MCP.

Assignment - 2

① Deterinants....

Q.1 $A = \begin{vmatrix} 8 & 5 & 2 \\ 3 & 4 & 5 \\ 1 & 2 & 3 \end{vmatrix}$ and $B = \begin{vmatrix} 6 & -3 & 2 \\ 5 & 4 & 5 \\ 1 & 2 & -1 \end{vmatrix}$ then find det A and det B.

Solve:- a) $\text{Det } A = 8\begin{vmatrix} 4 & 5 \\ 2 & 3 \end{vmatrix} - 5\begin{vmatrix} 3 & 5 \\ 1 & 3 \end{vmatrix} + 2\begin{vmatrix} 3 & 4 \\ 1 & 2 \end{vmatrix}$

$= 8|12-10| - 5|9-5| + 2|6-4|$

$= 8\times 2 - 5\times 4 + 2\times 2$

$= 16 - 20 + 4$

$\boxed{= 0}$

b) $\text{Det } B = 6\begin{vmatrix} 4 & 5 \\ 2 & -1 \end{vmatrix} + 3\begin{vmatrix} 5 & 5 \\ 1 & -1 \end{vmatrix} + 2\begin{vmatrix} 5 & 4 \\ 1 & 0 \end{vmatrix}$

$= 6|-4-10| + 3|-5-5| + 2|10-4|$

$= 6\times -14 + 3\times -10 + 2\times 6$

$= -84 - 30 + 12$

$\boxed{= -102}$

Q.2 Prove that : $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$

Solve| Taking $C_1 = C_1 - C_2$ and $C_2 = C_2 - C_3$

$$\begin{vmatrix} 0 & 0 & 1 \\ a-b & b-c & c \\ a^2-b^2 & b^2-c^2 & c^2 \end{vmatrix}$$

$$\begin{vmatrix} 0 & 0 & 1 \\ a-b & b-c & c \\ (a+b)(a-b) & (b+c)(b-c) & c^2 \end{vmatrix} \quad \{a^2-b^2 = (a+b)(a-b)\}$$

Taking $C_2 = C_2 - C_1$, common from C_1 & C_2

$$(a-b)(b-c)\begin{vmatrix} 0 & 0 & 1 \\ 1 & 1 & c \\ a+b & b+c & c^2 \end{vmatrix}$$

<u>Now, $C_2 = C_2 - C_1$</u>

$$= (a-b)(b-c)\begin{vmatrix} 0 & 0 & 1 \\ 1 & 0 & c \\ a+b & c-a & c^2 \end{vmatrix}$$

$= (a-b)(b-c)\{(c-a)\times 1 - (a+b)\times 0\}$

$= (a-b)(b-c)\{(c-a)-0\}$

$= (a-b)(b-c)(c-a)$

$= R.H.S$

Proved.

Q.3 Find value of x $\begin{vmatrix} x & 1 & 3 \\ -15 & 5 & 15 \\ 101 & 2 & 22 \end{vmatrix} = 0$

Solve:- $x \begin{vmatrix} 5 & 15 \\ 2 & 22 \end{vmatrix} - 1 \begin{vmatrix} -15 & 15 \\ 101 & 22 \end{vmatrix} + 3 \begin{vmatrix} -15 & 5 \\ 101 & 2 \end{vmatrix} = 0$

$\therefore x\,|110 - 30| - 1\,|-330 - 1515| + 3\,|-30 - 505| = 0$

$\therefore 80x + 1845 - 1605 = 0$

$\therefore 80x + 240 = 0$

$\therefore 80x = -240$

$x = -\frac{240}{80}$

$[\therefore x = -3]$

$\therefore$ The value of x is -3.

Q.4 Find the value of k if,

$\begin{vmatrix} 1 & 2 & 5 \\ 2 & k & 0 \\ 7 & 14 & 9 \end{vmatrix} = \begin{vmatrix} 16 & 8 & 26 \\ 6 & 3 & 7 \\ 2 & 1 & 4 \end{vmatrix}$

solve: $1\begin{vmatrix} k & 0 \\ 14 & 9 \end{vmatrix} - 2\begin{vmatrix} 2 & 0 \\ 7 & 9 \end{vmatrix} + 5\begin{vmatrix} 2 & k \\ 7 & 14 \end{vmatrix} = 16\begin{vmatrix} 3 & 7 \\ 1 & 4 \end{vmatrix} - 8\begin{vmatrix} 6 & 7 \\ 2 & 4 \end{vmatrix} + 26\begin{vmatrix} 6 & 3 \\ 2 & 1 \end{vmatrix}$

