Assignment - 1

Matrix :

Que. 1 Define the following:

(1) Squre matrix :-
A matrix of order $m \times n$ is called a squre matrix.
- In a squre matrix the number of rows equals the number of colums.

Ex :- $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $[1]$

$= 3 \times 3$ $= 2 \times 2$ $= 1 \times 1$

(2) Diagonal matrix :-
A squre matrix in which each element expect the diagonal element is zero, is called a diagonal matrix.

Ex :- $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $i = j$ $[m = n]$

(3) Transpose of a matrix:-
The matrix obtained from any given matrix A by changing its rows into corresponding columns is called the transpose of A and it is denoted by A' or A^T.

- Thus the transpose of $A = [a_{ij}]$ is $A' = (a_{ji})$

Ex:- $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \longrightarrow A^T = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$

[4] Symmetric Matrix:-
If for a squre matrix $A = [a_{ij}]$, $A' = A$, then A is called a symmetric matrix.

- In a symmetric matrix; $a_{ij} = a_{ji}$ for each pair (i, j) $m \times n = 3 \times 3$

[5] skew - symmetric matrix:-
If for a squre matrix $A = [a_{ij}]$, $A' = -A$, then A is called a skew symmetric matrix.

Thus all the diagonal elements of a skew symmetric matrix are zero

$A = -A^T$

Ex :- $B = \begin{bmatrix} 0 & 2 & 4 \\ -2 & 0 & 3 \\ -4 & -3 & 0 \end{bmatrix}$ $-B^T = \begin{bmatrix} 0 & -2 & -4 \\ 2 & 0 & -3 \\ 4 & 3 & 0 \end{bmatrix}$

Que:- 2 If $A = \begin{bmatrix} 6 & 7 & 8 \\ 1 & 7 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} -4 & 5 & 6 \\ 7 & -8 & -2 \end{bmatrix}$

then find $A+B$ and $A-B$.

Solve:- $A+B$

$A = \begin{bmatrix} 6 & 7 & 8 \\ 1 & 7 & 5 \end{bmatrix} + B \begin{bmatrix} -4 & 5 & 6 \\ 7 & -8 & -2 \end{bmatrix}$

$A+B = \begin{bmatrix} 6+(-4) & 7+5 & 8+6 \\ 4+7 & 7+(-9) & 5+(-2) \end{bmatrix}$

$A+B = \begin{bmatrix} 2 & 12 & 14 \\ 8 & -1 & 3 \end{bmatrix}$

$\Rightarrow$ $A-B$

$= A = \begin{bmatrix} 6 & 7 & 8 \\ 1 & 7 & 5 \end{bmatrix} - B \begin{bmatrix} -4 & 5 & 6 \\ 7 & -8 & -2 \end{bmatrix}$

$A-B = \begin{bmatrix} 6-(-4) & 7-5 & 8-6 \\ 1-7 & 7-8 & 5+2 \end{bmatrix}$

$A-B = \begin{bmatrix} 10 & 2 & 2 \\ -6 & -1 & 7 \end{bmatrix}$

Que. 3 If $A = \begin{bmatrix} 4 & 6 \\ 2 & 3 \\ 5 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 3 \\ 0 & 1 \\ 5 & -1 \end{bmatrix}$ then find

$4A - 2B =$

Solve:- $4A - 2B = 4\begin{bmatrix} 4 & 6 \\ 2 & 3 \\ 5 & -1 \end{bmatrix} - 2\begin{bmatrix} 1 & 3 \\ 0 & 1 \\ 5 & -1 \end{bmatrix}$

$$= \begin{bmatrix} 16 & 24 \\ 8 & 12 \\ 20 & -4 \end{bmatrix} - \begin{bmatrix} 2 & 6 \\ 0 & 2 \\ 10 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 16-2 & 24-6 \\ 8-0 & 12-2 \\ 20-10 & -4-2 \end{bmatrix}$$

$$4A - 2B = \begin{bmatrix} 14 & 18 \\ 8 & 10 \\ 10 & -6 \end{bmatrix}$$

Que. 4 If $A = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 2 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 6 \\ 2 & 5 \\ -2 & -1 \end{bmatrix}$ then find AB and BA

