UNIT - II CONDUCTORS & SUPERCONDUCTORS

2.1. CONDUCTORS

2.1.1. Introduction

The materials which can conduct electricity and heat easily through them are called as conductors. They have large electrical and thermal conductivity values. Metals are examples of conductors. The materials which do not conduct electricity and heat through them are called as insulators. They have large electrical and thermal resistivity values. Non-metals are examples of insulators. Recall, the resistance (R) of a conductor is directly proportional to the length (L) and inversely proportional to the area of cross-section (A) of the given conductor. Unit of 'R' is Ohms. (i.e) $R\ \alpha \frac{L}{A}$(or) $R = \frac{\rho L}{A}$, where the proportionality constant (ρ) is called as the electrical resistivity of the material of the given conductor and its unit is given by Ohm-m, since ρ= $R.A.L.$

Electrical resistivity (ρ) of the material is the reciprocal of the electrical conductivity (σ). (i.e) $\rho = \frac{1}{\sigma}$

2.1.2. Various theories developed to explain the structure and properties of solids.

a) **Classical Free Electron theory:** In 1900, Drude and Lorentz developed this theory which assumes that metals contain free electrons obeying the laws of classical mechanics. Here the valence electrons are free to move in a uniform potential provided by the positive ion lattice. The velocity and energy distribution of electrons obey the Maxwell-Boltzmann statistics.

b) **Quantum Free Electron Theory:** In 1928, Sommerfeld proposed this theory by retaining most of the concepts of classical free electron theory and with the quantum concepts. The velocity and energy distribution of electrons obey the Fermi-Dirac distribution function. It is understood quantum concepts deal with dual wave-particle behavior, superposition, and correlation between two or more atoms/ions whatever may be the distance between them (entanglement).

c) **Zone Theory (or) Band Theory:** In 1928, Bloch proposed this theory and it retains most of the postulates of Sommerfeld's and according to this theory electrons move in a periodic potential provided by the solid lattice.

2.1.3. Postulates of classical free electron theory

- According to this model, it is assumed that when metal atoms come together to form a solid, the valence electrons get liberated and move freely with in the metal and the remainder of the atom is a positive ion carrying the major portion of the atomic mass, which means the positive ions are heavy and immobile. Here positive ions refer to 'nucleus with core electrons' and the outermost electrons are referred as valence electrons.
- The valence electrons move freely in the uniform potential provided by the positive ions, like molecules of a perfect gas in a container. Hence they are called as **electron gas.** Hence these valence electronshave only kinetic energy and their potential energy is zero.
- In the **absence of external electric field**, the electrons move in random directions making collisions from time to time with positive ion lattice or with other free electrons. But all these collisions are considered to be elastic. Hence there is no loss in energy due to these collisions.
- In the **presence of external electric field**, the free electrons are accelerated. The accelerated electrons move in a direction opposite to the external electric field direction. The constant velocity acquired by the free electrons of the given metal in the presence of applied electric field is known as drift velocity (V_d).
- Mutual Repulsion between the free electrons is neglected and the electrons obey the classical kinetic theory of gases. The electron velocities and energy distribution follow the Maxwell – Boltzmann statistics.

Success of the Drude Lorentz Free electron theory

- Verifies ohm's law
- Used to derive mathematical expressions for electrical conductivity and thermal conductivity of metals and hence to verify Wiedmann-Franz law
- For impure metals, $\sigma\ \alpha \frac{1}{\sqrt{T}}$, where σ is the electrical resistivity and T, the absolute temperature of the given conducting material.
- For pure metals, $\sigma\ \alpha \frac{1}{T}$
- Used to explain optical properties of metals.

