CSCE 222 [505] Discrete Structures for Computing
Fall 2015 – Philip C. Ritchey

Problem Set 9

Due dates: Electronic submission of LaTeX and PDF files of this homework is due on **18 November 2015 (Wednesday) before 11:30 a.m.** on gradescope (`http://gradescope.com`).

Names of Group Members
Mitesh Patel UIN: 124002210 11-16-15

Resources. Discrete mathematics and its application chapter 10 and 11
Lecture slides
https://en.wikipedia.org/wiki/K-ary_tree

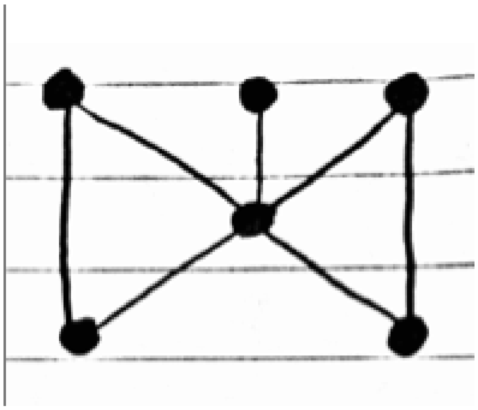
Problem 1. (10 points) Section 10.1, Exercise 24.

Solution. :
A) To model electronic mail messages we can use graphs with directed edges, with multiple edges, and loops. Multiple edges and directed edges because we want to have emails being sent back and forth, thats just how it works. Loops because we can indeed send emails to our selves.

B) A graph that models the electronic mail send in a network would be described as a directed multigraph. You can send mail to yourself which would be a loop, there would be directed edges since we can send mail back and forth, there would be multiple edges since we can send as many emails to one person as we would like, and they can do the same. Vertices would be the email addresses.

Problem 2. (15 points) Section 10.2, Exercise 40

Solution. :
$\Sigma_{v=v}$ deg(v) = 2m
= 4 + 3 + 3 + 2 + 2 = 2m
14 = 2m
m = 7 vertices



Problem 3. (15 points) Section 10.2, Exercise 64

Solution. :
G = (V,E)
V = $V_1 \cup V_2$
if |V| is even then $|V_1| = |V_2|$
if |V| is odd then $|V_1|$ - $|V_2|$ = 1

When |V| is even
$|V_1| = |V_2|$ = v/2
Max edges = $(v/2)^2$
therefore e$\leq$ v^2 / 4

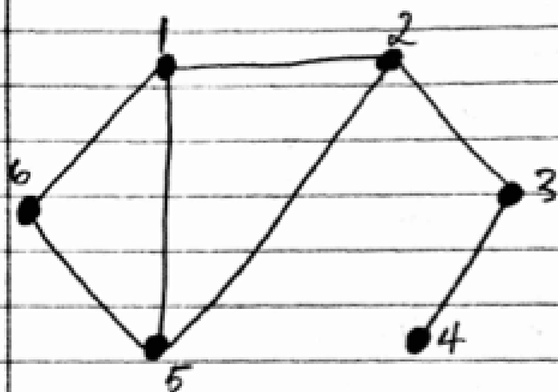
When |V| is odd
$|V_1| = \frac{v+1}{2}$ and $|V_2| = \frac{v-1}{2}$
so $(\frac{v+1}{2})(\frac{v-1}{2})$
e $\leq \frac{v^2-1}{4}$
e $\leq v^2/4$ so proved...

Problem 4. (15 points) Section 10.8, Exercise 18

Solution. :

adjacency matrix :

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$



Chromatic number is 3 so therefore 3 channels are needed.

Problem 5. (15 points) Section 11.1, Exercise 22

Solution. :
5 - ary tree so:
m=5
i = 10,000

n = mi + 1
n = i + L

i = 10,000 -1 = 9999 internal vertices
n = 5(9999) + 1
n = 49996

n - i = L
49,996 - 9,999 = 39,997 people who did not send out
n - 1 = 49,996 - 1 = 49,995 received the letter

Problem 6. (15 points) Section 11.2, Exercise 8

Solution. :

