(1) Mitesh Agrawal

Assignment → 1

1 $\begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix} \xrightarrow[\substack{R_3 \to R_3 - 3R_1 \\ R_4 \to R_4 - 6R_1}]{R_2 \to R_2 - 2R_1} \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 0 & -4 & -11 & 5 \end{bmatrix}$

$\downarrow R_4 \to R_4 - R_3 - R_2$

$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & -3 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \xleftarrow{R_3 \leftrightarrow R_2} \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

→ $\boxed{\text{Rank} = 3}$ as 3 non-zero rows

2. ATQ, every matrix of vector space W is of the form $\begin{bmatrix} a & b \\ \cancel{c} & \cancel{c} \end{bmatrix}$

W: vector space of 2×2 symmetric matrices.

$W = \{A : \text{where. } A^T = A\}$

$T: W \to P_2$

where

$T\begin{bmatrix} a & b \\ c & d \end{bmatrix} = (a-b) + (b-c)x + (c-a)x^2$

rank(T) = dim (Im T) and Nullity(T) = dim(Ker T)

As $w \in W$, $w = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ → symmetric matrix

$\Downarrow$

$\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a & c \\ b & d \end{bmatrix} \Rightarrow \boxed{b = c}$

And hence the $\boxed{\dim(W) = 3}$

Mitesh

(2)

For Rank (T)

standard Basis :- $\left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$

so, $T\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} = 1 + 0x + (-1)x^2 = 1 - x^2$

$T\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = -1 + 0x + 1x^2 = -1 + x^2$

$T\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} = 0 + 0x + 0x^2 = 0$

so, $M = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{R_2 \to R_2 + R_1} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$\Downarrow$

rank (M) = 1
= dim (Im T)

rank (T) = 1 = dim (Im T) = rank (M)

$\therefore$ $\boxed{\text{rank}(T) = 1}$ and $\boxed{\text{nullity}(T) = 2}$

$\hookrightarrow$ rank + nullity = 3

3. $\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$ $|A - \lambda I| = 0$

$\Downarrow$

$\lambda^2 - (Tr(A))\lambda + |A| = 0$

$\lambda^2 - 4\lambda + 3 = 0$

$\lambda^2 - 3\lambda - \lambda + 3 = 0$

$\lambda(\lambda - 3) - 1(\lambda - 3) = 0$

$(\lambda - 3)(\lambda - 1) = 0$

$\boxed{\lambda = 1, 3}$ → eigen values of A

$\Downarrow$

$\boxed{\text{eigen values of } A^{-1} = 1/3, 1}$

$\boxed{\text{eigen values of } A + 4I = 5, 7}$

Mitesh

③

As eigenvectors of A & A^{-1} are same

$\therefore$ for $\lambda = 1$

$$\begin{bmatrix} 2-\lambda & -1 \\ -1 & 2-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$R_2 \to R_2 + R_1$

$$\begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow x_1 - x_2 = 0$$

$\Downarrow$

$c \begin{bmatrix} 1 \\ 1 \end{bmatrix} \to$ eigenvector of A^{-1}

for $\lambda = 3$

$$\begin{bmatrix} 2-3 & -1 \\ -1 & 2-3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = c \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

(ii) $B = A + 4I \longrightarrow \lambda = 5, 7]$

$$= \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} + \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} 6 & -1 \\ -1 & 6 \end{bmatrix}$$

$$[B - \lambda I][X] = [0]$$

$$\begin{bmatrix} 6-\lambda & -1 \\ -1 & 6-\lambda \end{bmatrix}$$

Mitesh

For $\lambda = 7$ $[B - 7I][x] = [0]$

$$\begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$\Downarrow$

$\boxed{\text{eigen vector} \rightarrow (1, -1)}$ $\rightarrow$ for $A + 4I$

For $\lambda = 5$, $[B - 5I][x] = [0]$

$$\begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$\Downarrow$

$x_1 = x_2$

$\boxed{\text{eigen vector} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}}$ $\rightarrow$ for $A + 4I$

