CSCE 222 [505] Discrete Structures for Computing
Fall 2015 – Philip C. Ritchey

Problem Set 3

Due dates: Electronic submission of LaTeX and PDF files of this homework is due on **23 September 2015 (Wednesday) before 11:30 a.m.** on eCampus (`http://ecampus.tamu.edu`).

Name	Problems
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9-22-15	

Resources. Discrete Structures and its Applications - Rosen Chapter 3.
https://rob-bell.net/2009/06/a-beginners-guide-to-big-o-notation/
http://www.trans4mind.com/personal-development/mathematics/series/sumNaturalSquares.htm
http://www.ee.ryerson.ca/ courses/coe428/sorting/insertionsort.html
http://codeyarns.com/2012/06/29/inserting-an-algorithm-in-latex/

Problem 1. (10 points) Section 3.1, Exercise 4

Solution. :

Algorithm 1 Find largest difference in a list of integers

procedure DIFF($x_1....x_n : integers$)
    $large := x_1 - x_2$
    for i := 2 to n-1 **do**
        if $large < x_{i+1} - x_i$ **then** large := $x_{i+1} - x_i$
        end if
    end for
    Return *large*
end procedure

Problem 2. (10 points) Supplementary Exercise 2

Solution. :
A)

Algorithm 2 Find first and second largest element in list of integers

```
procedure FIND TWO(x_1....x_n : integers)
    first := x_1
    for  i := 2 to n  do
        if first < x_i  then first := x_i
        end if
    end for
    second := x_1
    for  i := 2 to n  do
        if second < x_i and second ¡ first then second := x_i
        end if
    end for
    Return first
    Return second
end procedure
```

B)
O(n) for both for loops so number of comparisons is O(n).

Problem 3. (10 points) Section 3.1, Exercise 38

Solution. :
Insertion Sort
Original list: 6, 2, 3 , 1 , 5 , 4
Checks: $6 > 2$
After first check: 2, 6, 3 , 1 , 5 , 4
Checks: $3 > 2, 3 > 6$
After second check: 2, 3, 6 , 1 , 5 , 4
Checks: $1 > 2, 1 < 3, 1 < 6$
After third check: 1, 2, 3 , 6 , 5 , 4
Checks: $5 > 1, 5 > 2, 5 > 3, 5 < 6$
After fourth check: 1, 2, 3 , 5 , 6 , 4
Checks: $4 > 1, 4 > 2, 4 > 3, 4 < 5, 4 < 6$
After fifth check: 1, 2, 3 , 4 , 5 , 6

List is now sorted after the fifth check.

Problem 4. (10 points) Section 3.1, Exercise 56

Solution. :
We can use proof by counterexample:
Our example could be : 20 cents.
We first start with the normal way of counting change.
20 cents is two dimes which is two coins.
Next, we use the 12 cent method to see how many coins it takes to satisfy 20 cents.
Using the 12 cent method, we get 1 12 cent, 1 nickel, and 3 pennies which is 5 coins.

Therefore, it does not produce change using the fewest coins possible by our proof by counterexample.

Problem 5. (10 points) Supplementary Exercise 18

Solution. :
$\Sigma_{j=1}^{n}$ j(j+1)
$= \Sigma_{j=1}^{n}$ j^2 + $\Sigma_{j=1}^{n}$ j using: $\Sigma_{j=1}^{n}$ j = $\frac{n(n+1)}{2}$
and $\Sigma_{j=1}^{n}$ j^2 = $\frac{n(n+1)(2n+1)}{6}$

$= \frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}$
$= \frac{n(n+1)}{2} \left(\frac{2n+1}{3} + 1\right)$ by factoring
$= \frac{n(n+1)(n+2)}{3}$
So: f(n) = $\frac{n(n+1)(n+2)}{3} \leq 2n^3$
We know c = 2 and k =1
Therefore the Big O notation would be O(n^3)

Problem 6. (10 points) Supplementary Exercise 26

Solution. :
For this problem we need to know that n grows faster then log(n) and that log(log(n)) < log(n). We can see this by graphing these functions on a graphing calculator.
Looking at the growth rate of the functions given to order. We only care about the exponents so:
$n^{log(n)}, nlog(n)loglog(n), n(log(n))^{3/2}, n^{4/3}log(n)^2, n^{3/2}, 2^{100n}, 2^{n^2}, 2^{2n}, 2^{n!}$

