CSCE 222 [505] Discrete Structures for Computing
Fall 2015 – Philip C. Ritchey

Problem Set 4

Due dates: Electronic submission of LaTeX and PDF files of this homework is due on **30 September 2015 (Wednesday) before 11:30 a.m.** on eCampus (`http://ecampus.tamu.edu`).

Name	**Problems**
Mitesh Patel	1–10
9-30-15	
UIN: 124002210	

Resources. Discrete Mathematics and its Applications by Rosen Chapter 5
http://math.stackexchange.com/questions/241060/structural-inductions
http://highered.mheducation.com/sites/0073383090/student-view0/index.html
https://www.youtube.com/watch?v=dMn5w4-ztSw
https://www.youtube.com/watch?v=IFqna5F0kW8
https://www.cs.cmu.edu/ adamchik/21-127/lectures/induction-1-print.pdf
http://www.inf.ed.ac.uk/teaching/courses/dmmr/slides/13-14/Ch5.pdf
http://math.stackexchange.com/questions/855680/discrete-math-induction-problem
http://math.stackexchange.com/questions/517440/whats-the-difference-between-simple-induction-and-strong-induction

Problem 1. (10 points) Section 5.1, Exercise 4

Solution. :
p(n) $= 1^3 + 2^3 + n^3 = (\frac{n(n+1)}{2})^2$
A) $p(1) = 1^3 = (\frac{1(1+1)}{2})^2$
B) Basis Step:$1^3 = (\frac{1(1+1)}{2})^2$
1=1 so p(1) is True
C) IH : Assume p(k) $= 1^2 + ... + k^3 = (\frac{k(k+1)}{2})^2$
D) Prove: $p(k) \rightarrow p(k+1)$
Show: $p(k) = 1^3 + ... + k^3 + (k+1)^3 = (\frac{k+1(k+2)}{2})^2$
E) $p(k) = k^3 = (\frac{k(k+1)}{2})^2$
so: $k^3 + (k+1)^3 = (\frac{k(k+1)}{2})^2 + (k+1)^3$ by IH
$= (\frac{k(k+1)}{2})(\frac{k(k+1)}{2}) + (k+1)^3$
$= (\frac{(k^2+1)(k^2+1)}{4}) + (k+1)^3$
$= 1/4k^2(k+1)^2 + (k+1)^2 + (k+1)$
$= 1/4(k+1)^2(k^2 + 4k + 4)$
$= 1/4(k+1)^2(k+2)^2$
$= (\frac{k+1(k+2)}{2})^2$
Therefore p(k+1) is true by PMI.
F) The basis step is true and our inductive step is also true, so by PMI the formula is true whenever n is a positive number.

Problem 2. (10 points) Supplementary Exercise 10

Solution. :
$p(n) = n^3 + (n+1)^3 + (n+2)^3 divisibleby 9$
Basis: p(0) = 9 divisible by 9? YES so p(0) is true
Inductive: p(k) → p(k+1)
Assume p(k) = $k^3 + (k+1)^3 + (k+2)^3$ divisible by 9
Show p(k+1) is true :
p(k+1) - p(k) using IH
$= (k+1)^3 + (k+2)^3 + (k+3)^3 - k^3 - (k+1)^3 - (k+2)^3$
$= (k+3)^3 - k^3$
$= (k^2 + 6k + 9)(k+3) - k^2$ by expanding $(k+3)^3$
$= 9k^2 + 27k + 27$
$= 9(k^2 + 3k + 3)$
so we can see it is divisible by 9.
therefore p(k) and p(k+1) are divisible by 9.

