CSCE 222 [505] Discrete Structures for Computing
Fall 2015 – Philip C. Ritchey

Problem Set 7

Due dates: Electronic submission of LaTeX and PDF files of this homework is due on **30 October 2015 (Friday) before 11:30 a.m.** on eCampus (`http://ecampus.tamu.edu`).

Name	Problems
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10-24-15	

Resources. http://www.inf.ed.ac.uk/teaching/courses/dmmr/slides/13-14/Ch2.pdf
http://web.ift.uib.no/Teori/KURS/WRK/TeX/symALL.html
http://math.stackexchange.com/questions/1496817/relation-in-diagram-is-it-reflexive-symmetric-transitive-and-antisymmetric
http://www.math.cornell.edu/ levine/18.312/alg-comb-lecture-7.pdf

Problem 1. (15 points) Section 9.1, Exercise 44 (a,c,d,f)

Solution. :
16 different relations on set {0,1}
AxA = {(0,0),(0,1),(1,0),(1,1)}

$R_1 = \emptyset$
R_2 = {(0,0)}
R_3 = {(0,1)}
R_4 = {(1,0)}
R_5 = {(1,1)}
R_6 = {(0,0),(0,1)}
R_7 = {(0,0),(1,0)}
R_8 = {(0,0),(1,1)}
R_9 = {(0,1),(1,0)}
R_{10} = {(0,1),(1,1)}
R_{11} = {(1,0),(1,1)}
R_{12} = {(0,0),(0,1),(1,0)}
R_{13} = {(0,0),(0,1),(1,1)}
R_{14} = {(0,0),(1,0),(1,1)}
R_{15} = {(0,1),(1,0),(1,1)}
R_{16} = {(0,0),(0,1),(1,0),(1,1)}

A) $R_8, R_{13}, R_{14}, R_{16}$, because contain all pairs in form of (a,a)

C)$R_1, R_2, R_5, R_8, R_9, R_{12}, R_{15}, R_{16}$ symmetric

D) R_1 through R_{14} anti symmetric because no pair of elements a and b with a != b such that both (a,b) and (b,a) belong to the relation

F) R_1 through R_{14} and R_{16} are transitive by showing if (a,b) and (b,c) belong to the relation then (a,c) does.

Problem 2. (15 points) Section 9, Supplementary Exercise 2

Solution. :

A) R = {(a,a),(b,b),(c,c),(d,d) , (a,b)(b,a)(c,b)(b,c)}
reflexive since our pairs include all ordered pairs such that (x,x) $\in$ R
Symmetric since (b,a) $\in$ R whenever (a,b) $\in$ R and (c,d) $\in$ R whenever (b,c) $\in$ R
Not Transitive since (a,b) $\in$ R and (b,c) $\in$ R, but (a,c) $\notin$ R

B) R = $\emptyset$ since nothing in this set, nothing is related.

C) R = {(a,b),(b,c)}
irreflexive because (a,a) $\in$ R
anti symmetric since (b,a) $\in$ R and False implying True is always True therefore antisymmetric
Not transitive since (a,c) $\notin$ R

D) R = {(a,a),(b,b),(c,c),(d,d),(a,b),(b,a),(c,a),(b,c)}
reflexive since all x (x,x) $\in$ R
not symmetric since (b,c) $\in$ R but not (c,b)
not antisymmetric since a != b
transitive since (a,b) $\in$ R and (b,c) $\in$ R then (a,c) $\in$ R is true

E) R = (a,c),(b,a),(c,c),(a,c)
(a,a) $\in$ R so not reflexive and (c,c) $\in$ R so irreflexive
Not symmetric since (a,c) $\in$ R but (c,a) $\notin$ R. Not anti symmetric because a != b
Not transitive since (b,a) $\in$ R and (a,c) $\in$ R then (b,c) $\notin$ R

Problem 3. (15 points) Section 9, Supplementary Exercise 8

Solution. :
Yes, by proof by contradiction,
let A = $\{(a,b)\}$
Constructing a symmetric relation: R = $\{(b,a),(a,b)\}$