$\therefore 1|9k - 0| - 2|18 - 0| + 5|28 - 7k| = 16|12 - 7| - 8|24 - 14| + 26|6 - 6|$

$\therefore 9k - 36 + 105k = 80 - 80 + 0$

$\therefore 114k - 36 = 0$

$k = \dfrac{\cancel{36}}{\cancel{114}} \quad \dfrac{\cancel{2} \times 18}{\cancel{2} \times 57}$

$k = \dfrac{\cancel{18}}{\cancel{57}} \quad \dfrac{6 \times \cancel{3}}{19 \times \cancel{3}}$

$\boxed{\therefore k = \dfrac{6}{19}}$

$\therefore$ The value of k is $\underline{\dfrac{6}{19}}$

Q.5 Evaluate a) $\begin{vmatrix} 3x+9 & 3x+8 & 3x+7 \\ 3x+13 & 3x+12 & 3x+11 \\ 1999 & 1998 & 1997 \end{vmatrix}$

b) $\begin{vmatrix} a+2b & a+3b & a+4b \\ a+4b & a+5b & a+6b \\ a+6b & a+7b & a+8b \end{vmatrix}$

Solve:- a) $\begin{vmatrix} 3x+9 & 3x+8 & 3x+7 \\ 3x+13 & 3x+12 & 3x+11 \\ 1999 & 1998 & 19997 \end{vmatrix}$

Let $R_2 - R_1$ in R_2

$= \begin{vmatrix} 3x+9 & 3x+8 & 3x+7 \\ 4 & 4 & 4 \\ 1999 & 1998 & 1997 \end{vmatrix}$

Now, $C_1 - C_2$ in C_1 and $C_2 - C_3$ in C_2

$= \begin{vmatrix} 1 & 1 & 3x+7 \\ 0 & 0 & 4 \\ 1 & 1 & 1997 \end{vmatrix}$

when two Row's and columns are same then determinant is 0.

$\therefore$ Ans = 0

b) $\begin{vmatrix} a+2b & a+3b & a+4b \\ a+4b & a+5b & a+6b \\ a+6b & a+7b & a+8b \end{vmatrix}$

Taking $C_2 - C_1$ in C_1 and $C_3 - C_2$ in C_2

$= \begin{vmatrix} b & b & a+4b \\ b & b & a+6b \\ b & b & a+8b \end{vmatrix}$

when, two row's and column's are same then determinant is 0.

Ans is = 0

Q.6 If $a+b+c=0$ then show that

$$\begin{vmatrix} 2a+b+c & b & c \\ c & 2b+c+a & a \\ b & a & 2c+a+b \end{vmatrix} = 0$$

→ Taking $C_2 - C_3 = C_1$ additions.

$$\therefore \begin{vmatrix} 2a+2b+2c & b & c \\ 2a+2b+2c & a+2b+c & a \\ 2a+2b+2c & a & 2c+a+b \end{vmatrix}$$

$$= 2(a+b+c) \begin{vmatrix} 1 & b & c \\ 1 & a+2b+c & a \\ 1 & a & 2c+a+b \end{vmatrix}$$

$$= 2(a+b+c) \begin{vmatrix} 1 & 1 & 1 \\ b & a+2b+c & a \\ c & a & 2c+a+b \end{vmatrix}$$

$\therefore$ Now $C_1 = C_1 - C_2$, $C_2 - C_3 = C_2$

$$\therefore 2(a+b+c) \begin{vmatrix} 0 & 0 & 0 \\ a+b+c & 2b+c & a-b \\ c-a & 2c+b & a+b+c \end{vmatrix} = 0$$

$$\therefore 2(0) \begin{vmatrix} 0 & 0 & 0 \\ a-b-c & 2b+c & a-b \\ c-a & 2c+b & a+b+c \end{vmatrix}$$

