Maxwell-Gl.:

① $\vec\nabla\cdot\vec E = \frac{\rho}{\varepsilon_0}$; $\phi_E = \int_{\partial V}\vec E\, d\vec f = \int_V d^3r\, \vec\nabla\cdot\vec E = \int_V \frac{\rho}{\varepsilon_0} = \frac{Q}{\varepsilon_0}$

② $\vec\nabla\times\vec E = -\frac{\partial\vec B}{\partial t}$; $\oint_{\partial A}\vec E\, d\vec s = -\int_A d\vec f\, \frac{\partial \vec B}{\partial t}$

③ $\vec\nabla\cdot\vec B = 0$; $\oint_{\partial V} d\vec f\, \vec B = 0$

④ $\vec\nabla\times\vec B = \mu_0\vec j + \mu_0\varepsilon_0\frac{\partial\vec E}{\partial t}$; $\oint_{\partial A}\vec B\, d\vec s = \underbrace{\int_A d\vec f\, \mu_0\vec j}_{=\mu_0 I} + \frac{d}{dt}\int_A \varepsilon_0\, \vec E\, d\vec f$

Stetigkeit:

$\hat n(\vec E_1 - \vec E_2) = \frac{\sigma}{\varepsilon_0}$ $\quad \hat t(\vec E_1 - \vec E_2) = 0$

$\hat n(\vec B_1 - \vec B_2) = 0$ $\quad \hat t(\vec B_1 - \vec B_2) = \mu_0\vec{\dot{j}}$

Im Leiter: $\vec E = 0$, $\phi = const.$ $\vec E_a \neq 0 \Rightarrow$ induzierte Ladung

$\phi(\infty) = 0$; $\phi(0)$ nicht singulär

$\phi(r=R)$ ist stetig und $\phi'(r=R)$ ist stetig

Randwertprobleme: ϕ auf $\partial V \to$ Dirichlet, $\hat n\,\vec\nabla\phi$ auf $\partial V \to$ Neumann

Separationsansatz: $\phi(\vec r) = \phi_x(x)\phi_y(y)\phi_z(z) \Rightarrow \frac{\phi_i''}{\phi_i} =$ konstante $k_c^2 \in \mathbb{C}$

DGL Legendre Polynome: $(1-x^2)P'' - 2xP' + \lambda P = 0 \Rightarrow \lambda = \ell(\ell+1)$

Separation Kugelkoord.: $\phi(\vec r) = \frac{U(r)}{r}P(\cos\theta)Q(\varphi)$, Zylinder: $Q = const.$

$L_\Delta \to \frac{r^2}{U}\frac{d^2U}{dr^2} + \frac{1}{P\sin\theta}\frac{d}{d\theta}\left(\sin\theta\frac{dP}{d\theta}\right) + \frac{1}{Q\sin^2\theta}\frac{d^2Q}{d\varphi^2} = 0$; $x = \cos\theta$ $\frac{dx}{d\theta} = -\sin\theta$

① Zylindersymmetrie: $U(\vec r) = A_\ell r^{\ell} + B_\ell r^{-\ell+1} \Rightarrow \phi(\vec r) = \sum_{\ell=0}^\infty U_\ell(r)P_\ell\cos\theta$

② Kugel: $\phi(r=R) = \phi_0\cos\theta$; $U(\vec r) = A_\ell r^\ell \Rightarrow \sum_\ell U_\ell(R)P_\ell\cos\theta = \phi(r=R)$

↳ Orthogonalität: $\sum_\ell U_\ell(r)P_\ell\cos\theta = \phi(R) \Rightarrow A_\ell r^\ell = \frac{2\ell+1}{2}\int_{-1}^1 dx\, P_\ell(x)\phi_0 x$

↳ Koeff. Vergl.; Lösung: $A_\ell = 0$ für $\ell \neq 1$, $\phi(\vec r,\theta) = \phi\frac{z}{r}$

③ Allgemein: $\frac{1}{Q}\frac{dQ^2}{d\varphi^2} = -m^2$; $\frac{\sin\theta}{P}\frac{d}{d\theta}\left(\sin\theta\frac{dP}{d\theta}\right) = m^2 \Rightarrow Q(\varphi) = e^{im\varphi}$

↳ P_ℓ^m DGL: $\frac{d}{dx}(1-x^2)\frac{dP}{dx} + \left(\lambda - \frac{m^2}{1-x^2}\right)P(x) = 0$

