1. **[2]**

$$\begin{aligned} C_r^n &= \frac{P_r^n}{r!} = \frac{n!}{(n-r)!}/r! \quad \text{(theorem 1)} \\ &= \frac{n!}{r!\,(n-r)!} \end{aligned}$$

2. **[5]**

$$\begin{aligned} P(A \cup B \cup C) &= P[A \cup (B \cup C)] \quad \text{Associative} \\ &= P(A) + P(B \cup C) - P[A \cap (B \cup C)] \quad \text{Theorem 2} \\ &= P(A) + \underbrace{P(B) + P(C) - P(B \cap C)}_{\text{Theorem 2}} \\ &\quad -P[A \cap (B \cup C)] \qquad \text{(eq.1)} \end{aligned}$$

where

$$\begin{aligned} P[A \cap (B \cup C)] &= P[(A \cap B) \cup (A \cap C)] \quad \text{Distributive law} \\ &= P(A \cap B) + P(A \cap C) \\ &\quad -P[(A \cap B) \cap (A \cap C)] \quad \text{Theorem 2} \\ &= P(A \cap B) + P(A \cap C) - P(A \cap B \cap C) \qquad \text{(eq.2)} \end{aligned}$$

Therefore the theorem is proven by substituting (eq. 2) into (eq.1).

3.

(a) $\dfrac{n!}{n_1!n_2!n_3!} = \dfrac{20!}{9!6!5!} = 77\ 597\ 520$ (2)

(b) $C_3^6 = 20$ (1)

(c) $\dbinom{9}{2}\dbinom{11}{2} = 36\,(55) = 1980$ (2)

(d)

i.

A. $P(A_1 \cap A_2 \cap A_3) = \left(\dfrac{9}{15}\right)\left(\dfrac{9}{15}\right)\left(\dfrac{9}{15}\right) = 0.2160$ (1)

B. $P(A_1 \cap B_2 \cap B_3) = \left(\dfrac{9}{15}\right)\left(\dfrac{6}{15}\right)\left(\dfrac{6}{15}\right) = 0.096$ (1)

ii.

A. $P(A_1 \cap A_2 \cap A_3) = \left(\dfrac{9}{15}\right)\left(\dfrac{8}{14}\right)\left(\dfrac{7}{13}\right) = 0.1846$ (2)

B. $P(A_1 \cap B_2 \cap B_3) = \left(\dfrac{9}{15}\right)\left(\dfrac{6}{14}\right)\left(\dfrac{5}{13}\right) = 0.0989$ (2)

(e)

i. $P(X=3) = \dfrac{\binom{9}{7}\binom{6}{3}}{\binom{15}{10}} = \dfrac{36(20)}{3003} = 0.2398$ (2)

ii. $P(2 \le X < 5) = P(1 < X \le 4) = F(4) - F(1)$
$= 0.7063 - 0.0020 = 0.7043$ (2)

iii. $me = 4$ (1)

iv. $f(4.5) = 0$ (1)

v. $F(4.5) = F(4) = 0.7063$ (1)

vi.

A. $P(K) = P(X > 2) = 1 - P(X \le 2) = 1 - 0.0470 = 0.9530$ (1)

B. $P(K \cap L) = f(3) + f(4) = 0.2397 + 0.4196 = 0.6593$
OR $F(4) - F(2) = 0.7063 - 0.0470 = 0.6593$ (1)

C. $P\left(\overline{K} \cup \overline{L}\right) = P\left(\overline{K \cap L}\right) = 1 - P(K \cap L) = 1 - 0.6593 = 0.3407$ (1)

D. $P(K|\, L) = \dfrac{P(K \cap L)}{P(L)} = \dfrac{f(3) + f(4)}{F(4)} = \dfrac{0.6593}{0.7063} = 0.9335$ (1)

E. $P\left(\overline{K} \cap L\right) = F(2) = 0.0470$ (1)

F. For K and L independent $P(K \cap L) = P(K)\,P(L)$ (2)

$$P(K \cap L) = 0.6593,\ P(K) = 0.9528 \text{ and } P(L) = 0.7063$$

$$P(K)\,P(L) = 0.9528\,(0.7063) = 0.6730$$

$$P(K \cap L) \ne P(K)\,P(L)$$

Therefore K and L are dependent events.

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4.

(a) (4)

$$\begin{aligned} P\left(\overline{A} \cap \overline{B}\right) &= P\left(\overline{(A \cup B)}\right) \text{ DeMorgan's rule} \\ &= 1 - [P(A) + P(B) - P(A \cap B)] \\ &= 1 - P(A) - P(B) + P(A)\,P(B) \text{ since } A \text{ and } B \text{ are independent} \\ &= (1 - P(A)) - P(B)(1 - P(A)) \\ &= (1 - P(A))(1 - P(B)) \\ &= P\left(\overline{A}\right) P\left(\overline{B}\right) \ \therefore \text{ independent} \end{aligned}$$

(b)

i. $P\left(\overline{A}\cup B\right)=P\left(\overline{A}\right)+P\left(B\right)-P\left(\overline{A}\cap B\right)=0.2+0.4-\left(0.2\right)\left(0.4\right)=0.52$ (2)

ii. $P\left(\left.\overline{B}\right|\overline{A}\right)=\dfrac{P\left(\overline{A}\cap\overline{B}\right)}{P\left(\overline{A}\right)}=\dfrac{\left(0.2\right)\left(0.6\right)}{0.2}=0.6=P\left(\overline{B}\right)$

Note: $\overline{A}$ and $\overline{B}$ are independent (2)

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5.

(a) (2)

- The events B_1 and B_2 are mutually exclusive, therefore $B_1\cap B_2=\emptyset$.
- $B_1\cup B_2=S$

(b)

i. (2)

$$\begin{aligned} A &= A\cap S \\ &= A\cap\left(B_1\cup B_2\right) \quad \text{Partition of } S \\ &= \left(A\cap B_1\right)\cup\left(A\cap B_2\right) \quad \text{Distributive law} \end{aligned}$$

ii. (2)

$$\begin{aligned} P\left(A\right) &= P\left[\left(A\cap B_1\right)\cup\left(A\cap B_2\right)\right] \quad \left(A\cap B_1\right) \text{ and } \left(A\cap B_2\right) \text{ are mutually exclusive} \\ &= P\left(A\cap B_1\right)+P\left(A\cap B_2\right) \quad \text{Axiom 3} \\ &= P\left(B_1\right)P\left(\left.A\right|B_1\right)+P\left(B_2\right)P\left(\left.A\right|B_2\right) \quad \text{Theorem 3} \end{aligned}$$

(c)

i. $P\left(B_1\cap A\right)=P\left(\left.A\right|B_1\right)P\left(B_1\right)=0.35\left(0.25\right)=0.0875$ (1)

ii. $P\left(A\right)=P\left(B_1\right)P\left(\left.A\right|B_1\right)+P\left(B_2\right)P\left(\left.A\right|B_2\right)$

$=0.35\left(0.25\right)+0.05\left(0.75\right)=0.125$ (2)

iii. $P\left(\left.B_1\right|A\right)=\dfrac{P\left(A\cap B_1\right)}{P\left(A\right)}=\dfrac{0.0875}{0.125}=0.7$ (2)

iv. $P\left(\left.B_2\right|A\right)=1-0.70=0.30$ OR: $P\left(\left.B_2\right|A\right)=\dfrac{P\left(A\cap B_2\right)}{P\left(A\right)}=\dfrac{0.0375}{0.1250}=0.3$ (1)

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