$$AB = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 2 & 5 \end{bmatrix} \times \begin{bmatrix} -2 & 6 \\ 2 & 5 \\ -2 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 2\times(-2)+3\times2+4\times(-2) & 2\times3+3\times5+4\times(-1) \\ 1\times(-2)+2\times2+5\times(-2) & 1\times6+2\times5+5\times(-1) \end{bmatrix}$$

$$= \begin{bmatrix} (-4)+6+(-8) & 6+15+(-4) \\ (-2)+4+(-10) & 6+10+(-5) \end{bmatrix}$$

$$AB = \begin{bmatrix} -6 & 17 \\ -8 & 11 \end{bmatrix}$$

$$\Rightarrow \quad BA = \begin{bmatrix} -2 & 6 \\ 2 & 5 \\ -2 & -1 \end{bmatrix} \times \begin{bmatrix} 2 & 3 & 4 \\ 1 & 2 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} -2\times2+6\times1 & -2\times3+6\times2 & -2\times4+6\times5 \\ 2\times2+5\times1 & 2\times3+5\times2 & 2\times4+5+5 \\ -2\times2+-1\times1 & -2\times3+-1\times2 & -2\times4+1\times5 \end{bmatrix}$$

$$= \begin{bmatrix} -4+6 & -6+12 & -8+30 \\ 4+5 & 6+10 & 8+25 \\ -4-1 & -6+(-2) & -8+-5 \end{bmatrix}$$

$$BA = \begin{bmatrix} 2 & 6 & 22 \\ 9 & 16 & 33 \\ -5 & -8 & -13 \end{bmatrix}$$

Que. 5 If $A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & -1 \\ 3 & -1 & 1 \end{bmatrix}$ then prove that $A^3 - 3A^2 - A + 9I = 0$

$$A^3 = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & -1 \\ 3 & -1 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & -1 \\ 3 & -1 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & -1 \\ 3 & -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1\times1 + 2\times0 + 1\times3 & 1\times2 + 2\times1 + 1\times(-1) & 1\times1 + 2\times(-1) + 1\times1 \\ 0\times1 + 1\times0 + (-1)\times3 & 0\times2 + 1\times1 + (-1)\times(-1) & 0\times1 + 1\times(-1) + (-1)\times1 \\ 3\times1 + (-1)\times0 + 1\times3 & 3\times2 + (-1)\times1 + 1\times(-1) & 3\times1 + (-1)\times(-1) + 1\times1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+0+3 & 2+2+(-1) & 1+(-2)+1 \\ 0+0+(-3) & 0+1+(-1) & 0+(-1)+(-1) \\ 3+0+3 & 6+(-1)+(-1) & 3+1+1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 3 & 0 \\ -3 & 2 & -2 \\ 6 & 4 & 5 \end{bmatrix} \times \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & -1 \\ 3 & -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4\times1 + 3\times0 + 0\times3 & 4\times2 + 3\times1 + 0\times(-1) \\ (-3)\times1 + 2\times0 + (-2)\times3 & (-3)\times2 + 2\times1 + (-2)\times(-1) \\ 6\times1 + 4\times0 + 5\times3 & 6\times2 + 4\times1 + 5\times(-1) \end{bmatrix}$$

$$\begin{matrix} 4\times1 + 3\times(-1) + 0\times1 \\ -3\times1 + 2\times(-1) + (-2)\times1 \\ 6\times1 + 4\times(-1) + 5\times1 \end{matrix}\Bigg]$$

$$= \begin{bmatrix} 4+0+0 & 8+3+0 & 4+(-3)+0 \\ -3+0+(-6) & -6+2+2 & -3+(-2)-2 \\ 6+0+15 & 12+4-5 & 6+(-4)+5 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 4 & 11 & 1 \\ -9 & -2 & -7 \\ 21 & 11 & 7 \end{bmatrix}$$

