

# Prediction Market 2

CSCI 1440/2440

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## 1 Prediction Markets

A **prediction market** is a market in which securities are traded based on the predicted outcomes of future events. Prediction markets can be based on the future outcomes of sporting events, elections, or even the weather. They are interesting because they effectively aggregate the beliefs of all participants, so that the market price conveys the collective belief/probability of an outcome.

## 2 Market with One Decoy

Recall that in the first market, we flipped two coins:  $C$ , the true coin; and  $D$ , the decoy coin. Both were fair coins. We then independently and uniformly at random chose one of the two coin flips to reveal to each student.

In this second market, we will again flip the true coin,  $C$ , but we will then flip a **unique** decoy coin,  $D$ , for each student in the class. The coins, once again, are fair. We will then independently and uniformly at random choose whether to reveal to each student the true coin flip or their decoy.

You, the students, will then begin trading contracts that are worth \$100 if the true coin is heads, and \$0, if it is tails. Once trading subsides, we will reveal the value of  $C$  and  $D$ , and settle all contracts.

## 3 Questions

The questions are a subset of the ones we asked about the first market. Again, assume that you are told “heads”.

$$(1/2 * 1) + (1/2 * 1/2) = 3/4$$

1. What is the probability that  $C$  is heads?
2. Let's say you find out that another person was also told “heads”. What is the probability that  $C$  is heads?
3. Let's say you find out that one other person was told “tails”. What is the probability that  $C$  is heads?
4. Let's say you find out that 1000000 other people were told “tails”. What is the probability that  $C$  is heads?

If I have real coin ( $1/2$ ):  
1  
If I have fake coin ( $1/2$ ):  
If they have real coin ( $1/2$ ): 1 -> they said heads  
If they have fake coin ( $1/2$ ):  $1/2$   
 $1/2 + 1/4 + 1/8$

If I have real coin ( $1/2$ ):  
1  
If I have fake coin ( $1/2$ ):  
If they have real coin ( $1/2$ ): 1 -> they said heads  
If they have fake coin ( $1/2$ ):  $1/2$   
 $1/2 + 1/4 + 1/8$   
CD HH ->  $1/4$  \*

C D M O  
H T C H D T  
H T C H D H  
H T C T D T  
H T C T D H  
H H C H D H  
H H