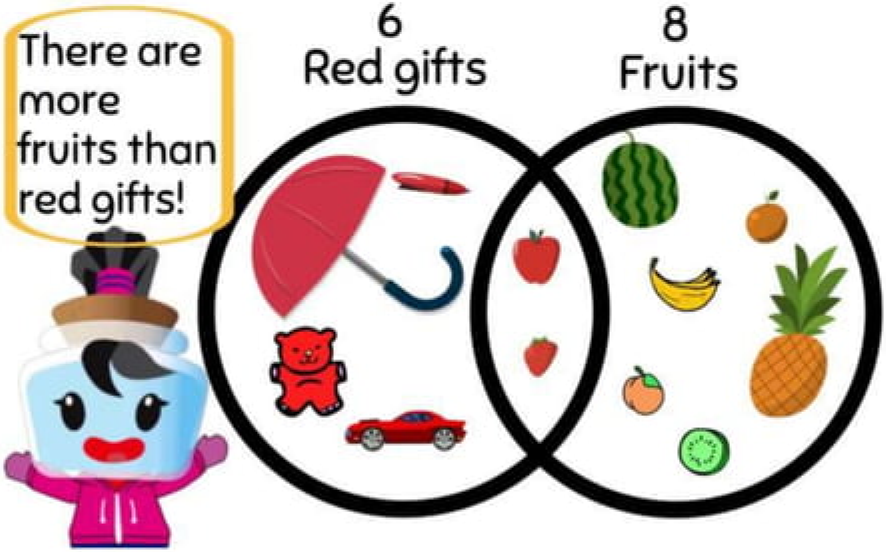


Sets

Collection or Group

What is common in all of them ?

Collection or Group of :

- Boys in your Class
- Girls in your Class
- Vowels in English Alphabet
- The rivers of India
- Odd numbers
- Even numbers

TYPE OF COLLECTION

Not well defined

- Top 3 actors of India
- Top 3 Punjabi Singers
- Top 3 Hindi Songs

Well Defined

- All Vowels in English Alphabet
- Name of all days in a week
- Name of all Months in a year

Note: In Well defined collection, we can definitely decide whether a given object belongs to the collection or not.

Set: Well Defined Collection of objects

- Vowels in English Alphabet
- The rivers of India
- Odd numbers
- Even numbers
- Name of days in a week
- Names of Months in a year

Things to Remember

Objects, elements and members of a set are synonymous terms

Sets are usually denoted by Capital Letters like A,B,C,D,E etc.

The elements of a set are represented by small letters like a,b,c,d,e etc

Commonly used Sets in Maths

- $\mathbf{N}$ **:** the set of all natural numbers
- $\mathbf{Z}$ **:** the set of all integers
- $\mathbf{Q}$ **:** Set of all rational numbers
- $\mathbf{R}$ **:** Set of real numbers
- $\boldsymbol{Z}^{+}$**:** Set of positive integers
- $\boldsymbol{Z}^{-}$**:** Set of negative integers

Notation

- A: set of odd numbers
- 3 is a member of set A.
- 2 is not a member of set A.

- A: set of odd numbers
- $3 \in A$
- $2 \notin A$

- B: set of vowels in English Alphabet
- 'a' is a member of set B.
- 'd' is not a member of set B.

- B: set of vowels in English Alphabet
- $a \in A$
- $b \notin A$

$\in \rightarrow$ is a member of (Belongs to)

$\notin \rightarrow$ is not a member of (does not Belong to)

WAYS OF REPRESENTING A SET

Roaster/Tabular form

- List all elements of a set.
- A is set of natural numbers less than 10
- is → =   set of → {}
- A = {1, 2, 3, 4, 5, 6, 7, 8, 9}
- {} → Braces   , → Comma
- List elements using ellipsis.
- A = {1, 2, 3,…,9}
- … → Ellipsis

Set builder form

- Based on common property between all elements of a set.
- A is set of natural numbers less than 10
- A = {1, 2, 3, 4, 5, 6, 7, 8, 9}
- A = {x : x is natural number and less than 10}
- A = {x : x <10 and $x \in$ N}
- where N is a Set of natural numbers and : → such that

EXAMPLES

Roaster/Tabular form

- A = {2, 3, 5, 7}
- B = {R, O, Y, A, L}
- C = {-2, -1, 0, 1, 2}
- D {L, O, Y, A}

Order/Arrangement of elements is not specific here.