4. $3x - 0.1y - 0.2z = 7.85$

$0.1x + 7y - 0.3z = -19.3$

$0.3x - 0.2y + 10z = 71.4$

Initially $x^0 = 0$, $y^0 = 0$, $z^0 = 0$

1st iteration

$$x^1 = \frac{7.85 + 0.1y^0 + 0.2z^0}{3}$$

$$x^1 = \frac{7.85}{3} \quad \boxed{x^1 = 2.61}$$

$$y^1 = \frac{-19.3 + 0.3z^0 - 0.1x^1}{7}$$

$$= \frac{-19.3 - (0.1)(2.61)}{7}$$

$$\boxed{y^1 = -2.79}$$

$$z' = \frac{71.4 + 0.2y' - 0.3x'}{10} = \frac{71.4 + 0.2(-2.79) - 0.3(7.1)}{10}$$

$$\boxed{z' = 7.00}$$

2nd iteⁿ

$$x^{(2)} = \frac{7.85 + 0.2z^{(1)} + 0.1y^{(1)}}{3} = \frac{7.85 + 0.2(7) + 0.1(-2.79)}{3}$$

$$\boxed{x^{(2)} = 2.99}$$

$$y^{(2)} = \frac{-19.3 + 0.3z^{(1)} - 0.1x^{(2)}}{7}$$

$$y^{(2)} = -2.49$$

$$z^{(2)} = \frac{71.4 + 0.2y^{(2)} - 0.3x^{(2)}}{10}$$

$$= \frac{71.4 + 0.2(-2.49) - 0.3(2.99)}{10}$$

$$\boxed{z^{(2)} = 7}$$

3rd itⁿ

$$x^{(3)} = \frac{7.85 + 0.2z^{(2)} + 0.1y^{(2)}}{3}$$

$$= \frac{7.85 + 0.2(7) + 0.1(-2.49)}{3}$$

$$\boxed{x^{(3)} = 3.00}$$

$$y^{(3)} = \frac{-19.3 + 0.3z^{(2)} - 0.1x^{(3)}}{7}$$

$$= \frac{-17.3 + 0.3(7) - 0.1(3)}{7}$$

$$= -2.5$$

$$z^{(3)} = \frac{71.4 + 0.2y^{(3)} - 0.3x^{(3)}}{10}$$

$$= \frac{71.4 + 0.2(-2.5) - 0.3(3)}{10}$$

$$\boxed{z^{(3)} = 7}$$

$\therefore$ 1st iteⁿ $(x^{(1)}, y^{(1)}, z^{(1)}) = (2.61, -2.79, 7.00)$
2nd iteⁿ $(x^{(2)}, y^{(2)}, z^{(2)}) = (2.99, -2.49, 7.001)$
3rd iteⁿ $(x^{(3)}, y^{(3)}, z^{(3)}) = (3.00, -2.5, 7.00)$

(5) $x + 3y + 2z = 0$
$2x - y + 3z = 0$
$3x - 5y + 4z = 0$
$x + 17y + 4z = 0$

$$A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & -1 & 3 \\ 3 & -5 & 4 \\ 1 & 17 & 4 \end{bmatrix}$$

$R_2 \to R_2 - 2R_1$
$R_3 \to R_3 - 3R_1$
$R_4 \to R_4 - R_1$

$$\begin{bmatrix} 1 & 3 & 2 \\ 0 & -7 & -1 \\ 0 & -14 & -2 \\ 0 & 14 & 2 \end{bmatrix}$$

$R_3 \to R_3 - 2R_2$
$R_4 \to R_4 + 2R_2$

$$\begin{bmatrix} 1 & 3 & 2 \\ 0 & -7 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$\rho(A) < 3$
$\Downarrow$
∞ solⁿ (Consisted)

$-7y - z = 0$
$z = -7y$

$$\boxed{\begin{aligned} y &= k \\ z &= -7k \end{aligned}}$$

$x + 3y + 2z = 0$
$x + 3k + 2(-7k) = 0$
$\boxed{x = 11k}$
$k \in R$

6) To determine
$T(a+bx+cx^2) = (a+1)+(b+1)x+(c+1)x^2$ is a linear Transformation or not.

(i) $T(P_1)+T(P_2) = T(P_1+P_2)$

$T(a_1+b_1x+c_1x^2) + T(a_2+b_2x+c_2x^2)$

$= (a_1+1)+(b_1+1)x+(c_1+1)x^2 + (a_2+1)+(b_2+1)x+(c_2+1)x^2$

$= (a_1+a_2+2)+(b_1+b_2+2)x+(c_1+c_2)x^2$

$T(P_1+P_2) = T((a_1+a_2)+(b_1+b_2)x+(c_1+c_2)x^2)$
$= (a_1+a_2+1)+(b_1+b_2+1)x+(c_1+c_2+1)x^2$

