CSCE 222 [505] Discrete Structures for Computing
Fall 2015 – Philip C. Ritchey

Problem Set 5

Due dates: Electronic submission of LaTeX and PDF files of this homework is due on **14 October 2015 (Wednesday) before 11:30 a.m.** on eCampus (`http://ecampus.tamu.edu`).

Name	**Problems**
Mitesh Patel	1–10
10-10-15	
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Resources. Discrete Mathematics and Its Applications Chapter 6
http://www.texample.net/tikz/examples/feature/trees/
http://tex.stackexchange.com/questions/5447/how-can-i-draw-simple-trees-in-latex
http://www.mycstutorials.com/articles/sorting/quicksort
https://en.wikipedia.org/wiki/Merge-sort
http://tex.stackexchange.com/questions/213770/how-to-combine-a-top-down-and-bottom-up-binary-tree-in-one-picture

Problem 1. (10 points) Section 5.4, Exercise 8

Solution. :

Algorithm 1 Recursive algorithm for finding sum of first n positive integers

```
procedure SUM(n : integer)
    if n == 1 then return 1
    else
        return n + sum(n-1)
    end if
end procedure
```

Problem 2. (10 points) Section 5.4, Exercise 16

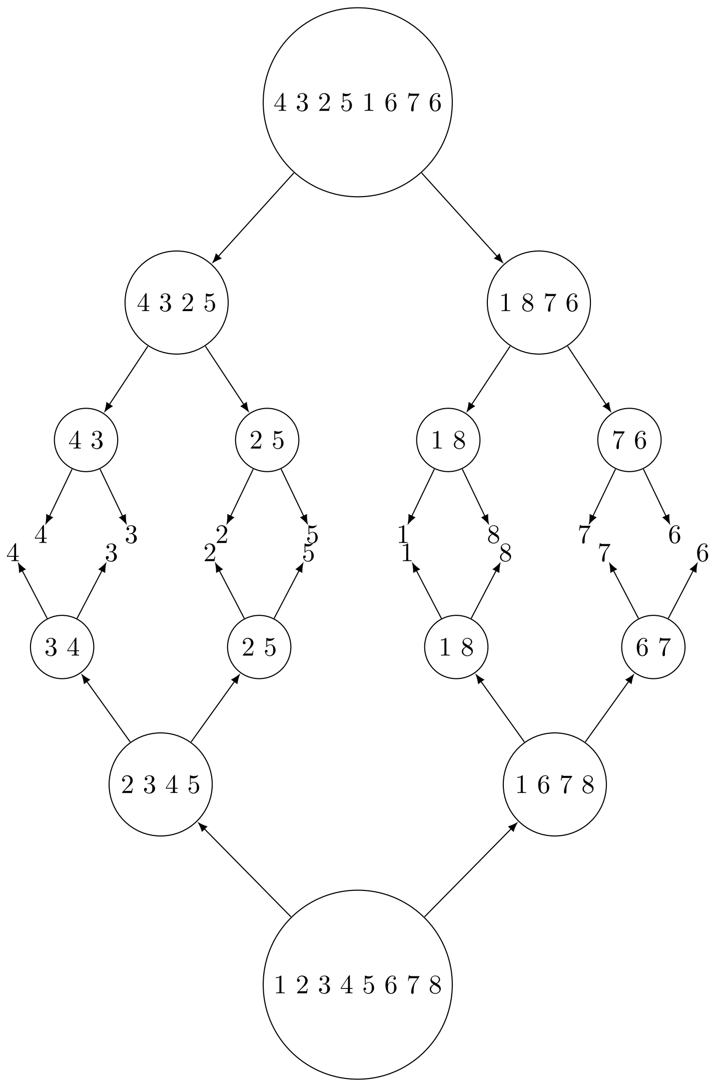
Solution. :
Proof by Mathematical Induction sum(n) = n + sum(n-1)

Basis:
If n=1 then the first step of the algorithm tells us that 1 + sum(1-1) = 1, and this is true because the sum of 1 positive integer is just 1, therefore our basis step checks out.