Set builder form

- A = {x: x is a prime number less than 10}
- B = {x: x is a letter in the word "ROYAL"}
- C = {x: x is an integer and $-3 < x < 3$}
- D = {x: x is a letter in the word "LOYAL"}

Note: In Roaster/Tabular Form, repetition is generally not allowed.

Empty Set

- If a set doesn't have any element, it is known as an empty set or null set or void set. This set is represented by φ or {}.

Example:

- A : Set of prime numbers between 24 and 28
- B : Set of even prime numbers greater than 2
- C = { }
- D = {x: x is natural number less than 1}

Singleton Set

- If a set contains only one element, then it is called a singleton set.

Example:

- A : Set of prime numbers between 8 and 12
- B : Set of even prime numbers
- C = {1}
- D = $\{x : x$ is natural number less than 2$\}$

Finite Set

If a set contains no element or fixed number of elements, it is called a finite set.

Example:

A : Set of months in a year

B : Set of prime numbers less than 10

C = {1, 2, 3, 4, 5}

D = {x: x is natural number less than 6}

Infinite Set

- If a set contains endless number of elements, then it is called an infinite set.

Example:

- A : Set of prime numbers
- B : Set of even numbers
- C = {1,3,5,7,...}
- D = $\{x \colon x$ is a negative integer$\}$

Cardinal Number of a Set

- The cardinal number of a finite set A is the number of distinct members of the set.
- It is denoted by n(A).
- The cardinal number of the empty set is 0.
- cardinal number of an infinite set is not defined.

Example:

- If A= {-3, -2, -1, 0, 1} then n(A) = 5
- If B : Set of months in a year, then n(B) = 12

Equivalent Sets

- Two finite sets with an equal number of members are called equivalent sets.
- If the sets A and B are equivalent, we write $A \leftrightarrow B$ and read this as
- "A is equivalent to B".
- $A \leftrightarrow B$ if $n(A) = n(B)$.

Example:

- $X = \{0, 2, 4\}$
- $Y = \{x : x$ is a letter of the word DOOR$\}$.
- As $n(X) = 3$ and $n(Y) = 3$. So, $X \leftrightarrow Y$.

Equal Sets

- If two sets contain exactly same elements, then sets are known as Equal sets.

Example:

- A : {x: x is a vowel in word "loyal"}
- B : {x: x is a vowel in word "oral"}
- A = B

- C : Set of positive integers
- D : Set of natural numbers
- C = D

Non-Equal Sets

- If two sets do not contain exactly same elements, then sets are known as Non-Equal sets.

Example:

- A : {x: x is a vowel in word "loyal"}
- B : {x: x is a vowel in word "towel"}
- $A \neq B$

- C : Set of negative integers
- D : Set of natural numbers
- $C \neq D$

Subset and Superset

- If every element of set A is also an element of set B, then A is called as subset of B or B is superset of A.
- It is denoted as $A \subseteq B$ (subset) or $B \supseteq A$ (superset)

Example:

A : set of vowels in English alphabet

B : set of letters in English alphabet

$A \subseteq B$ or $B \supseteq A$

$C = \{1, 2, 3, 4, 5\}$ $D = \{2, 3\}$

$D \subseteq C$ or $C \supseteq D$

Proper Subset and Proper Superset

- If A is a subset of B and $A \neq B$, then A is proper subset of B
- If B is a superset of A and $A \neq B$, then B is proper superset of A
- It is denoted as $A \subset B$ (subset) or $B \supset A$ (superset)

