CSCE 222 [505] Discrete Structures for Computing

Fall 2015 – Philip C. Ritchey

Name	**Problems**
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Resources. Discrete Mathematics and Its Application by Rosen (Chapter 2). http://math.stackexchange.com/questions/837318/if-a-function-has-a-inverse-that-is-well-defined-is-it-a-bijection. sharelatex.com and piazza.com

Problem 1. (10 points) Supplementary Exercise 4

Solution. :
A) E $\cup$ O = set that contains either even or odd integers = set Z.
B) E $\cap$ O = set containing both E and O = EMPTY set since even and odd are total opposites.
C) Z - E = set of all integers excluding even integers = set O.
D) Z - O = set of all integers excluding odd integers = set E.

Problem 2. (10 points) Supplementary Exercise 6

Solution. :
Given : $A \cap B = A$ which means that every element in set A also belongs in set B, and by definition of subset ($A \subseteq B$) which means that every element of A is also an element of B.
So: the quantification $\forall x(x \in A \to x \in B)$ is True.

Problem 3. (10 points) Section 2.2, Exercise 46

Solution. :

Using : $| A \cup B | = |A| + |B| - | A \cup B |$ (principle of inclusion exclusion)

$| A \cup B \cup C | = | A \cup (B \cup C) |$ ———— (associative law)
$= |A| + |B \cup C| - | A \cap (B \cup C) |$ —— (principle of inclusion exclusion)
$= |A| + |B| + |C| - | B \cup C | - | (A \cap B) \cup (A \cap C) |$ ———(distribution law)
$= |A| + |B| + |C| - | B \cup C | - | A \cap B | - | A \cap C | + | A \cap B \cap C |$
$= | A \cup B \cup C |$

Problem 4. (10 points) Supplementary Exercise 14

Solution. :
f(x) = x
x $\in$ S and f(x) $\in$ S
so x,y $\in$ S
if x=y then,
f(x)=f(y) therefore f is a one to one function, and
| f(S) | $\leq$ | S | is TRUE for all subsets of A.

Problem 5. (10 points) Supplementary Exercise 22

Solution. :
n $\in \mathbb{Z}$

n=2y for some integer y

$$[\frac{n}{2}]\,[\frac{n}{2}] = [\frac{2y}{2}]\,[\frac{2y}{2}]$$

$= y^2$

$[\frac{n^2}{4}] = [\frac{4y^2}{4}] = y^2$

Therefore: $[\frac{n}{2}]\,[\frac{n}{2}] = [\frac{n^2}{4}] = y^2$

Problem 6. (10 points) Section 2.3, Exercise 22

Solution. :
A) y = -3x +4
x = -3y + 4
f^{-1}(x) = (x-4) / -3
So it is a bijection since we can find the inverse.

B)Not a bijection because it fails the horizontal line test, and by failing the horizontal line test it means the function is not one to one. We can also not that f(2) = f(-2) to confirm.

C) Not a bijection from $\mathbb{R}$ to $\mathbb{R}$ because at x = -2 the function can not divide by 0.

D) y = x^5 + 1
f^{-1}(x) = $\sqrt[5]{x-1}$
So this is a bijection because we can take the inverse.

Problem 7. (10 points) Supplementary Exercise 30

Solution. :
Last three terms add up to be the next term so:
Given:
$a_o = 1$
$a_1 = 3$
$a_2 = 4$
Therefore the sequence is : a(n) = $a_{n-3} + a_{n-2} + a_{n-1}$
Next four terms :
$a_8 = a_5 + a_6 + a_7 = 27 + 50 + 92 = 169$
$a_9 = a_6 + a_7 + a_8 = 50 + 92 + 169 = 311$
$a_{10} = a_7 + a_8 + a_9 = 92 + 169 + 311 = 572$
$a_{11} = a_8 + a_9 + a_{10} = 169 + 311 + 572 = 1052$

Problem 8. (10 points) Section 2.4, Exercise 22

Solution. :
A) $a_n = a_{n-1} + 1000 + 0.05a_{n-1}$
$= 1.05a_{n-1} + 1000$
$a_o = 50000$

B) $a_8 = 1.05a_7 + 1000$
$a_o = 50000$
$a_1 = 1.05(50000) + 1000 = 53500$
$a_2 = 1.05(53500) + 1000 = 57175$
$a_3 = 1.05(57175) + 1000 = 61033.8$
$a_4 = 1.05(61033.8) + 1000 = 65085.5$
$a_5 = 1.05(65085.5) + 1000 = 69339.8$
$a_6 = 1.05(69339.8) + 1000 = 73806.8$
$a_7 = 1.05(73806.8) + 1000 = 78497.1$
$a_8 = 1.05(78497.1) + 1000 = 83422$

C) $a_n = 1.05^n\ a_o + \Sigma_{i=0}{}^{n-1}\ 1.05^i\ 1000$
$= 1.05^n\ 50000 + 1000\ \Sigma_{i=0}{}^{n-1}\ 1.05^i = 1.05^n\ 50000 + 1000\ (20\ 1.05^n - 20)$
$= 70000\ (1.05^n) - 20000$

Problem 9. (10 points) Section 2.4, Exercise 38

Solution. :
a$_k$ = k^3
telescoping: a$_k$ - a_{k-1} = k^3-$(k-1)^3$
expanding$(k-1)^3$ we then get:
= k^3 - (k^3-1^3-3k^2+3k)
= 1 + 3k^2 - 3k
k^2 = (3k-1) / 3
k^2 = (k^3 - $(k-1)^3$ + 3k - 1) / 3
$\Sigma_{k=1}^{n} k^2$ = (1/3) $\Sigma_{k=1}^{n}$ $(k^3 - (k-1)^3 + 3k - 1)$
= (1/3) $\Sigma_{k=1}^{n}$ $(k^3 - (k-1)^3)$ + 3 $\Sigma_{k=1}^{n}$ k - $\Sigma_{k=1}^{n}$ 1
by formula:
$\Sigma_{k=1}^{n}$ k = n(n+1) / 2
$\Sigma_{k=1}^{n}$ 1 = n
by telescoping: $\Sigma_{k=1}^{n}$ $(k^3 - (k-1)^3) = n^3 + 0$
by substituting: $\Sigma_{k=1}^{n}$ k^2 = (1/3)[n^3 - 0 + 3 (n(n+1) / 2) - n]
= (1/3)[n^3 + ((3n^2 + n) / 2)]
= n (n+1)(2n+1) / 6

Problem 10. (10 points) Supplementary Exercise 38

Solution. :
A=c**I**
We are trying to show that **AB**=**BA**.
Therefore, lets give values to c and the matrix I, and B.
c = 2
$\mathbf{I} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$
$\mathbf{B} = \begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix}$
So: $\mathbf{A} = 2 \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 6 & 8 \end{bmatrix}$
$\mathrm{AB} = \begin{bmatrix} 2 & 4 \\ 6 & 8 \end{bmatrix} \begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 8 & 12 \\ 12 & 8 \end{bmatrix}$
$\mathrm{BA} = \begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 2 & 4 \\ 6 & 8 \end{bmatrix} = \begin{bmatrix} 8 & 12 \\ 12 & 8 \end{bmatrix}$
Therefore we can conclude that $\mathbf{AB} = \mathbf{BA}$