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CSCE 222
MIDTERM III CORRECTIONS
Fall 2015

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Mitesh Patel

[10] **PROBLEM 1**

Definition. Expressions using operators + and $*$ with numbers and letters as operands.
Basis Step: Any number or letter is an expression.
Recursive Step: If E and F are expressions, then so are $(E + F)$ and $(E \times F)$.

Prove that every expression has an equal number of left and right parentheses.

E_l - # left parenthesis
E_r - # right parenthesis
$\forall_E : E_l = E_r$

Proof by structural induction:
Basis: E is number or letter
$E_l = 0$
$E_r = 0$
so $E_l = E_r$ therefore our basis step checks out.

Inductive step : Let E,F be expressions.
Assume: $E_l = E_r \wedge F_l = F_r$ E has same left and right parenthesis. (IH)

Show: G = (E+F) $G_l = G_r$
K = (E x F) $K_l = K_r$

$G_l = E_l + F_l + 1 = E_r + F_r + 1 = G_r$
$K_l = E_l + F_l + 1 = E_r + F_r + 1 = K_r$

Therefore $\forall_E : E_l = E_r$ by structural induction.

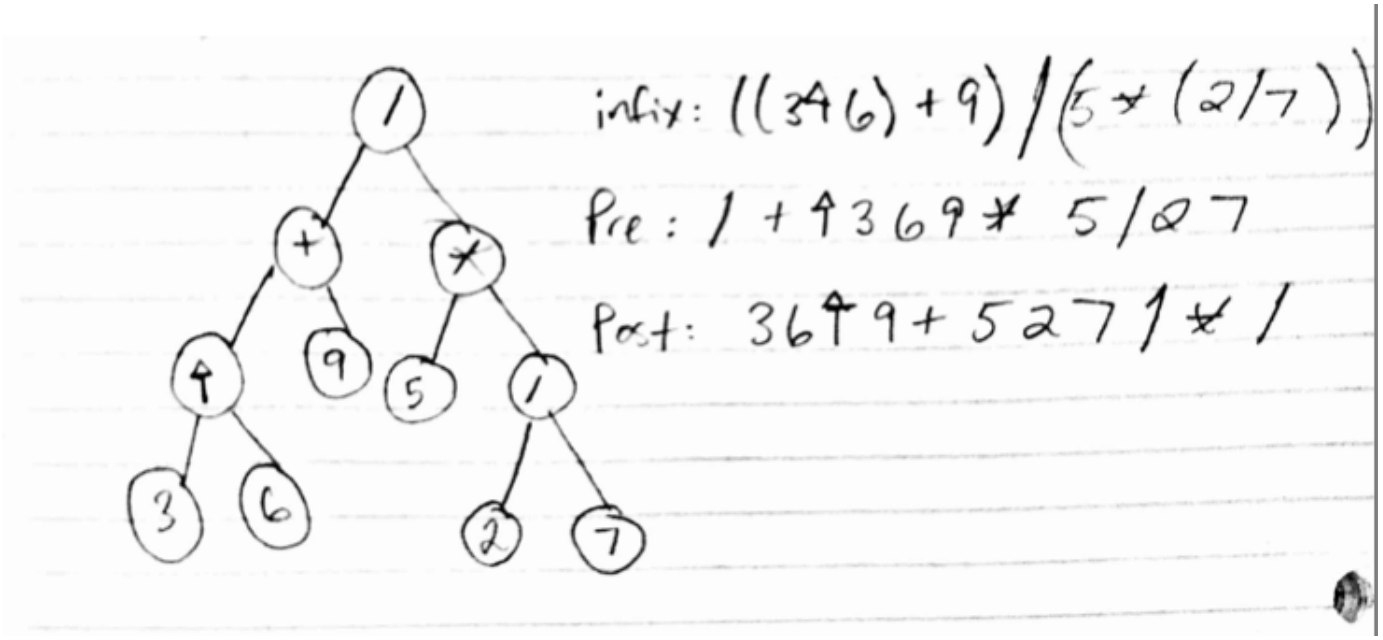
[10] **PROBLEM 2**

1. Find the number of vertices in an undirected graph with 54 edges, where every vertex has degree 3.
   Let:
   E = m - edges
   V = n - vertices

   m = 54
   Using handshaking theorem:

   $2m = \Sigma_{v=V} \deg(v)$
   $2(54) = \Sigma_{v=V} 3$
   $2(54) = 3n$
   $108 = 3n$
   $n = 108 / 3$
   $n = 36$

2. Draw a binary tree and give the prefix, infix, and postfix notations for $\frac{3^6 + 9}{5 \times \frac{2}{7}}$.



[10] **PROBLEM 3**

The zombie apocalypse starts at Texas A&M when a CSCE 222 exam goes horribly wrong and a single student is infected with a mutant strain of the Solanum virus. Anyone infected by the virus immediately begins seeking out, attacking, and eating the brains of the uninfected, who now become infected. Zombies who have consumed at least 1 brain are invulnerable while they are still hungry. However, after consuming 4 brains, a zombie will become full and begin to wander aimlessly. The survivors, armed with various implements of destruction and determined to put an end to the apocalypse, manage to find and eliminate all the zombies. In the end, 6046 zombies were found to have <u>not</u> eaten any brains.

1. How many zombies were able to eat the brains of the living?
   L = 6046 and m = 4
   Using: (1)n = mi + 1 and (2)n = i + L
   Substituting i+L into the first equation we get:
   i + L = mi + 1
   i + 6046 = 4i + 1
   3i = 6045
   i = 2015 zombies able to eat the brains of the living

2. How many people, in total, were zombified?

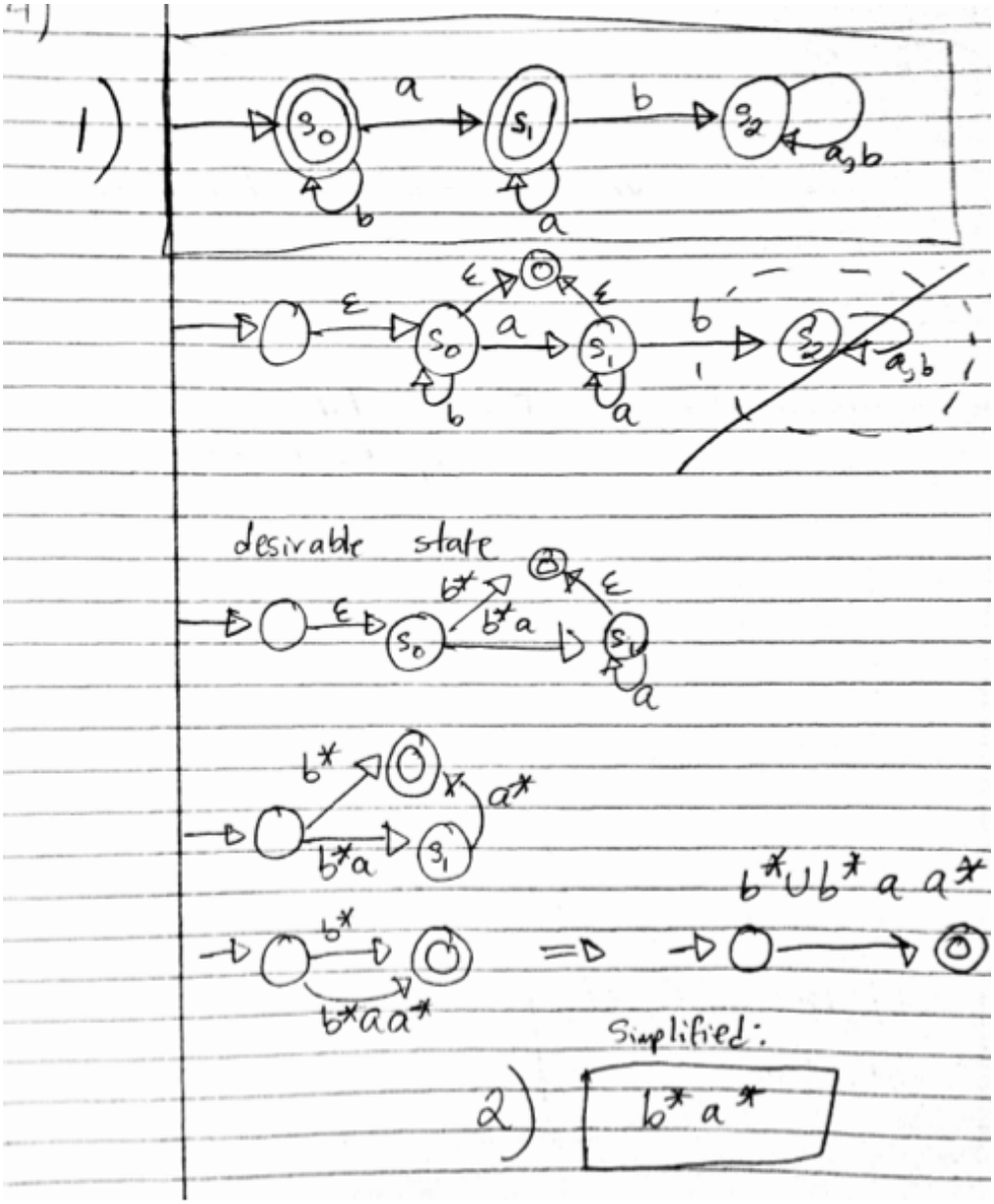
   We need to find n so using the second equation just plug in the i from above so:
   n = 2015 + 6046
   n = 8061 people in total were zombified.