$\therefore 2(0) = 0 = RHS.$

Q.7 Find minor of A and co-factor of A

$$A = \begin{vmatrix} 3 & 1 & -2 \\ 1 & 1 & -1 \\ -2 & 3 & 4 \end{vmatrix}$$

→ minor of $a_{11} = \begin{vmatrix} 1 & -1 \\ 3 & 4 \end{vmatrix} = 4+3$
$= 7$

$a_{12} = \begin{vmatrix} 1 & -1 \\ -2 & 4 \end{vmatrix} = 4-2$
$= 2$

$a_{13} = \begin{vmatrix} 1 & 1 \\ -2 & 3 \end{vmatrix} = 3+2$
$= 5$

$a_{21} = \begin{vmatrix} 1 & -2 \\ 3 & 4 \end{vmatrix} = 4+6$
$= 10$

$a_{22} = \begin{vmatrix} 3 & -2 \\ -2 & 4 \end{vmatrix} = 12-4$
$= 8$

$$a_{23} = \begin{vmatrix} 3 & 1 \\ -2 & 3 \end{vmatrix} = 9+2 = 11$$

$$a_{31} = \begin{vmatrix} 1 & -2 \\ 1 & -1 \end{vmatrix} = -1+2 = 1$$

$$a_{32} = \begin{vmatrix} 3 & -2 \\ 1 & -1 \end{vmatrix} = -2+3 = -1$$

$$a_{33} = \begin{vmatrix} 3 & 1 \\ 1 & 1 \end{vmatrix} = 2 \quad = \begin{vmatrix} 7 & 2 & 5 \\ 10 & 8 & 11 \\ 1 & -1 & 2 \end{vmatrix}$$

Co-factor of A :-

Co-factor of $a_{11} = (-1)^{1+1} \times 7$
$= 1 \times 7 = 7$

$a_{12} = (-1)^{1+2} \times 2$
$= -1 \times 2 = 2$

$a_{13} = (-1)^{1+3} \times 5$
$= 1 \times 5 = 5$

$a_{21} = (-1)^{2+1} \times 10$
$= -1 \times 10 = -10$

$a_{22} = (-1)^{2+2} \times 8$
$= 1 \times 8 \quad = 8$

$a_{23} = (-1)^{2+3} \times 11$
$= -1 \times 11 \quad = -11$

$a_{31} = (-1)^{3+1} \times 1$
$= 1 \times 1 \quad = 1$

$a_{32} = (-1)^{3+2} \times -1$
$= -1 \times -1 \quad = 1$

$a_{33} = (-1)^{3+3} \times 2$
$= 1 \times 2 \quad = 2$

$$= \begin{vmatrix} 7 & -2 & 5 \\ -10 & 8 & -11 \\ 1 & 1 & 2 \end{vmatrix}$$

8) Using the crammer's rule solve following equation

$2x + 3y = 11$ and $-5y + x = -14$

→ $2x + 3y - 11 = 0$
$-5y + x + 14 = 0$

$$\frac{x}{\begin{vmatrix} 3 & -11 \\ -5 & 14 \end{vmatrix}} = \frac{y}{\begin{vmatrix} -11 & 2 \\ 14 & 1 \end{vmatrix}} = \frac{1}{\begin{vmatrix} 2 & 3 \\ 1 & -5 \end{vmatrix}}$$

$$\frac{x}{42-55} = \frac{y}{-11-28} = \frac{1}{-10-3}$$

$$\frac{x}{-13} = \frac{y}{-39} = \frac{1}{-13}$$

$$x = \frac{\cancel{-13}^{1}}{\cancel{-13}} \qquad y = \frac{\cancel{-39}^{3}}{\cancel{-13}}$$

$$\therefore \boxed{x=1} \qquad \boxed{y=3}$$

$\therefore$ The solution is

$(x, y) = (1, 3)$

6) $\frac{y-5}{2} + \frac{x-3}{2} = 2$ —— (i)

$\frac{x+2}{3} + \frac{y+5}{4} = 7$ —— (ii)

Taking eq (i)

$$\frac{y-5}{2} + \frac{x-3}{2} = 2$$

$$\frac{y-5+x-3}{2} = 2$$

$$y-5+x-3 = 4$$
$$x+y-8-4=0$$
$$x+y-12=0 \quad \text{—— (i)}$$

eg. (ii) $\frac{4(x+2)+3(y+5)}{12} = 7$

$$4x+8+3y+15 = 84$$
$$4x+3y+23-84=0$$
$$4x+3y-61=0 \quad \text{—— (ii)}$$

$$x+y-12=0 \quad \text{—— (i)}$$
$$4x+3y-61=0 \quad \text{—— (ii)}$$

$$\frac{x}{\begin{vmatrix} 1 & -12 \\ 3 & -61 \end{vmatrix}} = \frac{y}{\begin{vmatrix} -12 & 1 \\ -61 & 4 \end{vmatrix}} = \frac{1}{\begin{vmatrix} 1 & 1 \\ 4 & 3 \end{vmatrix}}$$

$$\frac{x}{1-25} = \frac{y}{13} = \frac{1}{-1}$$

$\frac{x}{-25} = \frac{1}{-1}$ $\frac{y}{13} = \frac{1}{-1}$

$x = \frac{-25}{-1}$ $y = \frac{13}{-1}$

$\boxed{x = 25}$ $\boxed{y = -13}$

The solution is $(x, y) = (25, -13)$

9) Solve using crammer's rules:-

$3x + 5y + 6z = 4$
$2y + x + 3z = 2$
$5z + 4y + 2x = 3$

solve Arranging proper way.