Drawbacks Drude Lorentz free electron theory

- Fails to give correct mathematical expression for thermal conductivity of metal.
- Fails to explain electrical conductivity of semiconductors or insulators.
- Fails to explain super conductivity and ferromagnetism.
- Fails to explain Compton Effect, photo Electric Effect, black body Radiation, and Zeemann Effect.
- Fails to explain why certain materials have negative value for Hall Coefficient.
- Theoretical value of paramagnetic (χ) susceptibility is greater than the experimental value.
- Theory : Lorentz number (L) = 1.11×10^{-8} WΩK^{-2}
  But Experiment gives the value : L = 2.3×10^{-8} WΩK^{-2}
- Theory : $\sigma \;\alpha\; (1/T^{Y2})$
  Experiment: $\sigma \;\alpha\; (1/T)$
- Theory : electronic specific heat capacity (C_v)= 3R/2

Experiment :$C_v = 10^{-4}$ R.T

- Theory: Specific heat capacity = 4.5 R
  Experiment: Specific heat capacity = 3R.

where, T-Absolute temperature of the material, R-Universal gas constant.

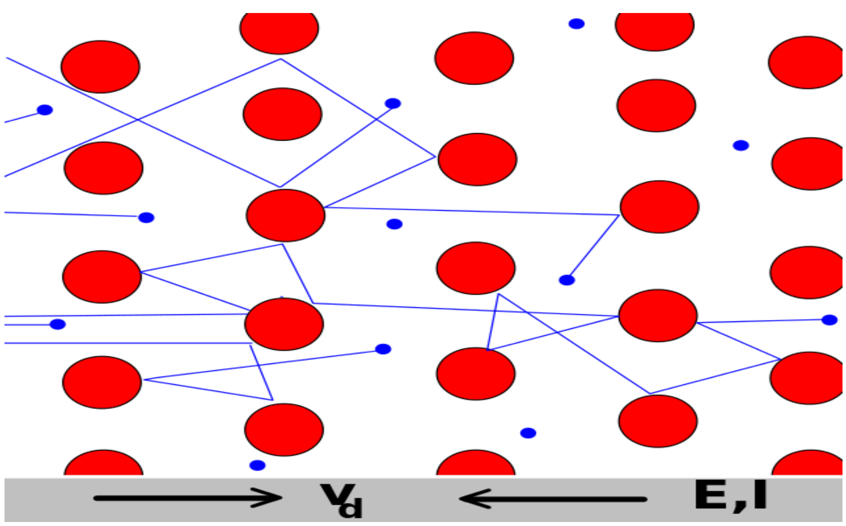


Fig.2.1. Visualization of lattice of a solid

Here Positive ions represent the Nucleus with core electrons and Valence electrons are the outermost electrons of the electron shells of atoms of the given solid.

2.1.4. Basic Terms/Keywords

- **Temperature gradient** refers to change in temperature with distance.
- **Potential gradient** refers to change in electric potential with distance.
- **Negative temperature gradient**
  Consider a metallic rod that is heated at one end then, as the distance from the hot end increases, the temperature at various sections of the rod decreases as one goes away from the hot end. This trend is referred as negative temperature gradient.
- **Drift Velocity (V_d) of a given solid.**
  It refers to the average velocity acquired by the electron between two successive collisions of the electron, in a particular direction due to the application of external electric field.

$V_d = -\frac{eE\tau}{m}$

where e-charge, Γ- relaxation time and m-mass of the electron and E-external applied electric field to the metal. The negative sign indicates that the electrons move away from the direction of external electric field.

- **Mean free path (λ) of the electron in a given solid.**

It is the average distance travelled by the electron between two successive collisions in the presence of external electric field. $\boldsymbol{\lambda} = V_d\boldsymbol{\tau}$, where $V_d$represents the root mean square velocity of the electron.

- **Mean free time or relaxation time (τ) of electron in solids.**
  It refers to the time taken by the electron to reach the equilibrium position from its disturbed position, in the presence of external electric field. It is of the order of 10^{-14} sec.
- **Mobility of an electron**
  Mobility of an electron (μ) is defined as the velocity of the free electron per unit applied electric field. Unit: $m^2v^{-1}sec^{-1}$

$\mu = \frac{\mathbf{Vd}}{E}$, Substituting for V_d, we get, $\mu = \frac{-e\Gamma}{m}$

- **Electrical resistivity (ρ)** is a measure of how strongly a material opposes the flow of electric current. The SI unit of electrical resistivity is ohm-meter (Ω–m). $R = \frac{\rho L}{A}$, where R is the electrical resistance, L is the length of the material, A is the cross – sectional area.