3. A zombie generation consists of all zombies who are equidistant in ancestry from patient zero. For example, the 1st generation of zombies are those who were infected by patient zero, the 2nd generation are those who were infected by a member of the 1st generation, and so on. The last generation of zombies is generation Z. What is the least possible value for Z in the outbreak above?

   Asking for height of tree:
   Z $\geq \lceil log_m(L) \rceil$
   Z = $\lceil log_m(L) \rceil = \lceil log_4(6046) \rceil = 7$

[10] **PROBLEM 4**

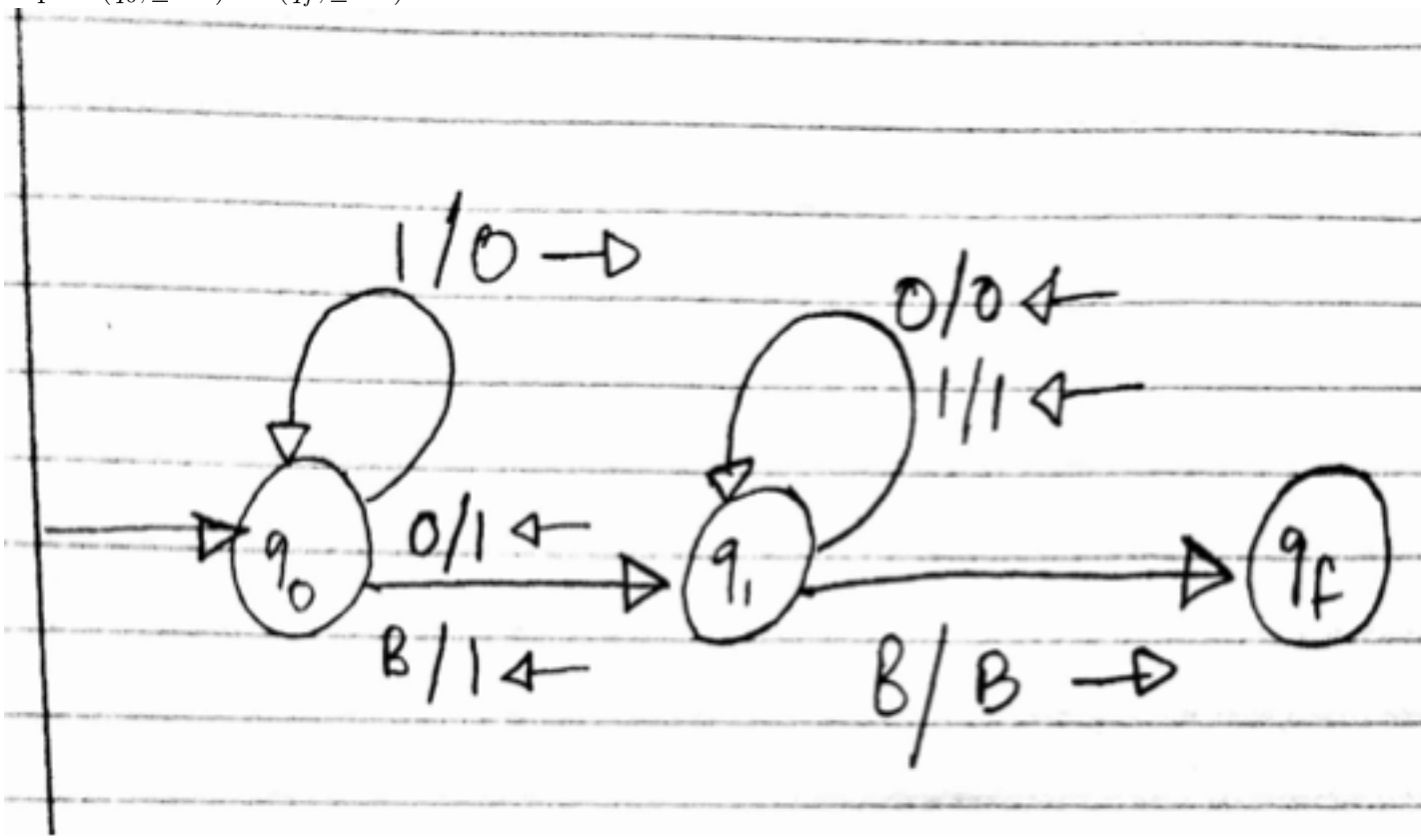
1. Construct a DFA that recognizes the language $L = \{w \mid w \in \{a, b\}^*$ **does not** contain $ab\}$.
2. Convert your DFA from Part 1 to a Regular Expression.



[10] **PROBLEM 5**