↳ Lösung: $\phi(\vec r,\theta,\varphi) = \sum_{\ell=0}^\infty\sum_{m=-\ell}^{\ell}(a_{\ell m}r^\ell + b_{\ell m}r^{-\ell-1})\, Y_{\ell m}(\theta,\varphi)$

Strahlung: Lorentz-Eichung $\Box\phi = -\frac{\rho}{\varepsilon_0}$; $\Box\vec A = -\mu_0\vec j$

Green-fkt. für Wellen: $\Box G(\vec r,\vec r', t,t') = -\delta(\vec r-\vec r')\,\delta(t-t')$

↳ $\phi(\vec r,t) = \frac{1}{\varepsilon_0}\int d^3r'\, dt'\, G(\vec r,t,\vec r',t')\,\rho(\vec r',t')$, $\vec A$ analog

$G(\vec k,\omega) = \frac{1}{k^2 - \frac{\omega^2}{c^2}}$; $G(\vec r,t) = \frac{1}{4\pi|\vec r-\vec r'|}\,\delta\left(t - t' \mp \frac{|\vec r-\vec r'|}{c}\right)$

$\Rightarrow \phi^+$ und $\vec A^+$ erfüllen Lorentz-Eichung

Abgestrahlte Energie: $\frac{dP}{d\Omega} = \frac{q^2}{(4\pi)^2\varepsilon_0 c}\dot\beta^2\sin^2\theta$, $\beta = \frac{|v|}{c}$

$P = \frac{q^2}{6\pi\varepsilon_0 c}\dot\beta^2$ (Larmor-Formel) $\quad \frac{dP}{d\Omega} = R^2\langle S\rangle_{r=R}\,\hat e_r$ (Sphäre)

Zeitabhängige Ladungsverteilung:

$\phi(\vec r,t) \approx \frac{1}{4\pi\varepsilon_0}\left[\frac{Q}{r} + \frac{\hat e_r\cdot\vec p(t_0)}{r^2} + \frac{\hat e_r\cdot\dot{\vec p}(t_0)}{rc}\right]$; $t_0 = t - \frac{|\vec r-\vec r'|}{c}$

$\vec A(\vec r,t) = \frac{\mu_0}{4\pi}\frac{\dot{\vec p}(t_0)}{r}$; $\vec S = \frac{1}{\mu_0}\vec E\times\vec B = \frac{1}{16\pi^2 r^2 c^3}\hat e_r\,|\ddot{\vec p}|\sin^2\theta$ $\quad \measuredangle\ddot{\vec p},\hat e_r$

Zusammenf. Wellen:

Maxwell-Gl.: $\Box\phi = -\frac{\rho}{\varepsilon_0}$; $\Box\vec A = -\mu_0\vec j$; $\vec B = \vec\nabla\times\vec A$; $\vec E = -\vec\nabla\phi - \frac{\partial}{\partial t}\vec A$

EM-Wellen: $\vec A_c(\vec r,t) = A_0\, e^{i\vec k\vec r - i\omega t}$ mit $\omega = c|\vec k|$

Zeitabhängige Ladungsverteilung: $\vec A(\vec r,t) = \mu_0\int d^3r'\,\frac{\vec j\left(\vec r, t - \frac{|\vec r-\vec r'|}{c}\right)}{4\pi|\vec r-\vec r'|}$

Abstrahlung: $\vec A(\vec r,t) = \frac{\dot{\vec p}(t_0)}{rc}$, $t_0 = t - \frac{r}{c}$, $|\vec A| \sim |\vec S| \sim \frac{1}{r}$

Hohlraumresonator: $\Box\vec E = 0$; $\Box\vec B = 0$; $\hat n\times\vec E = 0$; $\hat n\cdot\vec B = 0$

Separationsansatz: $\vec E = X(x)Y(y)Z(z)T(t)$

↳ $\underbrace{\frac{X''}{X}}_{-k_x^2} + \underbrace{\frac{Y''}{Y}}_{-k_y^2} + \underbrace{\frac{Z''}{Z}}_{-k_z^2} + \underbrace{\frac{T''}{T}}_{-\omega^2} = 0 \Rightarrow E_x = Re\left(C\sin(k_x x + \alpha)\sin\left(\frac{M\pi y}{L_y}\right)\sin\left(\frac{M\pi z}{L_z}\right)e^{-i\omega t}\right)$