Example:

$C = \{1, 2, 3, 4, 5\}$ $D = \{2, 3\}$

$D \subset C$ or $C \supset D$

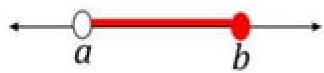
INTERVALS

- Let a, b ∈ R and a < b

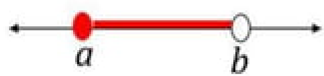
Open interval
(a , b)
$\{x : x \in \mathrm{R} \text{ and } a < x < b\}$

Closed interval
[a , b]
$\{x : x \in \mathrm{R} \text{ and } a \leq x \leq b\}$

Half open (or half closed)
(a , b]
$\{x : x \in \mathrm{R} \text{ and } a < x \leq b\}$



[a , b)
$\{x : x \in \mathrm{R} \text{ and } a \leq x < b\}$



Power Set

The power set is a set which includes all the subsets including the empty set and the original set itself.

The number of elements of a power set is written as |A|, If A has 'n' elements then it can be written as

$|P(A)| = 2^n$

Example:

Let us say Set A = { a, b, c }

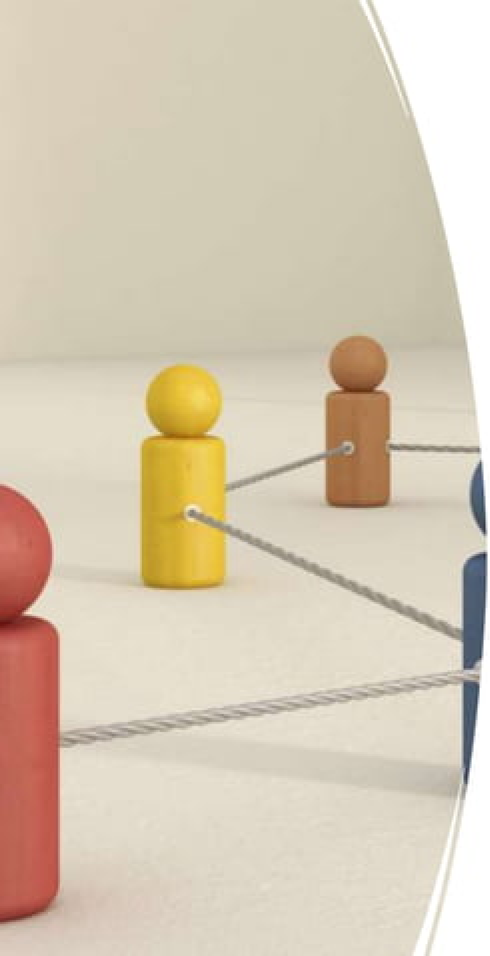
Number of elements: 3

Therefore, the subsets of the set are:

{ } empty set	{ c }	{ c, a }
{ a }	{ a, b }	{ a, b, c }
{ b }	{ b, c }	

Power set of A will be

P(A) = { { } , { a }, { b }, { c }, { a, b }, { b, c }, { c, a }, { a, b, c } }

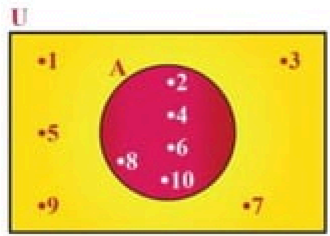


Universal Set

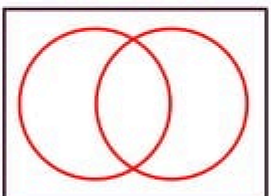
- A Universal Set is the set of all elements under consideration, denoted by U. All other sets are subsets of the universal set.
- **Example:**
- A : set of equilateral triangles
- B : set of scalene triangles
- C : set of isosceles triangles
- U : set of triangles (Universal set)
- $A \subset U$, $B \subset U$, $C \subset U$