$\Downarrow$

$T(P_1)+T(P_2) \neq T(P_1+P_2)$

$\Downarrow$

T is not a linear Transformation

7) For $S = \{(1,2,3),(3,1,0),(-2,1,3)\}$ be a basis of $V=\mathbb{R}^3$

$\Downarrow$

S should be LI & S should span V

$$S = \left[\begin{array}{ccc:c} 1 & 2 & 3 & x_1 \\ 3 & 1 & 0 & x_2 \\ -2 & 1 & 3 & x_3 \end{array}\right] \xrightarrow[R_2 \to R_2 - 3R_1]{R_3 \to R_3 + 2R_1} \left[\begin{array}{ccc:c} 1 & 2 & 3 & x_1 \\ 0 & -5 & -9 & x_2 - 3x_1 \\ 0 & 5 & 9 & x_3 + 2x_1 \end{array}\right]$$

$\Downarrow R_3 \to R_3 + R_2$

$$\left[\begin{array}{ccc:c} 1 & 2 & 3 & x_1 \\ 0 & -5 & -9 & x_2 - 3x_1 \\ 0 & 0 & 0 & x_3 + 2x_2 + x_2 - 3x_1 \end{array}\right]$$

Not fix $\Leftarrow$ Rank$(S) = 2$ if $(x_3+3x_2-3x_1=0)$
$R(S) = 3$ if $(x_3+3x_2-3x_1 \neq 0)$

$\Downarrow$

S is not LI

$\therefore$ S do not form basis of $V=\mathbb{R}^3$

Mitesh

(8)

Dime → 2

basis for the subspace spanned by $S = \{(1,3,3)(3,1,0)\}$

(8) $3x - 6y + 2z = 23$
$-4x + y - z = -15$
$x - 3y + 7z = 16$

1st itn :- $x^{(1)} = \frac{23 + 6\overset{0}{y} - 2\overset{0}{z}}{3} = 7.66$

$y^{(1)} = -15 + z + 4\underset{7.66}{x} = 15.64$

$z^{(1)} = \frac{16 + 3\overset{15.64}{y} - \overset{7.66}{x}}{7} = 6.46$

$(x^{(1)}, y^{(1)}, z^{(1)}) = (7.66, 15.64, 6.46)$

2nd iteration

$x^{(2)} = \frac{23 + 6y^{(2)} - 2z^{(2)}}{3} = \frac{23 + 6(15.64) - 2(6.46)}{3}$

$\boxed{x^{(2)} = 34.64}$

$y^{(2)} = \frac{4x^2 + z^{(1)} - 15}{}$ $= 130.02$

$z^{(2)} = \frac{16 + 3y^{(2)} - x^{(2)}}{7} = 53.06$

3rd itn

$x^{(3)} = \frac{23 + 6y^{(2)} - 2z^{(2)}}{3} = 232.33$

$y^{(3)} = -15 + z^{(2)} + 4x^{(2)} = 967.38$

$z^{(3)} = \frac{16 + 3y^{(2)} - x^{(3)}}{7} = 383.68$

(9) Matrix operations are extensively used in image processing. Like transpose of matrix is used to rotate the image in various directions and the blur matrix is used to blur certain area of image.

Apart from this, images are made up of matrix itself. Images are made up of pixels which are arranged in grid to produce image.

(10) Linear transformation plays very important role in computer vision. It is extremely used in manipulating image for various purposes.

One example is rotating image with θ angle about x-axis.

For this purpose we use famous rotation matrix in 2D, to do this task.

Here $T: V \to W$,

$$\text{where } T(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

If we have to rotate (x, y) about θ, then our new x' & y' are:

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

In this way, we perform this basic operation for each pixel of the image and find the rotated image.

This transformation is also used in object detection and image alignment.

Ass No 2

Mitra

{Class - Assignment on Linear Independence}

Page No. / Date / /

① $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \rightarrow R_2 \rightarrow R_2 - R_1, \; R_3 \rightarrow R_3 - R_1$

LI ⇓ Linearly Independent

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \rightarrow R_3 \rightarrow R_3 - R_2$

LD ⇓ Linearly Dependent

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \rightarrow \rho(A) = 3 \rightarrow 3$ LI cols

⇓

A is <u>LI</u>

② $A = \begin{bmatrix} 7 & -56 \\ -3 & 24 \\ 11 & -88 \\ -6 & 48 \end{bmatrix} \xrightarrow[\substack{R_2 \rightarrow R_2/3 \\ R_3 \rightarrow R_3/11 \\ R_4 \rightarrow R_4/6}]{R_1 \rightarrow R_1/7} \begin{bmatrix} 1 & -8 \\ -1 & 8 \\ 1 & -8 \\ -1 & 8 \end{bmatrix}$

$\downarrow R_2 \rightarrow R_2 + R_1, \; R_3 \rightarrow R_3 - R_1, \; R_4 \rightarrow R_4 + R_1$