$3x + 5y + 6z = 4$
$x + 2y + 3z = 2$
$2x + 4y + 5z = 3$

$$D = \begin{vmatrix} 3 & 5 & 6 \\ 1 & 2 & 3 \\ 2 & 4 & 5 \end{vmatrix}$$

$= 3(10-12) - 5(5-6) + 6(4-4)$
$= 3(-2) - 5(-1) + 0$
$= -6 + 5$
$= -1 \neq 0$

The solution exist

$$D_1 = \begin{vmatrix} 4 & 5 & 6 \\ 2 & 2 & 3 \\ 3 & 4 & 5 \end{vmatrix}$$

$= 4(10-12) - 5(10-9) + 6(8-6)$
$= 4(-2) - 5(1) + 6(2)$
$= -8 - 5 + 12$
$= -13 + 12$
$\boxed{= -1}$

$$D_2 = \begin{vmatrix} 3 & 4 & 6 \\ 1 & 2 & 3 \\ 2 & 3 & 5 \end{vmatrix}$$

$= 3(10-9) - 4(5-6) + 6(3-4)$
$= 3(1) - 4(-1) + 6(-1)$
$= 3 + 4 - 6$
$= 7 - 6$
$= 1$

$$D_3 = \begin{vmatrix} 3 & 5 & 4 \\ 1 & 2 & 2 \\ 2 & 4 & 3 \end{vmatrix}$$

$= 3(6-8) - 5(3-4) + 4(4-4)$

$= 3(-2) - 5(-1) + 0$

$= -6 + 5$

$= -1$

Then $x = \frac{D_1}{D}$ $\quad y = \frac{D_2}{D}$ $\quad z = \frac{D_3}{D}$

$= \frac{-1}{-1}$ $\quad = \frac{1}{-1}$ $\quad = \frac{-1}{-1}$

$\boxed{x = 1}$ $\quad \boxed{y = -1}$ $\quad \boxed{z = 1}$

The solution is $(x, y, z) = (1, -1, 1)$

10) Using the crammer's rule solve:-

$$\begin{vmatrix} x+3 & 4 \\ y-2 & 5 \end{vmatrix} = 10 \text{ and } \begin{vmatrix} 2x+1 & y+5 \\ 3 & 4 \end{vmatrix} = -8$$

eg. — (i) $\begin{vmatrix} x+3 & 4 \\ y-2 & 5 \end{vmatrix} = 10$

$5(x+3) - 4(y-2) = 10$
$5x+15 - 4y+8 = 10$
$5x - 4y + 23 - 10 = 0$
$5x - 4y + 13 = 0$ — (i)

eg - (ii) $\begin{vmatrix} 2x+1 & y+5 \\ 3 & 4 \end{vmatrix} = -8$

$4(2x+1) - 3(y+5) = -8$
$8x+4 - 3y+15 = -8$
$8x - 3y - 11 + 8 = 0$
$8x - 3y - 3 = 0$ — (ii)

equations $5x - 4y + 13 = 0$ — (i)
$8x - 3y - 3 = 0$ — (ii)

$$\frac{x}{\begin{vmatrix} -4 & 13 \\ -3 & -3 \end{vmatrix}} = \frac{y}{\begin{vmatrix} 13 & 5 \\ -3 & 8 \end{vmatrix}} = \frac{1}{\begin{vmatrix} 5 & -4 \\ 8 & -3 \end{vmatrix}}$$

$$\frac{x}{|-12+39|} = \frac{y}{|104+15|} = \frac{1}{|-15+32|}$$

$$\frac{x}{51} = \frac{y}{119} = \frac{1}{17}$$

$x = \frac{51}{17}$ | $y = \frac{119}{17}$

$\boxed{x = 3}$ | $\boxed{y = 7}$

$\therefore$ The solution is $(x, y) = (3, 7)$.