VENN DIAGRAM

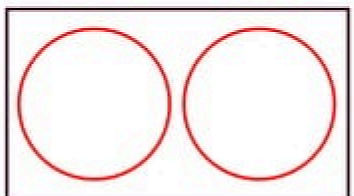
- A Venn diagram used to represent all possible relations of different sets. It can be represented by any closed figure, whether it be a Circle or a Polygon (square, hexagon, etc.). But usually, we use circles to represent each set.
- Example:
- U = {1,2,3,4,5,6,,8,9,10}
- A = {2,4,6,8,10}



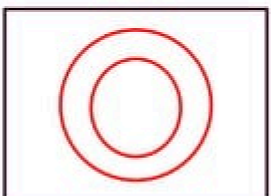
Intersecting sets



Non Intersecting sets



Subsets



Operation on sets

Operations on numbers:

Addition(+) Subtraction(−)

Multiplication(×) Division(÷)

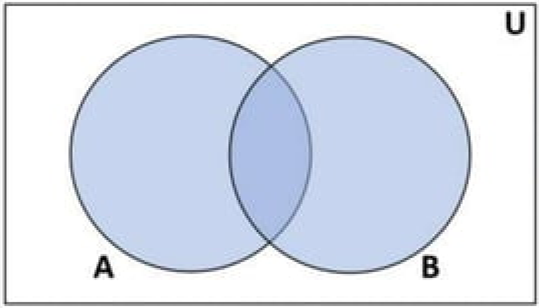
Set operations are the operations that are applied on two more sets to develop a relationship between them.

There are four main kinds of set operations which are:

- ❑ Union of sets
- ❑ Intersection of sets
- ❑ Complement of a set
- ❑ Difference between sets

Union

The union of sets A and B is the set of items that are in either A or B.



Notation: $A \cup B$

Examples:

$\{1, 2\} \cup \{1, 2\} = \{1, 2\}$

$\{1, 2, 3\} \cup \{4, 5, 6\} = \{1, 2, 3, 4, 5, 6\}$

$\{1, 2, 3\} \cup \{3, 4\} = \{1, 2, 3, 4\}$

Properties of Union of Sets

Commutative Law: The union of two or more sets follows the commutative law i.e., if we have two sets A and B then,

$$A \cup B = B \cup A$$

Example: $A = \{a, b\}$ and $B = \{b, c, d\}$

So, $A \cup B = \{a,b,c,d\}$

$B \cup A = \{b,c,d,a\}$

$A \cup B = B \cup A$

Hence, Commutative law proved.

Properties of Union of Sets

Associative Law: The union operation follows the associative law i.e., if we have three sets A, B and C then

$$(A \cup B) \cup C = A \cup (B \cup C)$$

Example: A = {a, b} and B = {b, c, d} and C = {a,c,e}

$(A \cup B) \cup C = \{a,b,c,d\} \cup \{a,c,e\} = \{a,b,c,d,e\}$

$A \cup (B \cup C) = \{a, b\} \cup \{b,c,d,e\} = \{a,b,c,d,e\}$

Hence, Associative law proved.

Properties of Union of Sets

Identity Law: The union of an empty set with any set A gives the set itself.

$$A \cup \phi = A$$

Example: $A = \{a,b,c\}$ and $\phi = \{\}$

$$\begin{aligned} A \cup \phi &= \{a,b,c\} \cup \{\} \\ &= \{a,b,c\} \\ &= A \end{aligned}$$

Hence, Identity law proved.

Properties of Union of Sets

Idempotent Law: The union of any set A with itself gives the set A.

$$A \cup A = A$$

Example: $A = \{1,2,3,4,5\}$

$A \cup A = \{1,2,3,4,5\} \cup \{1,2,3,4,5\}$

$= \{1,2,3,4,5\} = A$

Hence, Idempotent Law proved.