$\rho(A) = 1 \leftarrow \begin{bmatrix} 1 & -8 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$

1 LI col, 1 LD col

⇓

$\boxed{LD}$

③ $A = \begin{bmatrix} -1 & 16 & -64 \\ 5 & 8 & 56 \\ 0 & -3 & 9 \end{bmatrix} \xrightarrow{R_2 \rightarrow R_2 + 5R_1} \begin{bmatrix} -1 & 16 & -64 \\ 0 & 88 & -264 \\ 0 & -3 & 9 \end{bmatrix}$

$\downarrow R_2 \rightarrow R_2/88$

$\rho(A) = 2 \leftarrow \begin{bmatrix} -1 & 16 & -64 \\ 0 & 1 & -3 \\ 0 & 0 & 0 \end{bmatrix} \xleftarrow{R_3 \rightarrow R_3 + 3R_2} \begin{bmatrix} -1 & 16 & -64 \\ 0 & 1 & -3 \\ 0 & -3 & 9 \end{bmatrix}$

↓

1 LD col

⇓

LD

Mitesh

DOMS Page No. Date / /

(4) $\begin{bmatrix} 1 & 1 & -1 & 0 \\ -1 & 1 & 1 & 1 \\ 1 & -1 & 1 & 0 \end{bmatrix} \longrightarrow$ Max Rank = 3

$\Downarrow$

no. of cols = 4

$\Downarrow$

LD

(5) $\begin{bmatrix} 2 & 1 & 3 \\ -4 & 9 & 5 \end{bmatrix} \longrightarrow$ Max Rank = 2

no. of col = 3

$\Downarrow$

LD

(6) $\begin{bmatrix} 3 & 5 & -6 & 2 \\ -2 & 0 & 1 & 0 \\ 0 & 0 & 6 & 0 \\ 4 & 1 & 1 & 3 \end{bmatrix} \xrightarrow[R_3 \to R_3/6]{R_1 \to R_1/3} \begin{bmatrix} 1 & 5/3 & -2 & 2/3 \\ -2 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

$\Downarrow$

Max Rank = 3 $\rightarrow$ no. of cols = 4

$\downarrow$

max 3 non zero rows

$\Downarrow$

LD

(7) $\begin{bmatrix} 3 & 2 & 8 & 5 \\ 4 & 0 & 2 & 5 \\ 7 & 3 & 3 & 6 \end{bmatrix} \longrightarrow$ Max Rank = 3

& no. of cols = 4

$\Downarrow$

LD

(8) $\begin{bmatrix} 6 & 0 & 12 \\ 0 & -1 & 3 \\ 3 & 2 & 0 \\ 1 & 7 & -19 \\ 4 & 0 & 8 \\ 2 & 5 & -11 \end{bmatrix} \xrightarrow[R_5 \to R_5/4]{R_1 \to R_1/6} \begin{bmatrix} 1 & 0 & 2 \\ 0 & -1 & 3 \\ 3 & 2 & 0 \\ 1 & 7 & -19 \\ 1 & 0 & 2 \\ 2 & 5 & -11 \end{bmatrix} \xrightarrow{R_3 \to R_3 - 3R_1}$

$R_4 \to R_4 - R_1$

$R_5 \to R_5 - R_1$

$R_6 \to R_6 - 2R_1$

$\begin{bmatrix} 1 & 0 & 2 \\ 0 & -1 & 3 \\ 0 & 2 & -6 \\ 0 & 7 & -21 \\ 0 & 0 & 0 \\ 0 & 5 & -15 \end{bmatrix}$

$\downarrow$ $R_3 \to R_3 + 2R_2$

$R_6 \to R_6 + 5R_2$

$R_4 \to R_4 + 7R_2$

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & -1 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$\rho(A) = 2$ & 3 cols
$\Downarrow$
2 LI cols & 1 LD col
$\Downarrow$
$\boxed{A \rightarrow \text{Linearly Dependent}}$

Assignment No (3) → Eigen Values & Vectors

(1)
$$A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$$

→ eqⁿ $\lambda^3 - (tr(A))\lambda^2 + (\text{sum of Minors})\lambda - |A| = 0$

$|A| = -2(0-12) - 2(0-6) - 1(-12+3)$
$= 24 + 12 + 9$
$= 45$

$\Rightarrow \lambda^3 + \lambda^2 + (-12-3-6)\lambda - 45 = 0$
$\Rightarrow \lambda^3 + \lambda^2 - 21\lambda - 45 = 0$
$\downarrow \lambda = -3$ is one root
$\Downarrow$

