OPERATIONAL RESEARCH MINI PROJECT

⊛ A company manufactures three products namely X, Y and Z. Each of the products require processing on three machines, Turning, Milling and Grinding. Product X requires 10 hours of turning, 5 hours of milling and 1 hour of grinding. Product Y requires 5 hours of turning, 10 hours of milling and 1 hour of grinding and Product Z requires 2 hours of turning, 4 hours of milling and 2 hours of grinding. In the coming planning period, 2700 hours of turning, 2200 hours of milling and 500 hours of grinding are available. The profit contribution of X, Y and Z are Rs. 10, Rs. 15 and Rs. 20 per unit respectively. Find the optimal product mix to maximize the profit.

Solution:

Maximize $Z = 10x + 15y + 20z$

Subject to:

$$10x + 5y + 2z \leq 2700$$
$$5x + 10y + 4z \leq 2200$$
$$1x + 1y + 2z \leq 500$$

Let S_1, S_2, S_3 be three slack variables

$$\text{Maximize } Z = 10x + 15y + 20z + 0S_1 + 0S_2 + 0S_3$$
$$10x + 5y + 2z + 1S_1 + 0S_2 + 0S_3 = 2700$$
$$5x + 10y + 4z + 0S_1 + 1S_2 + 0S_3 = 2200$$
$$1x + 1y + 2z + 0S_1 + 0S_2 + 1S_3 = 500$$

$$\begin{bmatrix} 10 & 5 & 2 & 1 & 0 & 0 \\ 5 & 10 & 4 & 0 & 1 & 0 \\ 1 & 1 & 2 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ S_1 \\ S_2 \\ S_3 \end{bmatrix} = \begin{bmatrix} 2700 \\ 2200 \\ 500 \end{bmatrix}$$

		C_j	10	15	20	0	0	0	
Basic Variable	C_B	X_B	x	y	z	S_1	S_2	S_3	Min Ratio
S_1	0	2700	10	5	2	1	0	0	1350
S_2	0	2200	5	10	4	0	1	0	550
S_3	0	500	1	1	2*	0	0	1	250
		Δ_j	-10	-15	-20	0	0	0	

$R_3 \rightarrow \frac{R_3}{2}$; $R_2 \rightarrow R_2 - 4R_3$; $R_1 \rightarrow R_1 - 2R_3$

		C_j	10	15	20	0	0	
Basic Variable	C_B	X_B	x	y	z	S_1	S_2	Min Ratio.
S_1	0	1700	9	4	0	1	0	425
S_2	0	1200	3	8*	0	0	1	150
z	20	250	1/2	1/2	1	0	0	500
		Δ_j	0	-5	0	0	0	

$R_2 \to \frac{R_2}{8}$; $R_1 \to R_1 - 4R_2$ $R_3 \to R_3 - \frac{1}{2} R_2$.

		C_j	10	15	20	0	
Basic Variable	C_B	X_B	x	y	z	S_1	Min Ratio
S_1	0	1100	15/2	0	0	1	
y	15	150	3/8	1	0	0	
z	20	175	5/16	0	1	0	
		Δj	30/16	0	0	0	

As all the elements of Δj is 0 or positive, the solution is optimal.

The firm has to produce 150 units of y and 175 units of z.

The optimal profit will be

$$\begin{aligned} \text{Max } Z &= 10x + 15y + 20z \\ &= 10(0) + 15(150) + 20(175) \\ &= 2250 + 3500 \\ &= 5750 \end{aligned}$$

The optimal profit is 5750.