Properties of Union of Sets

Law of U : The union of a universal set U with its subset A gives the universal set itself.

$$A \cup U = U$$

Example: A = {1,2,4,7} and U = {1,2,3,4,5,6,7}

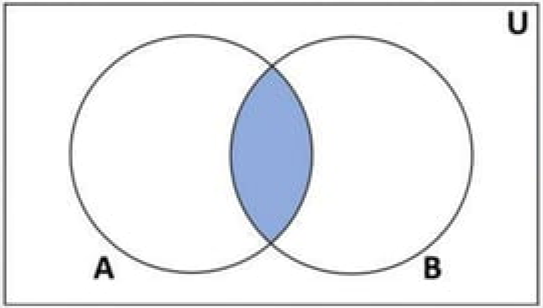
$A \cup U = \{1,2,4,7\} \cup \{1,2,3,4,5,6,7\}$

$= \{1,2,3,4,5,6,7\} = U$

Hence, Law of U proved.

Intersection

The intersection of sets A and B is the set of items that are in both A and B.



Notation: $A \cap B$

Examples:

$\{1, 2, 3\} \cap \{3, 4\} = \{3\}$

$\{1, 2, 3\} \cap \{4, 5, 6\} = \phi$ or $\{\}$

$\{1, 2\} \cap \{1, 2\} = \{1, 2\}$

Properties of Intersection of Sets

Commutative Law: The union of two or more sets follows the commutative law i.e., if we have two sets A and B then,

$$A \cap B = B \cap A$$

Example: $A = \{a, b\}$ and $B = \{b, c, d\}$

So, $A \cap B = \{b\}$

$B \cap A = \{b\}$

So, $A \cap B = B \cap A$

Hence, Commutative law proved.

Properties of Intersection of Sets

Associative Law: The union operation follows the associative law i.e., if we have three sets A, B and C then

$$(A \cap B) \cap C = A \cap (B \cap C)$$

Example: $A = \{a, b, c\}$ and $B = \{b, c, d\}$ and $C = \{a, c, e\}$

$(A \cap B) \cap C = \{b, c\} \cap \{a, c, e\} = \{c\}$

$A \cap (B \cap C) = \{a, b, c\} \cap \{c\} = \{c\}$

Hence, Associative law proved.

Properties of Intersection of Sets

Idempotent Law: The union of any set A with itself gives the set A.

$$A \cap A = A$$

Example: $A = \{1,2,3,4,5\}$

$$\begin{aligned} A \cap A &= \{1,2,3,4,5\} \cap \{1,2,3,4,5\} \\ &= \{1,2,3,4,5\} = A \end{aligned}$$

Hence, Idempotent law proved.

Properties of Intersection of Sets

Law of U : The union of a universal set U with its subset A gives the universal set itself.

$$A \cap U = A$$

A = {1,2,4,7} and U = {1,2,3,4,5,6,7}

Example:

$A \cap U = \{1,2,4,7\} \cap \{1,2,3,4,5,6,7\}$

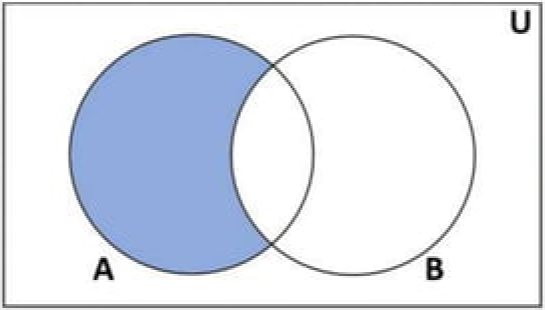
$= \{1,2,4,7\} = A$

Hence, Law of U proved.

Difference

The difference of sets A and B is the set of items that are in A but not B.