$(\lambda+3)(\lambda^2 - 2\lambda - 15) = 0$
$(\lambda+3)(\lambda+3)(\lambda-5) = 0$
$\Downarrow$
$\boxed{\lambda = 3, 3, 5}$

Eigen vector $[A - \lambda I][X] = [0]$

$$\begin{bmatrix} -2-\lambda & 2 & -3 \\ 2 & 1-\lambda & -6 \\ -1 & -2 & 0-\lambda \end{bmatrix}$$

Mitesh

13

For $\lambda = 3$

$[A - 3I][X] = [0]$

$$\begin{bmatrix} -5 & 2 & -3 \\ 2 & -2 & -6 \\ -1 & -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\downarrow R_1 \to R_1 / -5$

$$\begin{bmatrix} 1 & 2 & -3 \\ 2 & -2 & -6 \\ -1 & -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\downarrow R_3 \to R_3 + R_1$
$R_2 \to R_2 - 2R_1$

$$\begin{bmatrix} 1 & 2 & -3 \\ 0 & -6 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\hookrightarrow$ nullity = 1 $\Rightarrow$ 1 LI eigen vector

$x_2 = 0$

$x_1 + 2x_2 - x_3 = 0$

$\boxed{x_1 = x_3 = c}$

$$\Downarrow \quad c \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

For $\lambda = 5$

$[A - 5I][X] = [0]$

$$\begin{bmatrix} -7 & 2 & -3 \\ 2 & -4 & -6 \\ -1 & -2 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\downarrow R_1 \to R_1 / -7$

$$\begin{bmatrix} 1 & -2/7 & 3/7 \\ 2 & -4 & -6 \\ -1 & -2 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \longrightarrow \begin{matrix} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 + R_1 \end{matrix}$$

Mitesh

$$\begin{bmatrix} 1 & -2/7 & 3/7 \\ 0 & -24/7 & -48/7 \\ 0 & -33/7 & -38/7 \end{bmatrix} \begin{bmatrix} 1 & -2/7 & 3/7 \\ 0 & -24/7 & -48/7 \\ 0 & -16/7 & -38/7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$R_3 \to R_3 - 8R_2$

$\Downarrow$

~~$R_3 \to R_3 - 8R_1$~~

$$\begin{bmatrix} 1 & -2/7 & 3/7 \\ 0 & -24/7 & -48/7 \\ 0 & 0 & -62/7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \begin{bmatrix} 1 & -2/7 & 3/7 \\ 0 & 0 & -12 \\ 0 & 0 & -62/7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

nullity $= 0$

$\Downarrow$

no LI eigen vector

$-62/7\, x_3 = 0 \Rightarrow x_3 = 0$

$-\frac{24}{7} x_2 - \frac{48}{7} x_3 = 0 \Rightarrow x_2 = 0$

$x_1 - \frac{2}{7} x_2 + \frac{3}{7} x_3 = 0$

$\Downarrow$

$x_1 = 0$

eigenvector $= \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

(2) $\begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix} \xrightarrow{eq^n} \lambda^3 - 6\lambda^2 + (1+6+4)\lambda - 6 = 0$

$\lambda^3 - 6\lambda^2 + 11\lambda - 6 = 0$

$\Downarrow$

$\lambda = 1$ one root

$(\lambda-1)(\lambda-2)(\lambda-3)$

det

$4(1) + 2(0) - 2(0-1) = 6$

Mitesh

Eigen values → 1, 2, 3

Eigenvector for $\lambda = 1$

$$\begin{bmatrix} 4-\lambda & 0 & 1 \\ -2 & 1-\lambda & 0 \\ -2 & 0 & 1-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

$$\downarrow$$

$$\begin{bmatrix} 3 & 0 & 1 \\ -2 & 0 & 0 \\ -2 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\downarrow R_2 \to R_2 + \frac{2}{3} R_1, \quad R_3 \to R_3 + \frac{2}{3} R_1$$

$$\begin{bmatrix} 3 & 0 & 1 \\ 0 & 0 & 2/3 \\ 0 & 0 & 2/3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\downarrow R_3 \to R_3 - R_1$$

$$\begin{bmatrix} 3 & 0 & 1 \\ 0 & 0 & 2/3 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

nullity = 1 → 1 LI eigenvector

$x_3 = 0$

$3x_1 + x_3 = 0$

$x_1 = 0$

$x_2 = c$

$$\Rightarrow c \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

For $\lambda = 2$

$$\begin{bmatrix} 2 & 0 & 1 \\ -2 & -1 & 0 \\ -2 & 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\downarrow R_2 \to R_2 + R_1, \quad R_3 \to R_3 + R_1$$

$$\begin{bmatrix} 2 & 0 & 1 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

nullity $= 1 \to 1$ L.I eigen vector

$x_2 = x_3 = c$

$2x_1 + x_3 = 0$

$x_1 = -\frac{c}{2}$

$$\to c\begin{bmatrix} -1/2 \\ 1 \\ 1 \end{bmatrix}$$

for $\lambda = 3$

$$\begin{bmatrix} 1 & 0 & 1 \\ -2 & -2 & 0 \\ -2 & 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_2 \to R_2 + 2R_1$