Problem 7. (10 points) Section 3.2, Exercise 30

Solution. :
A) 3x+7 = O(x) b/c polynomial degree is 1
x $\leq$ 3x
x$\leq$ 3x+7
c = 3 , k=1
O(3x+7)
3x+7 and x are same order

B) $2x^2 + x - 7$ = O(x^2) b/c polynomial degree is 2
$x^2 < 2x^2$
$x^2 < 2x^2 + x - 7$
c=2 k=7
O($2x^2 + x - 7$)
$2x^2 + x - 7$ and x^2 are same order

C) $[x + \frac{1}{2}]$= O(x)
$x \leq [x + \frac{1}{2}]$
c=1 k =1
O($[x + \frac{1}{2}]$)
$[x + \frac{1}{2}]$ and x are same order

D)$log(x^2 + 1) \leq log(2x^2)$
$log(2x^2)$ = 2log(x)
c = 2 , k=1
= O(log(x))
log(x) $\leq$ $log(x^2 + 1)$
c=1 k=1
O($log(x^2 + 1)$)
$log(x^2 + 1)$ and log(x) are same order

E)$log_{10}(x) \leq$ log(x)
O(log(x))
log(x) $\leq$ $log_{10}(x)$
O($log_{10}(x)$)
$log_{10}(x)$ and log(x) are same order

Problem 8. (10 points) Supplementary Exercise 30

Solution. :
A)

Algorithm 3 sorts integers and checks for each pair of terms whether their difference is in the sequence

procedure CHECK($x_1....x_n : integers$)
    Sort ($x_1....x_n$) from high to low
    for i := 1 to n-1 **do** sub := $x_{i+1} - x_i$
        for k:= i+1 to n **do**
            if x_k := sub **then** Return true
            end if
        end for
    end for
end procedure

B) O(n^2) because nested for loops, and this algorithm is better than the brute force since it more efficient.

Problem 9. (10 points) Section 3.3, Exercise 16

Solution. :
86400 seconds in one day which is approximately 10^5
so: $10^5/10^{-11} = 10^{16}$
A) log(n) $\leq 10^{16}$
n $\leq 2^{10^{16}}$
$2^{10^{16}}$

B) 1000n $\leq 10^{16}$
n $\leq 10^{13}$
10^{13}

C) $n^2 \leq 10^{16}$
n $\leq 10^8$
10^8

D) $1000n^2 \leq 10^{16}$
$n^2 \leq 10^{13}$
$n \leq 10^{13/2}$
$10^{13/2}$

E) $n^3 \leq 10^{16}$
$10^{16/3}$

F) $2^n \leq 10^{16}$
n $\leq log 10^{16}$
$log 10^{16}$

G) $2^{2n} \leq 10^{16}$
n $\leq \frac{log 10^{16}}{2}$
$\frac{log 10^{16}}{2}$

H) $2^{2^n} \leq 10^{16}$
$2^n \leq log 10^{16}$
log($log 10^{16}$)

Problem 10. (10 points) Section 3.3, Exercise 46

Solution. :
A)
To use a brute-force algorithm we need to find a match for the first character of target then check each character afterwards. Using a while and a for loop to check for matching character of target and to check all the characters afterwards can help.

B)

Algorithm 4 Finding match for first character and checking successive characters for a match

procedure MATCH(x: target , y: text)
  i := 1
  while $x_1 = y_i$ AND $i \leq n$ **do** TorF := 1
    for k := 1 to m **do**
      if x_k != y_{i+k} **then** TorF := 0
      end if
    end fori := i+1
  end while
end procedure

C) Since outer loop runs n times and inner loop runs m times the big O is O(nm).

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Checklist:
1. Did you type your full name and UIN and those of any collaborators?
2. Did you abide by the Aggie Honor Code?
3. Did you solve all problems and start a new page for each?
4. Did you submit
   (a) your LaTeX source file?
   (b) your PDF file?