$\Rightarrow \omega^2 = c^2(k_x^2 + k_y^2 + k_z^2)$

Aus $\vec\nabla\cdot\vec E = 0$ folgt: $\cdot\, C k_x + C' k_y + C'' k_z = 0$

$\cdot \cos(k_x x + \alpha) = \sin\left(\frac{\ell\pi x}{L_x}\right) = \sin\left(\frac{\ell'\pi x}{L_x}\right) \Rightarrow \ell' = \ell$; $\alpha = -\frac{\pi}{2}$; $k_x = \frac{\ell\pi}{L_x}$

$\cdot\, e^{i\omega t} = e^{i\omega' t} = e^{i\omega'' t} \Rightarrow \omega$ identisch

↳ $\omega = c\pi\sqrt{\frac{\ell^2}{L_x^2} + \frac{m^2}{L_y^2} + \frac{n^2}{L_z^2}}$ $\quad \omega_c = \min\left(\frac{n\pi}{L_x}, \frac{m\pi}{L_y}\right)$

$\vec B_c = -\frac{i}{\omega}\vec\nabla\times\vec E$

Elektrostatik: $\vec F_{12} = \frac{q_1 q_2}{4\pi\varepsilon_0}\frac{\vec r_1 - \vec r_2}{|\vec r_1 - \vec r_2|^3}$ ⊕→⊖ Dipolmoment: $\vec p = q\cdot\vec a$

$\vec E = \frac{\vec F}{q} = \sum_i \frac{1}{4\pi\varepsilon_0}q_i\frac{\vec r - \vec r_i}{|\vec r - \vec r_i|^3} = \frac{1}{4\pi\varepsilon_0}\int d^3r'\,\rho(\vec r')\frac{\vec r - \vec r'}{|\vec r - \vec r'|^3} = \frac{1}{4\pi\varepsilon_0}\left[\frac{3\vec r(\vec r\cdot\vec p)}{|\vec r|^5} - \frac{\vec p}{|\vec r|^3}\right]$

$\rho(\vec r) = \sum_i q_i\,\delta(\vec r - \vec r_q)$ $\left(= \frac{Q}{A} = \frac{Q}{V}\right)$; $\phi_E = \int_F d\vec f\cdot\vec E$, Kugel: $\phi_E = \frac{Q}{\varepsilon_0}$

Poisson-Gl.: $\Delta\phi(\vec r) = \frac{1}{\varepsilon_0}\rho(\vec r)$

Green-Laplace: $\Delta G(\vec r,\vec r') = \Delta\left[-\frac{1}{4\pi}\frac{1}{|\vec r-\vec r'|} + F(\vec r,\vec r')\right] = \delta(\vec r - \vec r')$

Energie Verschiebung: $W = -\int_{r_1}^{r_2}\vec F d\vec\ell = -\int_{r_1}^{r_2} q\vec E d\vec\ell = q(\phi(\vec r_2) - \phi(\vec r_1))$

Energiedichte: $u_{em} = \frac{1}{2}\left(\varepsilon_0\vec E^2 + \frac{1}{\mu_0}\vec B^2\right)$

Stromdichte: $I = \frac{dQ}{dt} = \int_S d\vec f\cdot\vec j$ mit $\vec j(\vec r) = \vec v(\vec r)\cdot\rho(\vec r)$

Kontinuitätsgleichung: $\frac{\partial}{\partial t}\rho(\vec r,t) + \vec\nabla\vec j(\vec r,t) = 0$

↳ Strom durch ∂V = Ladungsänderung durch V

Lorentzkraft: $\vec F_L = q(\vec E + \vec v\times\vec B)$ $\quad [\vec B] = \left[\frac{F}{q\cdot v}\right]$

↳ $\vec B$-Feld ändert Richtung $\vec v$ aber nicht Betrag $|\vec v|$

↳ $W = \int\vec F d\vec\ell = \int m\cdot\vec a\cdot\vec v = 0$ (verrichtet keine Arbeit)

Multipole: $\phi = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q}{r} + \frac{\vec r\cdot\vec p}{r^3}\right]$

Potentiale $\vec E = -\vec\nabla\phi - \frac{\partial}{\partial t}\vec A$, $\vec B = \vec\nabla\times\vec A$

Eichtransformation: $\vec A \to \vec A' = \vec A + \vec\nabla\chi$; $\phi \to \phi' = \phi + \frac{\partial}{\partial t}\chi$

Coulomb-Eichung: $\vec\nabla\cdot\vec A = 0$; $\Delta\vec A = \mu_0\vec j$ (Poisson-Gl.)

