CSCE 222 [505] Discrete Structures for Computing
Fall 2015 – Philip C. Ritchey

Problem Set 6

Due dates: Electronic submission of LaTeX and PDF files of this homework is due on **21 October 2015 (Wednesday) before 11:30 a.m.** on eCampus (`http://ecampus.tamu.edu`).

Name	Problems
Mitesh Patel	1–10
10-20-15	
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Resources. www.piazza.com
https://en.wikipedia.org/wiki/Knapsack_problem
http://www.cplusplus.com/forum/beginner/69889/
https://users.cs.duke.edu/ ola/ap/recurrence.html
https://proofwiki.org/wiki/Inclusion-Exclusion_Principle

Problem 1. (15 points) Section 8.1, Exercise 16

Solution. :
A) We know that 0,1,2 can be in a ternary string.
Correct way: We can append these to a string with length n-1. Therefore we would get $3a_{n-1}$
Incorrect strings of length n-2 can be: $3^{n-2} - a_{n-2}$
We can add 00 or 11 to this string to make it a valid string: so: $2(3^{n-2} - a_{n-2})$
Therefore, $a_n = 3a_{n-1} + 2(3^{n-2}) - 2a_{n-2}$

B) initial conditions:
$a_o = 0$
$a_1 = 0$

C) $a_2 = 3a_1 + 2(3^0) - 2a_o = 3(0) + 2(1) - 2(0) = 2$
$a_3 = 3a_2 + 2(3^1) - 2a_1 = 3(2) + 2(3) - 2(0) = 12$
$a_4 = 3a_3 + 2(3^2) - 2a_2 = 3(12) + 2(9) - 2(2) = 50$
$a_5 = 3a_4 + 2(3^3) - 2a_3 = 3(50) + 2(27) - 2(12) = 180$
$a_6 = 3a_5 + 2(3^4) - 2a_4 = 3(180) + 2(81) - 2(50) = 602$

Problem 2. (15 points) Section 8, Supplementary Exercise 14

Solution. :
A) Since $w_j > w$
Weight of item j is greater than w. Using M(j,w) it means we cant have $w_j > w$
So: M(j,w) = M(j-1,w) - maximum weight in j items that don't exceed w

B) Consider $w_j \leq w$
Using item j:
the max weight that is subset of first j-1 items is M(j-1,w-w_j)
therefore, M(j,w) = max(M(j-1,w),w_j + M(j-1,w-w_j))

C)

Algorithm 1 dynamic programming

```
procedure MAXWEIGHT(w_1....w_n 0 ≤ w ≤ W)
    for w := 0 to W do
        M(0,W) = 0
    end for
    for j := 1 to n do
        for w:=0 to W do
            if W_j ≤ W then M(j,w) = max(M(j-1,w),w_j + M(j-1,w-w_j))
            else
                Return large
            end if
        end for
    end for
end procedure
```

D) We can use another function to store the values and set a temp variable to hold fixed weight limit. Then we can subtract this temp weight by item j.

Problem 3. (15 points) Section 8.2, Exercise 46

Solution. :

A) $a_o = 2$
$a_n = 2a_{n-1} + 100$

B) $r^n = 2r^{n-1}$
$r^n / r^n r^{-1} = 2$
r = 2
$a_n^{(h)} = \alpha r^n = \alpha 2^n$
$a_n = \alpha 2^n - 100$
$a_o = 2 = \alpha 2^0 - 100$
$\alpha = 102$
$a_n = 102(2^n) - 100$

C) $a_n = 2a_{n-1} - n$

D) $a_n = 2a_{n-1} - n$
$a_n = a_n^{(h)} + a_n^{(p)}$
first lets solve its associated linear homogeneous
$a_n = 2a_{n-1}$
r = 2
$a_n^{(h)} = \alpha 2^n$
F(n) = n
suppose p(n) = cn + d is such a solution
cn + d = 2(C(n-1)+d) - n
= 2(cn-c+d)-n
= 2cn-2c+2d-n
cn+n-2cn = -2c+2d-d
n(c+1-2c) = -2c + d
(-c+1)n + (2c+d) = 0
c = 1 , and d = -2
so plugging in:
$a_n^{(p)} = n - 2_n = n - 2 + \alpha 2^n$
$a_o = 2 = -2 + \alpha$
α= 4
Therefore, $a_n = n - 2 + 4(2^n)$