$R_3 \to R_3 + 2R_1$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & -2 & 2 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

1 L.I eigen vector

$x_2 = x_3 = c$

$x_1 = -c$

$$\Rightarrow c\begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$$

Mitesh

DOMS Page No.

Date / /

(17)

(1) $\begin{bmatrix} 5 & 0 & 0 \\ \bullet & 0 & 0 \\ -1 & 0 & 3 \end{bmatrix}$ → as it is lower triangular

∴ eigen values → 5, 0, 3

for $\lambda = 5$ eigen vector

$[A - 5\lambda][X] = [0]$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & -5 & 0 \\ -1 & 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_1 \to R_3$

$$\begin{bmatrix} -1 & 0 & -2 \\ 0 & -5 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$x_2 = 0$

$-x_1 - 2x_3 = 0$ $\Rightarrow$ $c\begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}$

$x_3 = c$

$x_1 = -2c$

$\lambda = 0$

$[A - 0][x] = [0]$

→ $R_2 \leftrightarrow R_3$ $\xrightarrow[\text{step}]{\text{Next}}$ $R_1 \leftrightarrow R_2$

$$\begin{bmatrix} -1 & 0 & 3 \\ 5 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\downarrow R_2 \to R_2 + 5R_1$

$$\begin{bmatrix} -1 & 0 & 3 \\ 0 & 0 & 15 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$c\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ $\Longleftarrow$

nullity = 1

1 LI eigen vector

$x_3 = 0$

$-x_1 + 3x_3 = 0$

$x_1 = 0$

$x_2 = c$

 Mitesh

For $\lambda = 3$

$[A - 3I][X] = [0]$

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & -3 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\downarrow R_3 \to R_3 + \frac{R_1}{2}$

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$x_2 = 0$ $\quad$ $x_3 = c$ $\quad$ $c \in R$

$x_1 = 0$ $\quad \Downarrow \quad$ $c\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

(4) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 3 & 4 \\ 0 & 0 & -2 \end{bmatrix}$ $\to$ upper triangular

$\therefore$ eigen value $\to 0, 3, -2$

eigen vector for $\lambda = 0$

$R_2 \leftrightarrow R_1 \xrightarrow{\text{Nextstep}} R_2 \leftrightarrow R_3$ $\begin{bmatrix} 0 & 3 & 4 \\ 0 & 0 & -2 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$\hookrightarrow x_3 = 0$

$3x_2 + 4x_3 = 0$

$x_2 = 0$

$x_1 = c$

$c\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

For $\lambda = 3$

$[A - 3I] = 0$

$$\begin{bmatrix} -3 & 0 & 0 \\ 0 & 0 & 4 \\ 0 & 0 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\hookrightarrow R_3 \to R_3 + \frac{5}{4} R_2$

Math

$$\begin{bmatrix} -3 & 0 & 0 \\ 0 & 0 & 4 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\Downarrow$

$x_1 = x_3 = 0 \quad , x_2 = c$

$\Downarrow$

$$c \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

For $\lambda = -2$

$$\begin{bmatrix} +2 & 0 & 0 \\ 0 & 5 & 4 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$x_1 = 0$

$5x_2 + 4x_3 = 0$

$x_3 = c$ CER

$x_2 = -\frac{4}{5} c$

$\Downarrow$

$$c \begin{bmatrix} 0 \\ -4/5 \\ 1 \end{bmatrix}$$

(5) As all rows are identical $\rightarrow$ det $= 0$

$\Downarrow$

Product of eigen values $= 0$

$\Downarrow$

One of eigen values $= 0$

Here all 3 rows are identical

$\Downarrow$

2 eigen values will be 0

+ as sum of eigen values = Trace

$\therefore$ 3rd eigen value $= 1+2+3$

$= 6$

Consistency Test Assignment → (4)