Design a Turing Machine that takes as input an integer $N \geq 0$ and adds 1 to it. The tape initially contains N in binary with blank symbols (B) on either side extending out to infinity. The machine is initially scanning the first bit of the input (for simplicity, the first bit is the least significant bit, the bits to its right increasing in significance). Your TM should halt in state q_f with $N+1$ in binary on its tape and the tape head scanning the leftmost non-blank symbol on the tape.

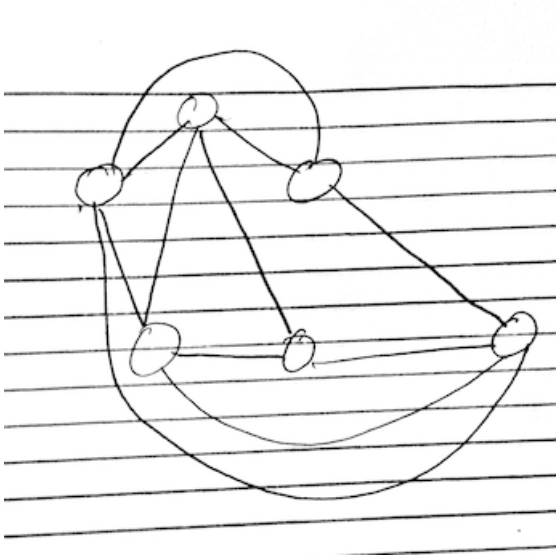
Example: $(q_0, \underline{1}011) \vdash^* (q_f, \underline{0}111)$



[5] **BONUS**

Note: a simple graph ***does not*** *have loops or multiple edges.*

[2 pts] Draw a simple planar graph that has degree sequence: 4,4,4,4,4,4.



[3 pts] Draw a simple planar graph that has degree sequence: 5,5,5,5,5,5,5,5,5,5,5,5.