Let (c,d) $\in \tilde{R}$ then (c,d) $\notin$ R
if (d,c) $\in$ R it would be contradiction and not symmetric
so (d,c) $\in \tilde{R}$
therefore, (d,c) $\in \tilde{R}$
finally: $\tilde{R} = \{(c,d),(d,c)\}$ which is symmetric

Problem 4. (15 points) Section 9.3, Exercise 34

Solution. :
Relation R on a set A can be a graph that has values of A as the vertices and ordered pairs (a,b) as edges

Since (a,b) $\in$ R iff (a,b) $\notin$ $\tilde{\text{R}}$ when there is an edge from a to b in graph of R, then the edge isnt drawn in $\tilde{\text{R}}$

So: digraph of $\tilde{\text{R}}$ is made by edges that aren't in digraph of R

Problem 5. (15 points) Section 9, Supplementary Exercise 20

Solution. :
A) x can have same zodiac sign with x so (x,x) $\in$ R - reflexive
x and y can have same zodiac sign and therefore y and x would have same sign so (x,y) $\in$ R $\implies$ (y,x) $\in$ R is true. So it is symmetric.
(x,y) $\in$ R, and (y,x) $\in$ R, x and y have same sign so y and z have same sign. therefore (x,z) $\in$ R so it is transitive. So it is a equivalence relation.

B) x can have same birth year as x so (x,x) $\in$ R - reflexive
x and y can have same birth year and therefore y and x would have same birth year so (x,y) $\in$ R $\implies$ (y,x) $\in$ R is true. So it is symmetric.
(x,y) $\in$ R, and (y,x) $\in$ R, x and y have same birth year, then y and z have same birth year, therefore (x,z) $\in$ R so it is transitive. So it is a equivalence relation.

C) x and x have been in same city so (x,x) $\in$ R is reflexive
x and y have been in same city so y and x have been in same city therefore (x,y) $\in$ R $\implies$ (y,x) $\in$ R is true so symmetric.
(x,y) $\in$ R, and (y,x) $\in$ R, x and y have been in same city, but it is not valid that x and z have been in same city, therefore (x,z) $\notin$ R so it is NOT transitive. So it is NOT a equivalence relation.

Problem 6. (15 points) Section 9.6, Exercise 6

Solution. :
A) a = a for every real number, so it is reflexive
(a=a $\wedge$ b=a) $\implies$ (a=b) is true so anti symmetric
(a=b $\wedge$ b=c) $\implies$ (a=c) is true so transitive
therefore it (R, =) is poset.

B) a $<$ a - NO this is not true, so it is not reflexive so just by checking this case we can conclude (R, $<$) not a poset.

C) a$\leq$a - yes for every a it will be = so true so reflexive
(a $\leq$ b $\wedge$ b $\leq$ a) $\implies$ (a=b) is true so anti-symmetric
(a $\leq$ b $\wedge$ b $\leq$c) $\implies$ (a$\leq$c) is true so transitive,
therefore (R,$\leq$) is poset.

D) a =a for all real number so it is not reflexive
therefore (R,$\neq$) is not a poset because a=a for all a.

Problem 7. (10 points) Section 9.2, Exercise 6

Solution. :
Professor and course number.
Professor and time.

This is because since no two professors with same name have same course number or time assessment.

Wildcard Quiz Problems (the quiz on Friday could also be one of these)
Section 9.1, Exercise 2
Section 9.2, Exercise 2
Section 9.3, Exercise 18/20/22
Section 9.5, Exercise 2
Section 9.6, Exercise 6

Aggie Honor Statement: On my honor, as an Aggie, I have neither given nor received any unauthorized aid on any portion of the academic work included in this assignment. Furthermore, I have disclosed all resources (people, books, web sites, etc.) that have been used to prepare this homework.

Checklist:

1. Did you type your full name and that of all collaborators?
2. Did you abide by the Aggie Honor Code?
3. Did you solve all problems and start a new page for each?
4. Did you submit
   (a) your LaTeX source file?
   (b) your PDF file?