Mitesh

(a) Test for consistency

(i) $[A\cancel{x}]\cancel{[B]}=[B]$

$$Ax=B$$

Check $[A:B]$ →
$$\begin{bmatrix} 2 & -3 & 7 & 5 \\ 3 & 1 & -3 & 13 \\ 2 & 19 & -47 & 32 \end{bmatrix}$$

$$\Big\downarrow \begin{array}{l} R_2 \to R_2 - \frac{3}{2}R_1 \\ R_3 \to R_3 - R_1 \end{array}$$

$$\begin{bmatrix} 2 & -3 & 7 & : & 5 \\ 0 & 11/2 & -27/2 & : & 11/2 \\ 0 & 22 & -54 & & 27 \end{bmatrix}$$

$$\Big\downarrow R_3 \to R_3 - 4R_2$$

$$\begin{bmatrix} 2 & -3 & 7 & : & 5 \\ 0 & 11/2 & -27/2 & : & 11/2 \\ 0 & 0 & 0 & & 5 \end{bmatrix}$$

↳ as $\rho(A) \neq \rho(A:B)$

⇓ ↓

Inconsistent ⇐ No sol^n

(ii)
$$\begin{bmatrix} 2 & -1 & 3 & : & 8 \\ -1 & 2 & 1 & : & 4 \\ 3 & 1 & -4 & : & 0 \end{bmatrix} \xrightarrow[R_3 \to R_3 - \frac{3}{2}R_1]{R_2 \to R_2 + \frac{R_1}{2}} \begin{bmatrix} 2 & -1 & 3 & : & 8 \\ 0 & 3/2 & 5/2 & : & 8 \\ 0 & 5/2 & -17/2 & : & -12 \end{bmatrix}$$

$$\Big\downarrow R_3 \to R_3 - \frac{5}{3}x_2$$

$$\begin{bmatrix} 2 & -1 & 3 & : & 8 \\ 0 & 3/2 & 5/2 & : & 8 \\ 0 & 0 & -\frac{38}{3} & : & -\frac{76}{3} \end{bmatrix}$$

$\rho(A) = \rho(A:B)$ ⟸

⇓

unique sol^n exists

$$-\frac{38}{3}x_3 = -\frac{76}{3}$$

$x_3 = 2$

$x_2 = 1$

$x_1 = 2$

Mitesh

(iii) $\left[\begin{array}{ccc|c} 4 & -1 & 0 & 12 \\ -1 & 5 & -2 & 0 \\ -2 & 0 & 4 & -8 \end{array}\right] \xrightarrow[R_3 \to R_3/-2]{R_1 \to R_1/4} \left[\begin{array}{ccc|c} 1 & -1/4 & 0 & 3 \\ 1 & -5 & 2 & 0 \\ 1 & 0 & -2 & 4 \end{array}\right]$

$R_2 \to R_2(-1)$

$\downarrow R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$

$\left[\begin{array}{ccc|c} 1 & -1/4 & 0 & 3 \\ 0 & -19/4 & 2 & -3 \\ 0 & 0 & \frac{-36}{19} & \frac{16}{19} \end{array}\right] \xleftarrow{R_3 \to R_3 + \frac{R_2}{19}} \left[\begin{array}{ccc|c} 1 & -1/4 & 0 & 3 \\ 0 & \frac{-19}{4} & 2 & -3 \\ 0 & 1/4 & -2 & 1 \end{array}\right]$

$\therefore -36x_3 = 16$

$x_3 = \frac{-4}{9}$ $x_2 = \frac{4}{9}$ $x_1 = \frac{28}{9}$

(b) $A:B = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 10 \\ 1 & 2 & \lambda & \mu \end{array}\right] \xrightarrow[R_3 \to R_3 - R_1]{R_2 \to R_2 - R_1} \left[\begin{array}{cccc} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 1 & \lambda-1 & \mu-6 \end{array}\right]$

$\downarrow R_3 \to R_3 - R_2$

$\left[\begin{array}{cccc} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & \lambda-3 & \mu-10 \end{array}\right]$

(i) for no sol^n

$\rho(A) \neq \rho(A:B)$

Case: $\lambda - 3 = 0 \Rightarrow \lambda = 3$

$\mu - 10 \neq 0$ $\quad \mu \neq 10$

(ii) for unique sol^n

$\rho(A) = \rho(A:B) = n = 3$

$\lambda - 3 \neq 0 \Rightarrow \lambda \neq 3$

$\mu - 10 \neq 0$ $\quad \mu \neq 10$

(iii) for ∞ sol^n

$\rho(A) = \rho(A:B) < n = 3$

$\lambda - 3 = 0$

$\mu - 10 = 0$

$\lambda = 3$

$\mu = 10$

DOMS Page No. Date / /

Mi'tesh

(c) $\left[\begin{array}{ccc:c} 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & \lambda \\ 1 & 4 & 10 & \lambda^2 \end{array}\right] \xrightarrow[R_3 \to R_3 - R_1]{R_2 \to R_2 - R_1} \left[\begin{array}{ccc:c} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & \lambda - 1 \\ 0 & 3 & 9 & \lambda^2 - 1 \end{array}\right]$