- **Electrical Conductivity (σ)** or specific conductanceis the reciprocal of electrical resistivity and it measures material's ability to conduct an electric current. Its SI unit is mho-meter^{-1}. $\sigma = 1/\rho$.
- **Charge density (n) and current density (J)**
  Generally, charge density refers to number of electrons per unit volume of the material and current density refers to electric current flowing per unit area of the material.
- **Various expressions for current density (J).**

$$J = I/A.$$

$$J = -nev_d$$

$$J = \sigma E.$$

  I-electric current, n-charge density, e-charge of the electron, v_d – drift velocity of the electron, σ-electrical conductivity, E-applied external electric field. The negative sign indicates that the electron moves in a direction opposite to the external electric field.
- **Phonons** are charge carriers for lattice vibrations (i.e.) lattice vibrations are quantised in terms of phonons.
- **Effective mass (**m***) refers to mass of the electron when it is in motion (i.e)** mass of the electron that is accelerated in a periodic potential.

- **Thermal velocity and drift velocity.**

Thermal Velocity	Drift Velocity
1. It is due to increase in temperature of the given material.	1. It refers to the average velocity acquired by the electron between two successive collisions of the electron, in the presence of external electric field.
2. The electrons move in random directions.	2. The electrons move in a direction opposite to the external electric field. i.e. they are directional.
3. It is of the order of 50 cm/sec.	3. It is of the order of 10^5 m/sec.

2.1.5. Comparison of electrical conduction with thermal conduction.

Thermal Conduction	Electrical Conduction
1. Heat flows from high temperature region to low temperature region.	1. Electric charge flows from high potential region to low potential region.
2. Thermal conductivity of the solid is determined both by free electrons as well as by phonons	2. Electrical conductivity of the solid is determined purely by free electrons per unit volume of the material.
3. In the case metals, thermal conductivity decreases with decrease in temperature.	3. In the case metals, electrical conductivity decreases with increase in temperature; but in semiconductors/insulators, electrical conductivity can be improved by heating and/or by doping.
4. Thermal conductivity is determined by mean free path, thermal velocity and specific heat.	4. Electrical conductivity is determined by electron density, relaxation time and the effective mass of the electron.
5. Coefficient of thermal conductivity is defined as the amount of heat flowing through unit area of the given metal in one second, maintaining unit temperature gradient. Unit :$Wm^{-1}K^{-1}$	5.Coefficient of electrical conductivity is defined as the amount of electric charge flowing through unit area of the given metal in one second maintaining unit potential gradient. Unit: $Ohm^{-1}m^{-1}$.
6. K= $-\frac{1}{2}nv_d k_B(v_d\tau_r)$	6. $\sigma = \frac{ne^2\tau_r}{m}$

2.1.6. Derivation of coefficient of thermal conductivity (K)

Coefficient of thermal conductivity (K) is defined as the amount of heat energy flowing through unit area of the given conductor in unit time, maintaining unit temperature gradient. Unit: $Wm^{-1}K^{-1}$

Consider a uniform metallic rod (AB) which is heated at one end A. Hence heat flows from the hot end 'A' to the other end 'B'. Let the temperatures at the ends of the conductor AB be T1 and T_2 respectively, where $T_1 > T_2$. Now consider a section 'C' which is at a distance equal to the mean free path of the electron between the ends A and B of the rod as shown in Figure 2.1. Hence length of the rod = 2λ.

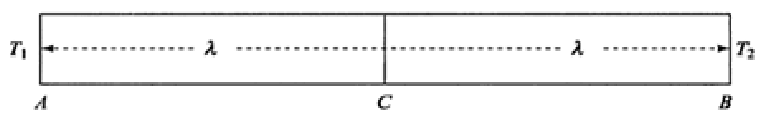


Fig.2.2. Thermal conduction in a metallic rod.