Inductive:
Assume: sum(k) = k + sum(k-1) , algorithm correctly computes first k positive integers.
Show: sum(k+1) = k + 1 + sum(k+1-1) , algoritm correctly computes first k + 1 integers.
= k + 1 + sum(k)
Using our IH we can conclude that our above statement is true for k+1. Therefore by PMI the algorithm is correct.

Problem 3. (10 points) Section 5.4, Exercise 44

Solution. :



Problem 4. (10 points) Section 5.4, Exercise 50

Solution. :
3,5,7,8,1,9,2,4,6
The median is 5 so that means out pivot is 5.

Now just looking at the first half of the numbers: 3,1,2,4 - our pivot is 3 therefore the numbers less than 3 go to the left of three and greater than 3 go to the right of 3. We get 1,2,3,4 as a result.

Now looking at the other half of the numbers: 7,8,9,6 - our pivot is 7 and using the same method above where the numbers less than 7 go to the left and the numbers greater go to the right. We get 6,7,8,9 as a result.

Therefore, as a result we get [1,2,3,4], 5 , [6,7,8,9] = 1,2,3,4,5,6,7,8,9 sorted.

Problem 5. (10 points) Section 6, Supplementary Exercise 8

Solution. :
A)
100 through 999 is a 3 digit positive integer
so: (1000-100) = 900
900 positive integers with three decimal digits (100 through 900)

B)
We can see that there are 9 ways for one positive integer with one decimal digit (1 through 9)
And from part A the number of positive 3 digit integers is 900
so: 900 + 9 = 909 numbers that have odd number of decimal digits

C)
The possibilities are:
A one digit number that is 9 = 1
A two digit number with the number 9 at the tens place and 0-9 at ones place is = 1 + 9 = 10
A two digit number with 9 at ones place and 1-8 at tens place = 1 + 7 = 8
A three digit number where 0-9 in the ones place and tens place, and 9 in 100s place = 10(10) = 100
A three digit number where 9 is in tens place and 1-8 in 100s place and 0-8 in ones place = 9(10) = 90
A three digit number where 9 in ones place, 0-8 in tens, and 1-8 in 100s = 9(9) = 81
Therefore by adding them all up we get 1+10+8+100+90+81 = 290 numbers have at least one decimal digit equal to 9

D)
1-9 on ones place - 1,2,3,4,5,6,7,8,9 - we have 4 even numbers
0-9 in tens place - 0,1,2,3,4,5,6,7,8,9 - we have 5 even numbers (counting 0)
0-9 in 100s place - - 0,1,2,3,4,5,6,7,8,9 - we have 5 even numbers (counting 0)
Therefore, $4(5^2) = 120$ numbers have no odd decimal digits.

E)
We can have 55 a two digit number where theres only one way in which the ones place has to be 0 - 1
We can have _ 55 a three digit number where 0-9 is in the blank so 10 ways - 10
We can have 55 _ a three digit number where 0-9 except 5 and 0 in the blank so 8 ways - 8
Therefore there are 1+10+8 = 19 numbers that have consecutive digits equal to 5

F)
There are 9 three digit palindromes : 111, 222, ... , 999
There are 9 two digit palindromes : 11, 22, ... , 99
3 digit palindromes could also be where middle value is numbers 0-9: 1 _ 1, 2 _ 2, ..., 9 _ 9 . Where the middle value can be a number from 0-9, so 10 ways to do this on 9 potential palindromes = 9(10) = 90
Therefore summing them up we get: 90+9+9 = 108 numbers that are palindromes.

Problem 6. (10 points) Section 6, Supplementary Exercise 12

Solution. :
There are 7 days in a week, and 12 months.
Using this we can multiple 12 by 7 which gives us 84. Now using the pigeon hole principle this gives us:
k = 84
k+1 = 84+1 = 85 people where at least two would be born on the same day of the week and on the same month.