Lorentz-Eichung: $\underbrace{\vec\nabla\cdot\vec A + \mu_0\varepsilon_0\frac{\partial}{\partial t}\phi}_{\text{in Elektrostatik: } = 0} = 0$

$\vec A(\vec r) = \mu_0\int d^3r'\,\frac{\vec j(\vec r')}{|\vec r - \vec r'|}$

$\vec B(\vec r) = \frac{\mu_0}{4\pi}\int d^3r'\,\frac{\vec j(\vec r')\times(\vec r - \vec r')}{|\vec r - \vec r'|^3}$ Biot-Savart-Gesetz

↳ Unendlicher Draht: $\vec B(\vec r) = \frac{\mu_0}{2\pi}I\frac{\hat e_\varphi}{\rho}$

Magnet. Dipolmoment: $\vec A(\vec r) = \frac{\mu_0}{4\pi}\frac{\vec m\times\vec r}{|\vec r|^3}$ mit $\vec m = \frac{1}{2}\int d^3r'\,\vec r'\times\vec j(\vec r')$

↳ $\vec B = \frac{\mu_0}{4\pi}\frac{3(\vec m\cdot\vec r)\vec r}{r^5} - \frac{\vec m}{r^3}$

Lorentz-Kraft: $\vec F = \vec j\times\vec B = -\vec\nabla u$ mit $u = -\vec m\cdot\vec B$

Drehmoment d. Magnetostatik: $\vec M = \vec m\times\vec B$

Faraday Induktionsgesetz: $U = -\int_S d\vec f\cdot\frac{d\vec B}{dt} \Leftarrow = \vec\nabla\times\vec E_{ind}$

Energiestromdichte: $\vec S = \frac{1}{\mu_0}\vec E\times\vec B$

Poynting-Theorem: $\frac{\partial}{\partial t}u_{em} + \vec\nabla\cdot\vec S = -\vec j\cdot\vec E$ (Kontinuitätsgl. für Energie)

Impulserhaltung: F_L für Impuls von Ladungen & Strömen verantwortlich

↳ $\frac{d\vec P}{dt} = F = \int_V d^3r\,\underbrace{(\rho\vec E + \vec j\times\vec B)}_{\text{Kraftdichte } f_L}$ $\quad \vec P = \int_V d^3r\,\underbrace{\vec\Pi_{em}}_{\text{Impulsdichte}}$

Spannungstensor: $\quad \vec\Pi_{em} = \varepsilon_0\vec E\times\vec B_0 = \varepsilon_0\mu_0\vec S$

$T_{ij} = \varepsilon_0(E_iE_j - E^2\delta_{ij}) + \frac{1}{\mu_0}(B_iB_j - B^2\delta_{ij})$

↳ Impulssatz: $f_{L_i} = -\frac{\partial}{\partial t}\Pi_{em_i} + \partial_j T_{ij}$

Wellengleichung: $\Box = \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \Delta$ mit $c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$ $\quad \omega = c|\vec k|$

Ebene Welle: $\vec A_c(\vec r,t) = \vec A_0\, e^{i\vec k\vec r - i\omega t}$, $\vec E$ und $\vec B$ analog

Coulomb-Eichung: $\phi = 0$; $\vec B_c = i\vec k\times\vec A_c$; $\vec E_c = i\omega\vec A_c$

Phasen: $\vec E(\vec r,t) = Re(\vec E_0\, e^{i\vec k\vec r - i\omega t})$ mit $\vec E_0 = |E_0|(\cos\alpha\,\hat e_1 + \sin\alpha\,\hat e_2)\, e^{i\varphi}$

$\vec B_0 = |B_0|(-\sin\alpha\,\hat e_1 + \cos\alpha\,\hat e_2)\, e^{i\varphi}$

Energie & Impuls:

$\langle\vec E\rangle = \frac{1}{2}\vec E_0\vec E_0^*$ $\quad \langle\vec B\rangle = \frac{1}{2}\vec B_0\vec B_0^*$ $\quad |B_0|^2 = \frac{1}{c^2}|E_0|^2$

$\langle w_{em}\rangle = \frac{1}{4}\varepsilon_0\vec E_0\vec E_0^* + \frac{1}{4}\mu_0\vec B_0\vec B_0^* = \frac{1}{2}\varepsilon_0\vec E_0\vec E_0^*$

$\langle\vec S\rangle = \langle\frac{1}{\mu_0}\vec E\times\vec B\rangle = \frac{1}{4\mu_0}\langle\vec E_0\vec B_0^* + \vec E_0^*\vec B_0\rangle = c\hat k\langle w_{em}\rangle$

Koax-Kabel: $\vec E = \vec E_0\, e^{ikz - i\omega t}$, $\vec B$ analog ; selbe Stetigkeitsbed.