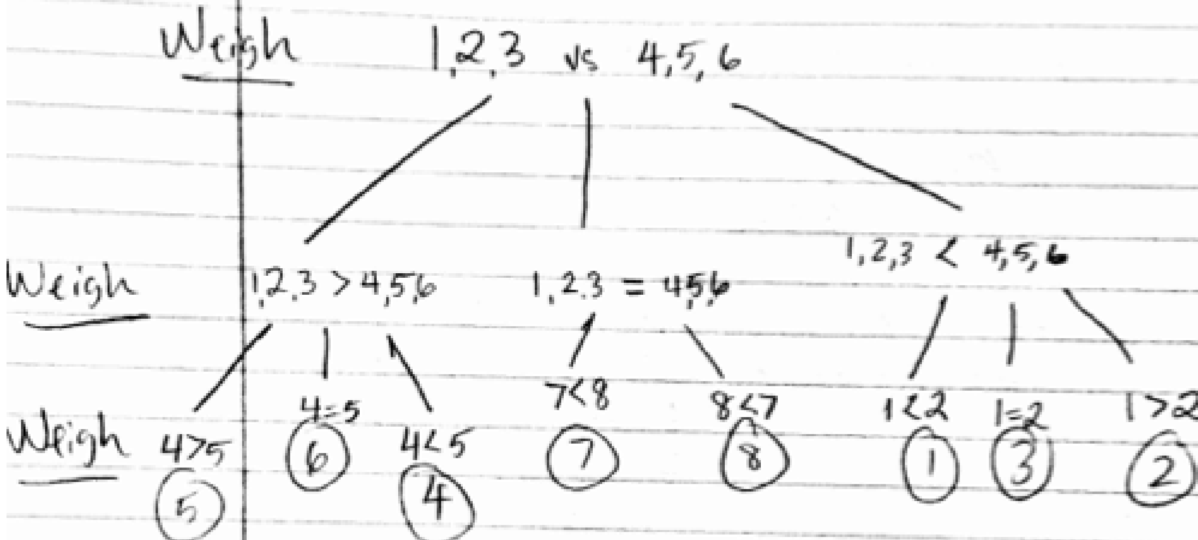
$L = 8$ leaves

3 outcomes - lighter, same, or heavier $(m=3)$

$h \geq \lceil \log_m L \rceil$

$h \geq \lceil \log_2 8 \rceil = \lceil 1.\sim \rceil = 2$

Drawing 3-ary decision tree:



So $\boxed{3 \text{ weighs}}$

We divide the pile of coins, and weigh them against each other.

Problem 7. (15 points) Section 11.3, Exercise 16

Solution. :

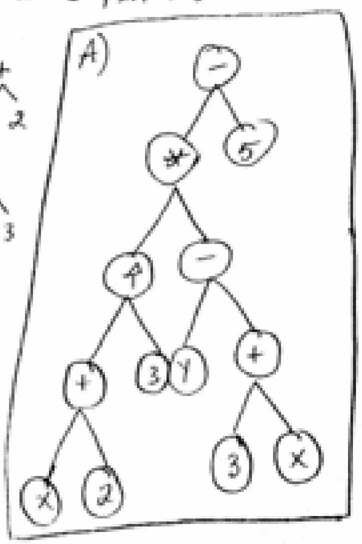
$((x+2)\uparrow 3) * (y-(3+x))-5$ ← Lets parse to get postfix

input		
(	(	
(	((	
x		x
+	((+	
2		x2
)	(	x2+
↑	(↑	
3		x2+3
)		x2+3↑
*	*	
(	*(	
y		x2+3↑y
-	*(-	
(	*(-(	
3		x2+3↑y3
+	*(-(+	
x		x2+3↑y3x
)	*(-	x2+3↑y3x+
)	*	x2+3↑y3x+-
-	-	x2+3↑y3x+-*
5		x2+3↑y3x+-*5-

Building tree:

Use stack:

A)



B) $- * \uparrow + x 2 3 - y + 3 x 5$

C) $x 2 + 3 \uparrow y 3 x + - * 5 -$

D) $((x+2)\uparrow 3) * (y-(3+x))-5$

Aggie Honor Statement: On my honor, as an Aggie, I have neither given nor received any unauthorized aid on any portion of the academic work included in this assignment. Furthermore, I have disclosed all resources (people, books, web sites, etc.) that have been used to prepare this homework.

Checklist:

1. Did you type your full name and that of all collaborators?
2. Did you abide by the Aggie Honor Code?
3. Did you solve all problems and start a new page for each?
4. Did you submit your PDF file?