$$\therefore A^3 - 3A^2 - A + 9I = 0$$

$$\begin{bmatrix} 4 & 11 & 1 \\ -9 & -2 & -7 \\ -21 & 11 & 7 \end{bmatrix} - 3\begin{bmatrix} 4 & 3 & 0 \\ -3 & 2 & -2 \\ 6 & 4 & 5 \end{bmatrix} - \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & -1 \\ 3 & -1 & 1 \end{bmatrix} + 9\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\therefore \begin{bmatrix} 4 & 11 & 1 \\ -9 & -2 & -7 \\ 21 & 11 & 7 \end{bmatrix} - \begin{bmatrix} 12 & 9 & 0 \\ -9 & 6 & -5 \\ 18 & 12 & 15 \end{bmatrix} - \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & -1 \\ 3 & -1 & 1 \end{bmatrix} + \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

$$\therefore \begin{bmatrix} 4-12-1 & 11-9-2 & 1-0-1 \\ -9+9-0 & -2-6-1 & -7+6+1 \\ 21-18-3 & 11-12+1 & 7-15-1 \end{bmatrix} + \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

$$\therefore \begin{bmatrix} -9 & 0 & 0 \\ 0 & -9 & 0 \\ 0 & 0 & -9 \end{bmatrix} + \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix} = 0$$

$$\therefore \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0$$ R.H.S Proved.

$\therefore$ The solution $A^3 - 3A^3 - A + 9I$ is 0 proved.

Que. 6 If $= A = \begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$ then prove that $adj\ A = 3A^T$.

$adj\ A = 3A^T$

$= 3 \begin{bmatrix} -1 & 2 & 2 \\ -2 & 1 & -2 \\ -2 & -2 & 1 \end{bmatrix}$

$3A^T = \begin{bmatrix} -3 & 6 & 6 \\ -6 & 3 & -6 \\ -6 & -6 & 3 \end{bmatrix} - (i)$

$adj = A$

$a_{11} = 1 - 4 = -3$

$a_{12} = 2 + 4 = 6$

$a_{13} = -4 - 2 = -6$

$a_{21} = -2 - 4 = -6$

$$adj\ A = \begin{bmatrix} -3 & 6 & 6 \\ -6 & 3 & -6 \\ -6 & -6 & 3 \end{bmatrix} \text{---(ii)}$$

$\therefore adj\ A = 3A^T$

$\therefore L.H.S = R.H.S$

$a_{22} = -1+4 = 3$

$a_{23} = 2+4 = 6$

$a_{31} = 4+2 = 6$

$a_{32} = 2+4 = 6$

$a_{33} = -1+4 = 3$

$$\text{Co-factor} = \begin{bmatrix} -3 & -6 & -6 \\ 6 & 3 & -6 \\ -6 & -6 & 3 \end{bmatrix}$$

Que.7 If $A = \begin{bmatrix} 2 & 7 \\ 5 & 3 \end{bmatrix}$ find $A + A^T + A^{-1}$

Solve- $A^T = \begin{bmatrix} 2 & 5 \\ 7 & 3 \end{bmatrix}$

$$A^{-1} = \frac{adj\ A}{|A|}$$

$$= \frac{1}{-29} \times \begin{bmatrix} 3 & -7 \\ -5 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -3/29 & 7/29 \\ -5/29 & -2/29 \end{bmatrix}$$

$|A| = \begin{bmatrix} 2 & 7 \\ 5 & 3 \end{bmatrix}$

$= 6 - 35$

$= -29 \neq 0$

$\therefore A^{-1}$ is exists.

$A^{-1} = \begin{bmatrix} 2 & 7 \\ 5 & 3 \end{bmatrix}$

$= \begin{bmatrix} 3 & -7 \\ -5 & 2 \end{bmatrix}$

$\therefore A + A^T + A^{-1}$

$$\therefore \begin{bmatrix} 0 & 7 \\ 5 & 3 \end{bmatrix} + \begin{bmatrix} 2 & 5 \\ 7 & 3 \end{bmatrix} + \begin{bmatrix} -3/29 & 7/29 \\ 5/29 & -2/29 \end{bmatrix}$$

$$\therefore \begin{bmatrix} \frac{58+58-3}{29} & \frac{203+145+7}{29} \\ \frac{145+203+5}{29} & \frac{87+87-2}{29} \end{bmatrix}$$

$$= \begin{bmatrix} 113/29 & 355/29 \\ 353/29 & 172/29 \end{bmatrix}$$

Que. 8 If $A = \begin{bmatrix} 5 & 2 \\ 6 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & -3 \\ 3 & -1 \end{bmatrix}$ then verify that $\text{adj}(AB) = (\text{adj} B) \times (\text{adj} A)$