↳ $\vec E_0 = f_\rho(\rho)\hat e_\rho + f_\varphi(\rho)\hat e_\varphi$; $\vec B$ analog ;

$\cdot\,\vec\nabla\vec E = 0 \Rightarrow f_\rho = \frac{c_1}{\rho}$; $\cdot\,\vec\nabla\vec B = 0 \Rightarrow g_\rho = \frac{c_2}{\rho}$; $(\vec\nabla\times\vec B)_z = \frac{1}{c^2}\frac{\partial}{\partial t}E_z = 0 \Rightarrow g_\varphi = \frac{c_3}{\rho}$

$\cdot\,\vec\nabla\times\vec E_z = 0 \Rightarrow f_\varphi = \frac{c_4}{\rho}$; $\cdot\,(\vec\nabla\times\vec E)_\varphi = \frac{\partial}{\partial z}E_r = -\frac{\partial}{\partial t}B_\rho \Rightarrow c_3 = \frac{k}{\omega}c_1$

$\cdot\,(\vec\nabla\times\vec B)_\rho = \frac{1}{c^2}\frac{\partial}{\partial t}E_\rho \Rightarrow c_1 = \frac{1}{c^2}\frac{k}{\omega}c_3 \Rightarrow k = \frac{\omega}{c}$

↳ $\vec E = \frac{c_1}{\rho}\hat e_\rho\, e^{ikz - i\omega t}$, $\vec B = \frac{1}{c}\vec E\cdot\frac{\hat e_\varphi}{\hat e_\rho}$

$\frac{1}{|\vec r - \vec r'|} \approx \frac{1}{r} + \frac{\vec r\cdot\vec r'}{r^3}$

4-Vektoren:

kontravariant: $a'^{\mu} = \Lambda^{\mu}{}_{\nu} a^{\nu} = \begin{pmatrix} ct \\ \vec{r} \end{pmatrix}$

kovariant: $a'_{\mu} = \Lambda_{\mu}{}^{\nu} a_{\nu}$

Skalarprodukt: $a \cdot b = a^{\mu} b_{\mu} = a^{\mu} \eta_{\mu\nu} b^{\nu}$

Minkowski-Metrik: $\eta = \mathrm{diag}(1, -1, -1, -1)$

$a^{\mu} = \eta^{\mu\nu} a_{\nu}$

$a_{\mu} = \eta_{\mu\nu} a^{\nu}$ $\quad \eta_{\mu\nu} = \eta^{\mu\nu} = \eta^{-1}$

Abstand: $s = x^{\mu} \eta_{\mu\nu} x^{\nu} = x^{\mu} x_{\mu}$ (Lorentz-invariant)

Lorentz-Transformation:

$ct' = \gamma ct - \beta\gamma x$

$x' = \gamma x - \beta\gamma ct$

$\beta = \frac{v}{c}$

$\gamma = \frac{1}{\sqrt{1-\beta^2}}$

Zeit: $\Delta t' = \gamma \Delta t > \Delta t$

Länge: $\Delta x' = \frac{1}{\gamma} \Delta x < \Delta x$

Speed: $u' = \frac{u' + v}{1 + \frac{u' v}{c^2}}$

4-Gradient:

$\partial_{\mu} = \frac{\partial}{\partial x^{\mu}} = \begin{pmatrix} \frac{1}{c}\frac{\partial}{\partial t} \\ \vec{\nabla} \end{pmatrix}$ ist 4-Vektor

$\partial_{\mu}' = \frac{\partial}{\partial x'^{\mu}} = \frac{\partial x^{\nu}}{\partial x'^{\mu}} \frac{\partial}{\partial x^{\nu}} = \Lambda_{\mu}{}^{\nu} \partial_{\nu}$ $\quad d^4x' = |\det \Lambda|\, d^4x$

$\partial_{\mu}\partial^{\mu} = \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \Delta = \Box$ ist Lorentzinvariant

4-Speed:

$d\tau = \sqrt{1-\frac{v^2}{c^2}}\, dt = \frac{1}{c}\sqrt{c^2 dt^2 - d\vec{r}^2} = \frac{1}{c} ds$ $\rightarrow = \frac{1}{\gamma} dt$

$u^{\mu} = \begin{pmatrix} \gamma c \\ \gamma \vec{v} \end{pmatrix} \Rightarrow u^{\mu} u_{\mu} = c^2$

4-Impuls:

