Common Derivatives Cheat Sheet

A Comprehensive List of Derivative Formulas in Calculus

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February 25, 2025

Introduction

This document provides a comprehensive list of common derivative formulas in single-variable calculus. It includes basic rules, derivatives of exponential and logarithmic functions, trigonometric functions, inverse trigonometric functions, hyperbolic functions, and other useful formulas. Use this as a reference when studying or solving calculus problems.

1. Basic Rules

Constant Rule

$$\frac{d}{dx}[c] = 0 \quad \text{(where } c \text{ is a constant)}$$

Power Rule

$$\frac{d}{dx}[x^n] = n\,x^{n-1} \quad \text{(for any real number } n\text{)}$$

Constant Multiple Rule

$$\frac{d}{dx}[c \cdot f(x)] = c\,f'(x)$$

Sum/Difference Rule

$$\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x)$$

Product Rule

$$\frac{d}{dx}[f(x)g(x)] = f'(x)g(x) + f(x)g'(x)$$

Quotient Rule

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

Chain Rule

If $y = f(g(x))$, then

$$\frac{dy}{dx} = f'(g(x)) \cdot g'(x)$$

2. Exponential and Logarithmic Functions

Exponential Functions

$$\frac{d}{dx}[e^x] = e^x,$$

$$\frac{d}{dx}[a^x] = a^x \ln(a) \quad (a > 0)$$

Logarithmic Functions

$$\frac{d}{dx}[\ln x] = \frac{1}{x} \quad (x > 0)$$

$$\frac{d}{dx}[\log_a x] = \frac{1}{x \ln(a)}$$

3. Trigonometric Functions

Sine and Cosine

$$\frac{d}{dx}[\sin x] = \cos x,$$

$$\frac{d}{dx}[\cos x] = -\sin x.$$

Tangent and Cotangent

$$\frac{d}{dx}[\tan x] = \sec^2 x,$$

$$\frac{d}{dx}[\cot x] = -\csc^2 x.$$

Secant and Cosecant

$$\frac{d}{dx}[\sec x] = \sec x \tan x,$$
$$\frac{d}{dx}[\csc x] = -\csc x \cot x.$$

4. Inverse Trigonometric Functions

Arcsine and Arccosine

$$\frac{d}{dx}[\arcsin x] = \frac{1}{\sqrt{1-x^2}} \quad (|x| < 1),$$
$$\frac{d}{dx}[\arccos x] = -\frac{1}{\sqrt{1-x^2}} \quad (|x| < 1).$$

Arctangent and Arccotangent

$$\frac{d}{dx}[\arctan x] = \frac{1}{1+x^2},$$
$$\frac{d}{dx}[\operatorname{arccot} x] = -\frac{1}{1+x^2}.$$

Arcsecant and Arccosecant

$$\frac{d}{dx}[\operatorname{arcsec} x] = \frac{1}{|x|\sqrt{x^2-1}} \quad (|x| > 1),$$
$$\frac{d}{dx}[\operatorname{arccsc} x] = -\frac{1}{|x|\sqrt{x^2-1}} \quad (|x| > 1).$$

5. Hyperbolic Functions

Hyperbolic Sine and Cosine

$$\frac{d}{dx}[\sinh x] = \cosh x,$$
$$\frac{d}{dx}[\cosh x] = \sinh x.$$

Hyperbolic Tangent, Cotangent, Secant, and Cosecant

$$\frac{d}{dx}[\tanh x] = \operatorname{sech}^2 x,$$
$$\frac{d}{dx}[\coth x] = -\operatorname{csch}^2 x,$$
$$\frac{d}{dx}[\operatorname{sech} x] = -\operatorname{sech} x \tanh x,$$
$$\frac{d}{dx}[\operatorname{csch} x] = -\operatorname{csch} x \coth x.$$

Inverse Hyperbolic Functions

$$\frac{d}{dx}[\operatorname{arcsinh} x] = \frac{1}{\sqrt{1+x^2}},$$
$$\frac{d}{dx}[\operatorname{arccosh} x] = \frac{1}{\sqrt{x^2-1}} \quad (x > 1),$$
$$\frac{d}{dx}[\operatorname{arctanh} x] = \frac{1}{1-x^2} \quad (|x| < 1).$$

6. Other Useful Formulas

Derivative of an Inverse Function

If $y = f^{-1}(x)$, then

$$\frac{dy}{dx} = \frac{1}{f'(f^{-1}(x))}.$$

General Exponential Function with a Function Exponent

If $y = a^{g(x)}$, then by the chain rule,

$$y' = a^{g(x)} \ln(a) \cdot g'(x).$$