Problem 4. (15 points) Section 8.3, Exercise 16

Solution. :
from 14: number of rounds from two sets is f(n/2), requires one more round to get winner.
therefore recurrence relation would be f(n) = f(n/2) + 1, f(1) = 0 where n = 2^k

Now 16:
$a_{(2^k)} = a_{(2^{k-1})}$ + 1
= $a_{(2^{k-1})}$ + 1
= $a_{(2^{k-2})}$ + 1 + 1
= $a_{(2^{k-3})}$ + 1 + 1 + 1
.
.
.
= $a_{(2^{k-i})}$ + i
i_{max} = k
so : $a_{(2^{k-k})}$ + k
= a_1 + k
from 14 : = 0 + k = k = a_{2^n}

Problem 5. (15 points) Section 8.4, Exercise 14

Solution. :

$0 \leq x \leq 3$
$x_1 + x_2 + x_3 + x_4 + x_5 = 12$
$(1 + x + x^2 + x^3)^5 = (1 - x^4)^5 \ / \ (1 - x)^5$
by expanding first couple of terms = $\frac{1-5x^4+10x^8-10x^{12}+...}{(1-x)^5}$
using: $\frac{1}{1-(ax)^n}$ = $\Sigma_{k=0}^{\infty}$ C(n+k-1,k)$a^k x^k$
n = 12, k = 4
n = 8, k = 4
n = 4, k = 4
n = 0, k = 4

$\binom{n+k-1}{k}$

$\binom{16}{4}$ - 5$\binom{12}{4}$ + 10$\binom{8}{4}$ - 10$\binom{1}{1}$

= 1820 - 2475 + 700 -10 = 35

Problem 6. (15 points) Section 8.5, Exercise 14

Solution. :
26! permutations total
|A| := set that contains "fish"
|B| := set that contains "rat"
|C| := set that contains "bird"

so:
|A| = 26 - 4 + 1 = 23! // we add one because the string is one block
|B| = 26 - 3 +1 = 24!
|C| = 26 - 4 + 1 = 23!

$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$

$|A \cap B|$ = 21! because fish and rat are 7 diff letters so 19 letters left and they are two blocks of string.
$|A \cup B|$ = empty set because repeated i letter
$|B \cap C|$ = empty set because repeated r letter therefore , $|A \cap B \cap C|$ is empty

$|A \cup B \cup C|$ = 24! + 23! + 23! -21!
finally subtract 26! from above so:
26! - 24! - 23! - 23! + 21! // —– answer

Problem 7. (10 points) Section 8.6, Exercise 2

Solution. :
P_1 := property for altitude sickness
P_2 := property for not in shape
P_3 := property for getting allergies

$$N(P_1'P_2'P_3') = N - N(P_1) - N(P_2) - N(P_3) + N(P_1P_2) + N(P_2P_3) + N(P_1P_3) - N(P_1P_2P_3)$$

N = 1000
$N(P_1) = 450$
$N(P_2) = 622$
$N(P_3) = 30$
$N(P_1P_2) = 111$ (altitude sickness and not in shape)
$N(P_2P_3) = 14$ (not in shape and allergies)
$N(P_1P_3) = 18$ (altitude sickness and allergies)
$N(P_1P_2P_3) = 9$ (all three)

Plugging in:
1000 - 450 - 622 - 30 + 111 + 14 + 18 - 9 = 32

Wildcard Quiz Problems (the quiz on Friday could also be one of these)
Section 8.1, Exercise 12
Section 8.2, Exercise 6
Section 8.3, Exercise 22
Section 8.3, Exercise 13

Aggie Honor Statement: On my honor, as an Aggie, I have neither given nor received any unauthorized aid on any portion of the academic work included in this assignment. Furthermore, I have disclosed all resources (people, books, web sites, etc.) that have been used to prepare this homework.

Checklist:
1. Did you type your full name and UIN and those of any collaborators?
2. Did you abide by the Aggie Honor Code?
3. Did you solve all problems and start a new page for each?
4. Did you submit
   (a) your LaTeX source file?
   (b) your PDF file?