$p^{\mu} = m u^{\mu} = \begin{pmatrix} \gamma m c \\ \gamma m \vec{v} \end{pmatrix} = \gamma m c \begin{pmatrix} 1 \\ \vec{\beta} \end{pmatrix} = m \frac{dx^{\mu}}{d\tau}$ $\rightarrow p^{\mu} p_{\mu} = m^2 c^2$

$cp^0 = \gamma m c^2 = mc^2 + \frac{1}{2} m v^2 \Rightarrow (p^0)^2 - \vec{p}^2 = mc^2$

Energie-Impuls-Erhaltung: $E = \sqrt{m^2c^4 - p^2c^2}$; $\sum p^{\mu} = \mathrm{const.}$

Lorentz-Transform:

$A'^{\mu} = \Lambda^{\mu}{}_{\nu} A^{\nu}$, $A'_{\mu} = \eta_{\mu\varsigma} \Lambda^{\varsigma}{}_{\sigma} \eta^{\sigma\nu} A_{\nu} = (\Lambda^{T})^{-1}{}_{\mu}{}^{\nu} A_{\nu}$

Rücktransform: $\Lambda^{-1} = \Lambda^{T}{}_{\mu}{}^{\nu} = \eta_{\mu\varsigma} \Lambda^{\varsigma}{}_{\sigma} \eta^{\sigma\nu}$

Regeln: $\eta \Lambda \eta^{-1} = (\Lambda^{-1})^{T}$; $\Lambda \Lambda^{-1} = \Lambda^{T}(\Lambda^{-1})^{T} = \mathbb{1} = \delta$ (Lorentz-Delta)

$x'^{\mu} x_{\mu} = x_{\mu} x^{\mu}$; $(\eta \Lambda \eta^{-1})^{T} = \Lambda^{-1}$; $\Lambda_{\mu}{}^{\nu} \Lambda^{\varsigma}{}_{\nu} = \delta_{\mu}{}^{\varsigma}$

Lorentz-Boost:

Rotation: $\Lambda = \begin{pmatrix} 1 & \\ & \hat{R} \end{pmatrix}$ 3x3 Matrix $\begin{pmatrix} \cos & -\sin \\ \sin & \cos \end{pmatrix}$

$$\Lambda = \begin{pmatrix} \gamma & -\beta\gamma & & \\ -\beta\gamma & \gamma & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$$

$\Rightarrow$ Beliebiger Boost d. Rotation mit x-Boost verketten

Lorentz-Kraft:

$\frac{dp^{\mu}}{d\tau}$ ist 4-Vektor

Annahme: $\frac{dp^{\mu}}{d\tau} = kq F^{\mu\nu} u_{\nu}$

$\rightarrow$ Räumlich: $\mu = i = 1,2,3$

$\hookrightarrow \frac{d\vec{p}}{dt} = q(\vec{E} + \vec{v} \times \vec{B})$ mit $k=1$

$\rightarrow$ Zeitlich: $\mu = 0$

$\hookrightarrow \frac{dE}{dt} = q\vec{E} \cdot \vec{v}$ (Lorentz-Leistung)

Lagrange: $S = \int_{t_1}^{t_2} dt\, L(\vec{r}, \dot{\vec{r}})$; $\mathcal{L} = -mc^2 - qA^{\mu}u_{\mu}$

$\hookrightarrow S = \int d^4x \left(\frac{1}{4\mu_0} F^{\mu\nu} F_{\mu\nu} - A^{\mu} j_{\mu} \right) = S \int d^4x\, \mathcal{L}$

$$F^{\mu\nu} = \begin{pmatrix} 0 & -\frac{E_x}{c} & -\frac{E_y}{c} & -\frac{E_z}{c} \\ & 0 & -B_z & B_y \\ & & 0 & -B_x \\ & & & 0 \end{pmatrix}$$

Kontinuitätsgleichung:

$\partial_{\mu} j^{\mu} = 0$ mit $j^{\mu} = \begin{pmatrix} c\varrho \\ \vec{j} \end{pmatrix}$

Dispersion: $k^{\mu} k_{\mu} = 0$ $\quad j^{\mu} = \varrho v^{\mu} = \frac{\varrho}{\gamma} u^{\mu}$ und $\frac{\varrho}{\gamma}$ ist lorentzinvariant

4-Potential: $\partial_{\mu}(\partial^{\mu} A^{\nu} - \partial^{\nu} A^{\mu}) = \mu_0 j^{\mu}$ mit $A^{\mu} = \begin{pmatrix} \frac{1}{c}\phi \\ \vec{A} \end{pmatrix}$ und $\partial^{\mu} = \eta^{\mu\nu}\partial_{\nu} = \begin{pmatrix} \frac{1}{c}\frac{d}{dt} \\ -\vec{\nabla} \end{pmatrix}$

