)$$

Plug into equation (24) $$i_L + \frac{L}{R}\frac{di_L}{dt} = i_s \quad (28a)$$

Rewrite using equation (5a) $$i_L + \tau_{RL}\frac{di_L}{dt} = i_s \quad (28b)$$

Second Order Series RLC circuit



KVL: $$v_L + v_R + v_C = v_s \qquad (29)$$

Write in terms of the loop current i, (equations (2a). (3a), (4a)).

$$v_L = L\frac{di}{dt} \qquad v_R = Ri \qquad v_C = \frac{1}{C}\int_{-\infty}^{t} i(\tau)\,d\tau \qquad (30)$$

Plug into (29) $\quad v_L + v_R + v_C = v_s$:

$$L\frac{di}{dt} + Ri + \frac{1}{C}\int_{-\infty}^{t} i(\tau)\,d\tau = v_s \qquad (31)$$

The integral is a problem so take the time derivative of every term in (31)

$$L\frac{d^2 i}{dt^2} + R\frac{di}{dt} + \frac{1}{C}i = \frac{dv_s}{dt} \qquad (32)$$

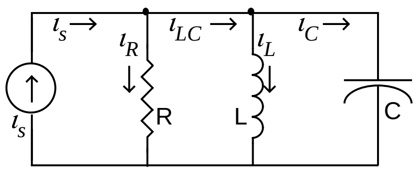
Divide by L

$$\frac{d^2 i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC}i = \frac{1}{L}\frac{dv_s}{dt} \qquad (33a)$$

Rewrite using equations (5a) and (6)

$$\frac{d^2 i}{dt^2} + \frac{1}{\tau_{RL}}\frac{di}{dt} + \omega_0^2 i = \frac{1}{L}\frac{dv_s}{dt} \qquad (33b)$$

Second Order Parallel RLC circuit



KCL: $i_{LC} + i_R = i_s$ and $i_L + i_C = i_{LC}$ Combine to give $i_L + i_R + i_C = i_s$ (34)

Write in terms of v, the voltage across all components (equations (2b). (3b), (4b)).

$$i_L = \frac{1}{L}\int_{-\infty}^{t} v_L(\tau)\,d\tau \qquad i_R = \frac{1}{R}v_R \qquad i_C = C\frac{dv_C}{dt} \tag{35}$$

Plug into (34) $i_L + i_R + i_C = i_s$:

$$\frac{1}{L}\int_{-\infty}^{t} v_L(\tau)\,d\tau + \frac{1}{R}v_R + C\frac{dv_C}{dt} = i_s \tag{36}$$

The integral is a problem so take the time derivative of every term in (36). Reorder.

$$C\frac{d^2v}{dt^2} + \frac{1}{R}\frac{dv}{dt} + \frac{1}{L}v = \frac{di_s}{dt} \tag{37}$$

Divide by C

$$\frac{d^2v}{dt^2} + \frac{1}{RC}\frac{dv}{dt} + \frac{1}{LC}v = \frac{1}{C}\frac{di_s}{dt} \tag{38a}$$

Rewrite using equations (5b) and (6)

$$\frac{d^2v}{dt^2} + \frac{1}{\tau_{RC}}\frac{dv}{dt} + \omega_0^2 v = \frac{1}{C}\frac{di_s}{dt} \tag{38b}$$

Summary

Circuit	Differential Equation Form	
First Order Series RC circuit	$\frac{dv_C}{dt}+\frac{1}{\tau_{RC}}v_C=\frac{1}{\tau_{RC}}v_s$	(11b)
First Order Series RL circuit	$\frac{di}{dt}+\frac{1}{\tau_{RL}}i=\frac{1}{L}v_s$	(17b)
First Order Parallel RC circuit	$\frac{dv}{dt}+\frac{1}{\tau_{RC}}v=\frac{1}{C}i_s$	(23b)
First Order Parallel RL circuit	$\frac{di_L}{dt}+\frac{1}{\tau_{RL}}i_L=\frac{1}{\tau_{RL}}i_s$	(28b)
First Order General Form	$\frac{dx}{dt}+\frac{1}{\tau}x=f(t)$	

Second Order Series RLC circuit	$\frac{d^2i}{dt^2}+\frac{1}{\tau_{RL}}\frac{di}{dt}+\omega_0^2 i=\frac{1}{L}\frac{dv_s}{dt}$	(33b)
Second Order Parallel RLC circuit	$\frac{d^2v}{dt^2}+\frac{1}{\tau_{RC}}\frac{dv}{dt}+\omega_0^2 v=\frac{1}{C}\frac{di_s}{dt}$	(38b)
Second Order General Form	$\frac{d^2x}{dt^2}+\frac{1}{\tau}\frac{dx}{dt}+\omega_0^2 x=f(t)$	

Time Constants	$\tau_{RL}=\frac{L}{R}$	(5a)
	$\tau_{RC}=RC$	(5b)
Natural frequency	$\omega_0^2=\frac{1}{LC}$	(6)