Notation: A − B



Examples:

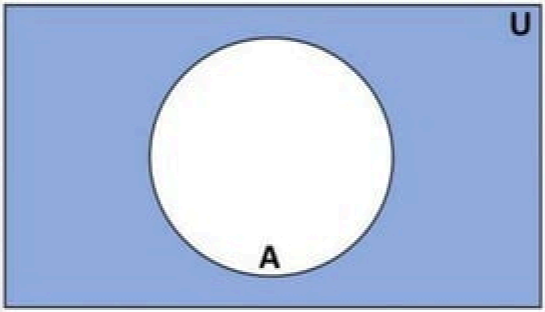
$\{1, 2, 3\} - \{2, 3, 4\} = \{1\}$

$\{1, 2\} - \{1, 2\} = \phi$

$\{1, 2, 3\} - \{4, 5\} = \{1, 2, 3\}$

Complement

The complement of set A is the set of items that are in the universal set U but are not in A.



Notation: A' or A^c

Examples:

- If $U = \{1, 2, 3\}$ and $A = \{1, 2\}$ then $A^c = \{3\}$
- If $U = \{1, 2, 3, 4, 5, 6\}$ and $A = \{1, 2\}$ then $A^c = \{3, 4, 5, 6\}$

Properties of Complement Sets

Complement Laws:

- $A \cup A' = U$
- $A \cap A' = \phi$

For Example:

- If U = {1 , 2 , 3 , 4 , 5 } and A = {1 , 2 , 3 } then
- $A' = \{4, 5\}$
- $A \cup A' = \{1, 2, 3, 4, 5\} = U$
- $A \cap A' = \{\} = \phi$

Properties of Complement Sets

Law of Double Complementation:

- (A')' = A

For Example:

- If U = {1 , 2 , 3 , 4 , 5 } and A = {1 , 2 , 3 } then
- A' = {4 , 5}
- (A')' = {1 , 2 , 3} = A
- (A')' = A

Properties of Complement Sets

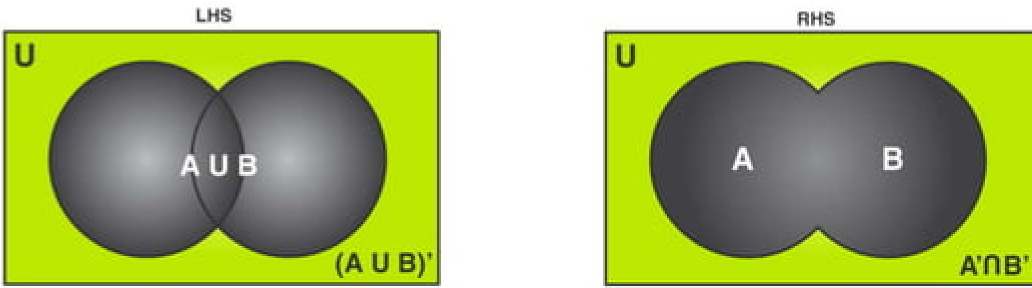
Law of empty set and universal set:

- $\phi' = U$
- $U' = \phi$

DE MORGAN'S LAW

The complement of the union of two sets A and B is equal to the intersection of the complement of the sets A and B.

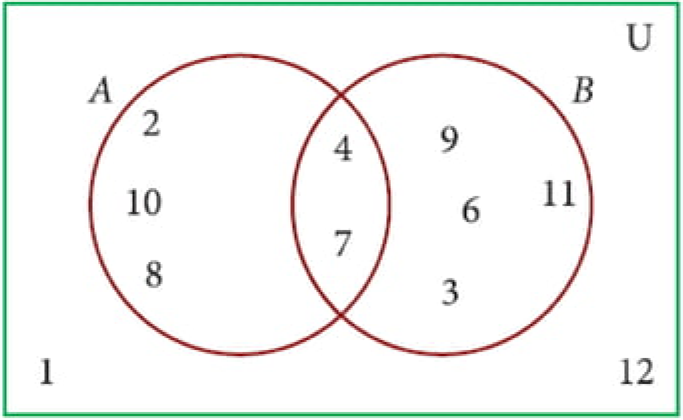
$$(A \cup B)' = A' \cap B'$$



INCLUSION EXCLUSION PRINCIPLE

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

- $n(A) = 5$
- $n(B) = 6$
- $n(A \cap B) = 2$
- $n(A \cup B) = 9$




thank you