$\hookrightarrow$ Eichtransform: $A^{\mu} \rightarrow A'^{\mu} = A^{\mu} - \partial^{\mu}\chi$, Lorentz-Eichung: $\partial_{\mu} A^{\mu} = 0 \Rightarrow \Box A^{\nu} = \mu_0 j^{\nu}$

Feldstärkentensor: $F^{\mu\nu} = \partial^{\mu}A^{\nu} - \partial^{\nu}A^{\mu} \rightarrow F^{ii} = 0$, $F^{i0} = \frac{1}{c}\vec{E}_{\nu} = -F^{0i}$; $F^{ij \neq 0} = -\varepsilon_{ijk} \vec{B}_k$ $\quad \boxed{\partial^{\mu} k^{\alpha} x_{\alpha} = k^{\mu}}$

$\hookrightarrow$ Maxwell-Gl.: $\partial_{\mu} F^{\mu\nu} = \mu_0 j^{\nu}$, Kontinuitätsgl.: $\partial_{\nu} j^{\nu} = \frac{1}{\mu_0}\partial_{\nu}\partial_{\mu} F^{\mu\nu} = 0$

Relativistische Einflüsse: $\vec{E}_{\parallel}' = \vec{E}_{\parallel}$, $\vec{B}_{\parallel}' = \vec{B}_{\parallel}$, $\vec{E}_{\perp}' = \gamma(\vec{E}_{\perp} + \vec{v} \times \vec{B})$, $\vec{B}_{\perp}' = \gamma(\vec{B}_{\perp} - \frac{1}{c}(\vec{v} \times \vec{E}))$

Phase:

$\varphi = \omega t - \vec{k} \cdot \vec{r} = k_{\mu} x^{\mu}$ mit $k_{\mu} = \begin{pmatrix} \omega/c \\ -\vec{k} \end{pmatrix}$, k_{μ} und $k_{\mu}k^{\mu}$ sind lorentz-invariant $\quad \cos\Theta' = \frac{k'_x}{\frac{\omega'}{c}} = \frac{\cos\Theta - \beta}{1 - \beta\cos\Theta}$

$\hookrightarrow$ Lorentz-Boost $\Lambda k_{\mu}' \Rightarrow \frac{\omega'}{c} = \gamma \frac{\omega}{c}(1 - \beta\cos\Theta)$ und $k_x' = \gamma |k| (\cos\Theta - \beta) \Rightarrow \omega' = |\vec{k}'| c$

$\hookrightarrow \omega' = \omega \cdot (\gamma(1 + \beta\cos\Theta'))^{-1}$ $\quad \bullet\, \Theta = 0$ (von Quelle weg) $\Rightarrow \omega' > \omega$ $\quad \bullet\, \Theta = \pi$ (zu Quelle): $\omega' < \omega$ $\quad \bullet\, \Theta = \frac{\pi}{2}$: $\omega' = \frac{\omega}{\gamma} = 1 - \frac{1}{2}\beta^2$ für $\beta \ll 1$

Integralsätze:

$\int_{\vec{a}}^{\vec{b}} d\vec{\ell}\, \vec{\nabla}\phi = \phi(\vec{b}) - \phi(\vec{a})$

Satz von Gauß: $\int_V d^3r\, \vec{\nabla} \cdot \vec{A} = \int_{\partial V} d\vec{f} \cdot \vec{A}$

-"- -"- Stokes: $\oint_{\partial S} d\vec{\ell} \cdot \vec{A} = \int d\vec{f}\, (\vec{\nabla} \times \vec{A})$

1. Green: $\oint_{\partial V} d\vec{f}\, (\psi \vec{\nabla}\phi) = \int_V d^3r\, [(\nabla\psi)(\nabla\phi) + \psi\Delta\phi]$

2. Green: $\oint_{\partial V} d\vec{f}\, (\psi\vec{\nabla}\phi - \phi\vec{\nabla}\psi) = \int_V d^3r\, [\psi\Delta\phi - \phi\Delta\psi]$

Fourier:

$\tilde{f}(\omega) = \int dt\, f(t)\, e^{-i\omega t}$; $f(t) = \int \frac{d\omega}{2\pi} \tilde{f}(\omega)\, e^{i\omega t}$

Geometrische Reihe: $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$ für $|x| < 1$