Problem 7. (10 points) Section 6.3, Exercise 30

Solution. :
A)
We have 7 women and 9 men.
Using C(n,r) = $\frac{n!}{r!(n-r)!}$
C(7,1)C(9,4) + C(7,2)C(9,3) + C(7,3)C(9,2) + C(7,4)C(9,1) + C(7,5)C(9,0) =
$\frac{7!}{1!(6!)}\frac{9!}{4!(5!)} + \frac{7!}{2!(5!)}\frac{9!}{3!(6!)} + \frac{7!}{3!(4!)}\frac{9!}{2!(7!)} + \frac{7!}{4!(3!)}\frac{9!}{1!(8!)} + \frac{7!}{5!(3!)}\frac{9!}{0!(9!)}$ = 7(126) + 21(84) + 35(36) + 35(9) + 21(1)
= 4242 ways

B)
C(7,1)C(9,4) + C(7,2)C(9,3) + C(7,3)C(9,2) + C(7,4)C(9,1) =
$\frac{7!}{1!(6!)}\frac{9!}{4!(5!)} + \frac{7!}{2!(5!)}\frac{9!}{3!(6!)} + \frac{7!}{3!(4!)}\frac{9!}{2!(7!)} + \frac{7!}{4!(3!)}\frac{9!}{1!(8!)}$ = 7(126) + 21(84) + 35(36) + 35(9) =
= 4221 ways

Problem 8. (10 points) Section 6.4, Exercise 8

Solution. :
We want to find the coefficient of x^8y^9
We are given $(3x+2y)^{17}$
By looking at these two equations we can conclude that:
n = 17
j = 9
$(3x+2y)^{17} = \Sigma_{j=0}^{17} \binom{17}{j} 3x^{17-j}2y^j$
Plugging in j we get: $\binom{17}{9} 3x^{17-9}2y^9$
$= \binom{17}{9} 3x^8 2y^9$
$= \binom{17}{9} 3^8 2^9 = \frac{17!}{9!(8!)} (3^8 2^9)$
= using a calculator we can conclude that the final answer is 81662929920

Problem 9. (10 points) Section 6, Supplementary Exercise 38

Solution. :
Given the fact that we need to pick 12 dozens 3 of each kind of apples.
There are 20 delicious apples in which we can pick 3 of the same kind.
There are 20 macintosh apples in which we can pick 3 of the same kind.
Finally, there are 20 granny smith apples in which we can pick 3 of the same kind.
Therefore, $3^3 = 27$ ways in which we choose dozen apples if at least three of each kind must be chosen.

Problem 10. (10 points) Section 6.6, Exercise 6

Solution. :
Steps to complete this problem is as follows:
1) look for LAST pair of integers where $a_j < a_{j+1}$
2) look for the least integer to the right of a_j that is greater than a_j
3) now swap the value found in step 2 above and place it in the position a_j
4) finally order the remaining elements

A) 1342
Using step 1 $a_2 = 3$ and $a_3 = 4$
Using step 2 $a_3 = 4$
Using step 3 $a_2 = 4$
Using step 4 our answer is 1423

B) 45321
Using step 1 $a_1 = 4$ and $a_2 = 5$
Using step 2 $a_2 = 5$
Using step 3 $a_1 = 5$
Using step 4 our answer is 51234

C) 13245
Using step 1 $a_4 = 4$ and $a_5 = 5$
Using step 2 $a_5 = 5$
Using step 3 $a_4 = 5$
Using step 4 our answer is 13254

D) 612345
Using step 1 $a_5 = 4$ and $a_6 = 5$
Using step 2 $a_6 = 5$
Using step 3 $a_5 = 5$
Using step 4 our answer is 612354

E) 1623547
Using step 1 $a_6 = 4$ and $a_7 = 7$
Using step 2 $a_7 = 7$
Using step 3 $a_6 = 7$
Using step 4 our answer is 1623574

F) 23587416
Using step 1 $a_7 = 1$ and $a_8 = 6$
Using step 2 $a_8 = 6$
Using step 3 $a_7 = 6$
Using step 4 our answer is 23587461

Wildcard Quiz Problems (the quiz on Friday could also be one of these)
Section 6, Supplementary Exercise 4
Section 6.2, Exercise 10
Section 6.3, Exercise 8
Section 6.5, Exercise 30

Aggie Honor Statement: On my honor, as an Aggie, I have neither given nor received any unauthorized aid on any portion of the academic work included in this assignment. Furthermore, I have disclosed all resources (people, books, web sites, etc.) that have been used to prepare this homework.