Solve $\text{adj}\, AB = \begin{bmatrix} -1 & 3 \\ -3 & 4 \end{bmatrix} \times \begin{bmatrix} 3 & -2 \\ 6 & -5 \end{bmatrix}$

$$= \begin{bmatrix} -3+18 & 2-15 \\ -9+24 & 6-20 \end{bmatrix}$$

$$\text{adj}\, AB = \begin{bmatrix} 15 & -13 \\ 15 & -14 \end{bmatrix} \quad \text{---} (i)$$

$$AB = \begin{bmatrix} -5 & 2 \\ -6 & 3 \end{bmatrix} \times \begin{bmatrix} 4 & -3 \\ 3 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} -5\times 4 + 2\times 3 & -5\times -3 + 2\times -1 \\ -6\times 4 + 3\times 3 & -3\times -3 + 3\times -1 \end{bmatrix}$$

$$= \begin{bmatrix} -20+6 & 15-2 \\ -24+9 & 18-3 \end{bmatrix}$$

$$= \begin{bmatrix} -14 & 13 \\ -15 & 15 \end{bmatrix} = \begin{bmatrix} 15 & -13 \\ 15 & -14 \end{bmatrix} \quad \text{— (ii)}$$

$\longrightarrow$ (i) = (ii)

Que. 9 If $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & 1 \\ 3 & 2 & 0 \end{bmatrix}$ then find A^{-1}

Solve $|A| = \begin{vmatrix} 1 & 2 & 3 \\ 0 & -1 & 1 \\ 3 & 2 & 0 \end{vmatrix} = 1(0-2) - 2(0-3) + 3(0+3)$

$= -2 + 6 + 9$

$= 13 \neq 0$

$\therefore A^{-1}$ exists

minor

$a_{11} = \begin{vmatrix} -1 & 1 \\ 2 & 0 \end{vmatrix} = 0 - 2$

$= -2$

$a_{12} = \begin{vmatrix} 0 & 1 \\ 3 & 0 \end{vmatrix} = 0-3$
$= -3$

$a_{13} = \begin{vmatrix} 0 & -1 \\ 3 & 2 \end{vmatrix} = 0+3$
$= 3$

$a_{21} = \begin{vmatrix} 2 & 3 \\ 2 & 0 \end{vmatrix} = 0-6$
$= -6$

$a_{22} = \begin{vmatrix} 1 & 3 \\ 3 & 0 \end{vmatrix} = 0-9$
$= -9$

$a_{33} = \begin{vmatrix} \cancel{2}1 & 2 \\ 3 & 2 \end{vmatrix} = 2-6$
$= -4$

$a_{31} = \begin{vmatrix} 2 & 3 \\ -1 & 1 \end{vmatrix} = 2+3$
$= 5$

$a_{32} = \begin{vmatrix} 1 & 3 \\ 0 & 1 \end{vmatrix} = 1-0$
$= 1$

$a_{33} = \begin{vmatrix} 1 & 2 \\ 0 & -1 \end{vmatrix} = -1-0$
$= -1$

$\begin{vmatrix} -2 & -3 & 3 \\ -6 & -9 & -4 \\ 5 & 1 & -1 \end{vmatrix}$

$\text{co-Factor} = \begin{vmatrix} -2 & 3 & 3 \\ 6 & -9 & 4 \\ 5 & -1 & 1 \end{vmatrix}$

$$\text{co-factor} = \begin{vmatrix} -2 & 3 & 3 \\ 6 & -9 & 4 \\ 5 & -1 & 1 \end{vmatrix}$$

$$A^{-1} = \frac{\text{adj} A}{|A|} = \frac{1}{13} \begin{bmatrix} -2 & 3 & 3 \\ 6 & -9 & 4 \\ 5 & -1 & 1 \end{bmatrix}$$

Que. 10 show that $\begin{bmatrix} -2/3 & 2/3 & 2/3 \\ 2/3 & 2/3 & 1/3 \\ -1/3 & -2/3 & 2/3 \end{bmatrix}$ is an orthogonal matrix.