Problem 3. (10 points) Supplementary Exercise 20

Solution. :
first few numbers of fibonacci:
(0,1,1,2,3,5,8,13,21)
Basis: f(0) = 0
0 is divisible by of 3. means position 0 of the sequence which is the first element in the sequence. So:
f(4) = 3 is a divisible of 3 so basis step is completed.
Inductive: assume f(k) is divisible by 3
show f(k+4) is divisible by 3
f(k+4) = f(k+3) + f(k+2)
= f(k+2) + f(k+1) + f(k+1) + f(k)
= f(k+1) + f(k) + f(k+1) + f(k+1) + f(k)
= 2f(k) + 3f(k+1)
by IH f(k) is divisible by 3 and we know f(k+1) is some integer times 3 which is divisible by 3
so f(k+4) is divisible by 3 by PMI

Problem 4. (10 points) Section 5.2, Exercise 4

Solution. :

A)Basis step:
p(18) true b/c 1 4 cent stamp and 2 7 cent stamps
p(19) true b/c 3 4 cent stamp and 1 7 cent stamp
p(20) true b/c 5 4 cent stamps
p(21) true b/c 3 7 cent stamps
B) Inductive: Assume p(i) for $18 \leq$ i $\leq$ k
C) Show (k+1) with 4 cent and 7 cent stamps
D) (k+1)cent = (k-3)cent and 4 cents.. k-3+4 = k+1 so holds true
E) True because both the basis and inductive steps were completed

Problem 5. (10 points) Section 5.2, Exercise 32

Solution. :
The flaw is in the induction step from the 4 cent case, we can form it using only 1 4 cent stamp so there is no need for 2 4 cent stamps.

Problem 6. (10 points) Supplementary Exercise 52

Solution. :
A) No, not well ordered because it has infinite integers.
B) Yes, it is well-ordered because least would be -99.
C) No, not well ordered because it has positive infinite rationals with no least element.
D) Yes, it is well ordered because least element would be 1/100.

Problem 7. (10 points) Section 5.3, Exercise 13

Solution. :
$p(n) = f_1 + f_3 + ... + f_{2n-1} = f_{2n}$
Basis step : p(1) $= f_1 = 1 = f_2$
Inductive step: p(k) $\rightarrow$ p(k+1)
Assume $p(k) : f_1 + f_3 + ... + f_{2k-1} = f_{2k}$
Show p(k+1): $f_1 + f_3 + ... + f_{2k-1} + f_{2k+1} = f_{2k} + f_{2k+1}$
by IH the right hand side $= f_{2k} + f_{2k+1}$
$= f_{2k+2}$
$= f_{2(k+1)}$
so:
p(k+1) is true by PMI

Problem 8. (10 points) Section 5.3, Exercise 32

Solution. :
$\Sigma = (0, 1)$
x $\in \Sigma$, t$\in \Sigma^*$
ones(tx) = ones(t) + x
B) Base step: t = x
ones(sx) = ones(s) + 0
= ones(s) + ones(x) b/c ones(x) = 0
Assume ones(st) = ones(s) + ones(t)
Recursive step: y $\in \Sigma$ s,t $\in \Sigma^*$
ones(s+y) = ones(st) + y
= ones(s) + ones (t) + y b/c from above we get
ones(s) + ones(ty)
There fore by structural induction it is proved, because of our assumption we used in recursive step.

Problem 9. (10 points) Section 5.3, Exercise 44

Solution. :
Basis step: since no internal vertices i(T) = 0
so l(T) = 1
= 1 + i(T)
l(T) = 1+i(T) is True
Assume : l(s) = 1 + i(S)
Recursive step:
If it is true for T_1 and T_2 then it is true for T_1
know: $i(T) = i(T_1) + i(T_2) + 1$
$l(T) = l(T_1) + l(T_2)$
$=i(T_1) + 1 + i(T_2) + 1$ from assumption
$= i(T) + 1$ using $i(T) = i(T_1) + i(T_2) + 1$
$l(T) = i(T) + 1$ is true by structural induction

Problem 10. (10 points) Section 5.3, Exercise 64

Solution. :
f(n) = n/2 ... it is basically f divided by 2
divide by 2^k to iterate through function so:
$f^k(n) = n/2^k$
$f^k(n) \leq 1$
$n/2^k \leq 1$
$n \leq 2^k$
$log_2(n) \leq k$
so:
$f_1^k(n) = log_2(n)$

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Checklist:

1. Did you type your full name and UIN and those of any collaborators?
2. Did you abide by the Aggie Honor Code?
3. Did you solve all problems and start a new page for each?
4. Did you submit
   (a) your LaTeX source file?
   (b) your PDF file?