$\sin(x) = \frac{1}{2i}(e^{ix} - e^{-ix})$; $\cos(x) = \frac{1}{2}(e^{ix} + e^{-ix})$

$\cosh^2(x) - \sinh^2(x) = 1$; $\sin(x) \approx x - \frac{x^3}{6}$; $\cos(x) \approx 1 - \frac{x^2}{2}$

$\cosh(x) + \sinh(x) = e^{x}$; $\cosh(x) - \sinh(x) = e^{-x}$

$\int_0^{L_x} \sin\left(\frac{n\pi x}{L_x}\right) \sin\left(\frac{n'\pi x}{L_x}\right) dx = \begin{cases} \frac{L_x}{2} & \text{für } n' = n \\ 0 & \text{sonst} \end{cases}$ $\quad \boxed{\nabla_r \rightarrow \nabla_k}$

$\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$; $\vec{H} = \frac{1}{\mu_0}\vec{B} - \vec{M}$

$\hat{n} \times (E_2 - E_1) = 0$; $\hat{n} \cdot (D_2 - D_1) = \sigma$;

$\hat{n} \cdot (B_2 - B_1) = 0$; $\hat{n} \times (H_2 - H_1) = j$

Legendre-Polynome:

$\ell=2$	$3(1-x^2)$	$-3x\sqrt{1-x^2}$	$\frac{1}{2}(3x^2-1)$	$\frac{1}{2}x\sqrt{1-x^2}$	$\frac{1}{8}(1-x^2)$
$\ell=1$		$-\frac{1}{2}\sqrt{1-x^2}$	x	$\frac{1}{2}\sqrt{1-x^2}$	
$\ell=0$			1		
	$m=-2$	$m=-1$	$m=0$	$m=1$	$m=2$

Kugelflächenfunktionen

$Y_{\ell 0} = [(2\ell+1)/4\pi]$

$\ell=2$	$\sqrt{\frac{15}{32\pi}} \sin^2\Theta\, e^{-2i\varphi}$	$\sqrt{\frac{15}{8\pi}} \sin\Theta\cos\Theta\, e^{-i\varphi}$	$\sqrt{\frac{5}{16\pi}}(3\cos^2\Theta - 1)$	$-\sqrt{\frac{15}{8\pi}} \sin\Theta\cos\Theta\, e^{i\varphi}$	$\sqrt{\frac{15}{32\pi}} \sin^2\Theta \cdot e^{2i\varphi}$
$\ell=1$		$\sqrt{\frac{3}{8\pi}} \sin\Theta\, e^{-i\varphi}$	$\sqrt{\frac{3}{4\pi}} \cos\Theta$	$-\sqrt{\frac{3}{8\pi}} \sin\Theta\, e^{i\varphi}$	
$\ell=0$			$\sqrt{\frac{1}{4\pi}}$		
	$m=-2$	$m=-1$	$m=0$	$m=1$	$m=2$

- $Y_{\ell 0} = \sqrt{\frac{2\ell+1}{4\pi}}\, P_{\ell}(\cos\Theta)$ $\quad \bullet\, Y_{\ell,-m} = (-1)^m Y_{\ell m}^{*}$
- $f(\Theta, \varphi) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} c_{\ell m} Y_{\ell m}(\Theta, \varphi)$ mit $c_{\ell m} = \int_0^{2\pi} d\varphi \int_0^{\pi} d\Theta\, \overset{\sin\Theta}{\vee}\, Y_{\ell m}^{*}(\Theta', \varphi') f(\Theta, \varphi)$
- $\sum_{m=-\ell}^{\ell} \frac{4\pi}{2\ell+1} Y_{\ell m}^{*}(\Theta', \varphi') Y_{\ell m}(\Theta, \varphi) = P_{\ell}\cos(\gamma)$ Additionstheorem
- $\Delta_{\Theta,\varphi} Y_{\ell m} + \ell(\ell+1) Y_{\ell m} = 0$
- $\int_0^{2\pi} d\varphi \int_0^{\pi} d\Theta \sin\Theta\, Y_{\ell' m'}^{*}(\Theta', \varphi') Y_{\ell m}(\Theta, \varphi) = \delta(\cos\Theta - \cos\Theta')\, \delta(\varphi - \varphi')$

Legendre-Polynome: $\bullet\, P_{\ell}(-x) = (-1)^{\ell} P_{\ell}(x)$; $\bullet\, P_{\ell}(1) = 1$; $\bullet \int_{-1}^{1} dx\, P_{\ell} P_{\ell'} = \frac{2}{2\ell+1}\delta_{\ell\ell'}$