Solve

$$A^T = \begin{bmatrix} -2/3 & 2/3 & 1/3 \\ 1/3 & 2/3 & -2/3 \\ 2/3 & 1/3 & 2/3 \end{bmatrix}$$

$$A \cdot A^T = \begin{bmatrix} -2/3 & 1/3 & 2/3 \\ 2/3 & 2/3 & 1/3 \\ 1/3 & -2/3 & 2/3 \end{bmatrix} \times \begin{bmatrix} -2/3 & 2/3 & 1/3 \\ 1/3 & 2/3 & -2/3 \\ 2/3 & 1/3 & 2/3 \end{bmatrix}$$

$$= \begin{bmatrix} -2/3 \times -2/3 + -1/3 \times 1/3 + 2/3 \times 2/3 & -2/3 \times 2/3 + 1/3 \times 2/3 + 2/3 \times 1/3 \\ 2/3 \times -2/3 + 2/3 \times 1/3 + 1/3 \times 2/3 & 2/3 \times 2/3 + 2/3 \times 2/3 + 1/3 \times 1/3 \\ 1/3 \times -2/3 + -2/3 \times -2/3 + 2/3 \times 2/3 & 1/3 \times 2/3 + -2/3 \times 2/3 + 2/3 \times 1/3 \end{bmatrix}$$

$$= \begin{bmatrix} -4/9 + 1/9 + 4/9 & -4/9 + 2/9 + 2/9 & -2/9 - 2/9 + 4/9 \\ -4/9 + 2/9 + 2/9 & 4/9 + 4/9 + 1/9 & 2/9 - 4/9 + 2/9 \\ -2/9 - 2/9 + 4/9 & 2/9 + 4/9 + 2/9 & 1/3 + 4/9 + 4/9 \end{bmatrix}$$

$$= \begin{bmatrix} 9/9 & 0/9 & 0/9 \\ 0/9 & 9/9 & 0/9 \\ 0/9 & 0/9 & 9/9 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = R.H.S.$$

Que 11 IF $A^{-1} = \begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix}$ then find matrix A such that $A \cdot A^{-1} = I$. Also satisfy given matrix is orthogonal matrix.

Solve:- $A \cdot A^{-1} = I$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \times \begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 3a+7b & 2a+5b \\ 3c+7d & 2c+5d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\therefore$ $3a+7b = 0$ — (i) $\quad 3c+7b = 0$ — (iii)

$2a+5b = 0$ — (ii) $\quad 2c+5d = 1$ — (iv)

equation (i) - (ii)

$3a + 7b = 1$

$-2a + 5b = 0$

$+ \quad - \quad -$

$a + 2b = 1$

$a - 1 - 2b$ — (v)

equation. 5 value put in (2)

$2(1-2b)+5b=0$

$2-4b+5b=0$

$0+b=0$

$\boxed{b=-2}$

$a=1-2(-2)$

$=1+4$

$\therefore \boxed{a=5}$

equation (iii) - (iv)

$3c+7d=0$

$-2c+5d=1$

$+ \quad - \quad -$

$c+2d=-1$

$c=-1-2d$ — (c)

eq (c) value put in equation (3)

$3(-1-2d)+7d=0$

$-3-6d+7d=0$

$-3+d=0$

$-3+d=0$

$\boxed{d=3}$

$c=7-2d$

$=-1-2(3)$

$-1-6$

$\boxed{c=-7}$

$A=\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix}$

$\therefore A \cdot A^T = I$

$$\begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix} \times \begin{bmatrix} 5 & -7 \\ -7 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 25+4 & -35-6 \\ -35-6 & 49+9 \end{bmatrix} = \begin{bmatrix} 29 & -41 \\ -41 & 58 \end{bmatrix} \neq I$$

$\therefore A \begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix}$ is not orthogonal matrix