$\downarrow R_3 \to R_3 - 3R_2$

$\left[\begin{array}{ccc:c} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & \lambda - 1 \\ 0 & 0 & 0 & \lambda^2 - 3\lambda + 2 \end{array}\right]$

If $\lambda^2 - 3\lambda + 2 = 0$

$\Downarrow$

If $\lambda = 1, 2 \rightarrow \rho(A) = \rho(A:B) < n$

$\Downarrow$

It will have ∞ soln

For $\lambda = 1$

$\left[\begin{array}{ccc:c} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

$x_2 + 3x_3 = 0$

$x_3 = k$

$x_2 = -3k$

$x_1 + x_2 + x_3 = 1$

$x_1 - 3k + k = 1$

$x_1 = 1 + 2k$

$\Downarrow$

soln $(1 + 2k, -3k, k)$

$k \in R$

For $\lambda = 2$

$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$\hookrightarrow x_2 + 3x_3 = 1$

$\hookrightarrow x_3 = k$

$x_2 = 1 - 3k$

$x_1 + x_2 + x_3 = 1$

$x_1 + 1 - 3k + k = 1$

$x_1 = 2k$

soln $\longrightarrow (2k, 1 - 3k, k)$ $k \in R$

Mitesh

(d) $\left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 3 & -1 & -11 & 0 \\ -2 & 4 & 14 & 0 \end{array}\right]$ homo eqn $\longrightarrow$ ∴ consistent

+ check rank of A

$R_2 \to R_2 - 3R_1$

~~$R_3 \to R_3 + 2R_1$~~

$\left[\begin{array}{ccc|c} 1 & 3 & -2 & 0 \\ 2 & -1 & 4 & 0 \\ 1 & -11 & 14 & 0 \end{array}\right]$

$\downarrow R_2 \to R_2 - 2R_1$
$R_3 \to R_3 - R_1$

$\left[\begin{array}{ccc|c} 1 & 3 & -2 & 0 \\ 0 & -7 & 8 & 0 \\ 0 & -14 & 16 & 0 \end{array}\right]$

$\downarrow R_3 \to R_3 - 2R_2$

$\left[\begin{array}{ccc|c} 1 & 3 & -2 & 0 \\ 0 & -7 & 8 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

As $\rho(A) = \rho(A:B) < n \Rightarrow \infty$ soln

$-7x_2 + 8x_3 = 0$

$x_3 = k$

$x_2 = +\frac{8k}{7}$

$x_1 + 3x_2 - 2x_3 = 0$

$x_1 + \frac{24k}{7} - 2k = 0$

$x_1 = -\frac{10k}{7}$ $\quad k \in R$

Mitesh

(c) $A = \begin{bmatrix} 3 & 1 & -\lambda \\ 4 & -2 & -3 \\ 2\lambda & 4 & \lambda \end{bmatrix}$ → homo eqⁿ

$R_1 \to \frac{R_1}{3}$

$$\begin{bmatrix} 1 & 1/3 & -\lambda/3 \\ 4 & -2 & -3 \\ 2\lambda & 4 & \lambda \end{bmatrix} \xrightarrow[R_3 \to R_3 - 2\lambda R_1]{R_2 \to R_2 - 4R_1} \begin{bmatrix} 1 & 1/3 & -\lambda/3 \\ 0 & -10/3 & -\frac{9-4\lambda}{3} \\ 0 & 4-\frac{2\lambda}{3} & \frac{\lambda+2\lambda^2}{3} \end{bmatrix}$$

$\Big\downarrow R_3 \to R_3 + \left(\frac{-6+\lambda}{5}\right) R_2$

$$\begin{bmatrix} 1 & 1/3 & -\lambda/3 \\ 0 & -10/3 & -\frac{9-4\lambda}{3} \\ 0 & 0 & \frac{6\lambda^2+20\lambda+54}{15} \end{bmatrix}$$

⇓

for non-trivial solⁿ

$6\lambda^2 + 20\lambda + 54 = 0$

⇓

$3\lambda^2 + 10\lambda + 27 = 0$

⇓

$D = \sqrt{9 - 4(3)(27)}$

⇓

$D < 0$

⇓

~~No real~~

No real solⁿ of λ exists for which the given eqⁿ has non-trivial solⁿ