The amount of heat (Q) conducted by the rod from the end A to B of length 2λ, having the area of cross section as A, in time t, maintaining the temperature difference as $(T_1 - T_2)$ is given by

$$Q \propto \frac{A(T_1 - T_2)}{2\lambda} t \quad \text{(or)}$$

$$Q = -KA\frac{(T_1 - T_2)}{2\lambda} t \text{ ----------- (1)}$$

where K- Coefficient of thermal conductivity and the negative sign indicates the negative temperature gradient i.e. temperature decreases as distance from the hot end increases.

From equation (1) we can write

Coefficient of thermal conductivity through unit area of the given metallic rod per unit time is given by

$$K = \frac{Q}{(T_1 - T_2)} 2\lambda \text{ ----------- (2)}$$

Let us assume that there is equal probability for the electrons to move in all the six directions as shown in Fig.2. Since each electron travels with thermal velocity 'v' and if 'n' is the free electron density (number of electrons per unit volume of the material), then on an average, $\frac{1}{6}nv$, number of electrons will travel in any one direction.

Therefore, the number of electrons crossing through unit area of the section 'C' in unit time is given by

$$= \frac{1}{6}nv \text{-------- (3)}$$

According to kinetic theory of gases, free electrons are assumed to move like gas molecules and the average kinetic energy of an electron at the hot end ‘A’ of temperature (T_1) $= \frac{3}{2} k_B T_1$

Similarly, the average kinetic energy of an electron at cold end ‘B’ of temperature (T_2) $= \frac{3}{2} k_B T_2$, where $k_B$is the Boltzmann constant $=1.380*10^{-23}$ J/K

Therefore now, the amount of heat energy transferred per unit area per unit time from end A to B across the section C is given by,

= Number of electrons x Average K.E. of electron moving from A to B

$= \frac{1}{6} nv \times \frac{3}{2} k_B T_1$

$= \frac{1}{4} n k_B T_1$------------------- (4)

At the same time, some of the electrons will move from ‘B’ to ‘A’. Hence the amount of heat energy transferred per unit area per unit time from the end B to A across the section C is given by $= \frac{1}{4} nv k_B T_2$--------------------- (5)

Therefore, the net amount of heat energy transferred from A to B per unit area per unit time (Q) across the section ‘C’ could be obtained by subtracting equation (5) from equation (4)

$= \frac{1}{4} nv\, k_B T_1 - \frac{1}{4} nv\, k_B T_2$

$= \frac{1}{4} nv k_B (T_1 - T_2)$ ---------------- (6)

Substituting equation (6) in equation (2) we have

Thermal conductivity $K= \frac{1}{4} \frac{nv\, k_B\, (T1-T2)}{(T_1 - T_2)} 2\lambda$

$K= \frac{nVk_B}{2} \lambda$ -------------- (7)

We know for metals. Relaxation time(τ) = Collision time (τ_c)

$\tau = \tau_c = \frac{\lambda}{V}$

$\lambda = \tau v$ ---------------- (8)

Substituting equation (8) in equation (7) we have $K = \frac{1}{2}$ nv $k_B \tau v$

Therefore, Coefficient of Thermal conductivity is given by,

$$\boxed{K= \frac{1}{2} nv^2 k_B \tau}$$

2.1.7. Derivation of coefficient of electrical conductivity (σ)

Coefficient of electrical conductivity (σ) is defined as the quantity of electricity flowing through unit area of the given conductor in unit time, maintaining unit potential gradient. Unit : $\Omega^{-1}m^{-1}$

In the absence of external electric field, the motion of electrons in a metal moves randomly in all directions. When an electric field (i.e.,) potential difference is maintained between the two ends of a metallic rod, the electrons will move towards the positive field direction and produces the current in the metallic rod.

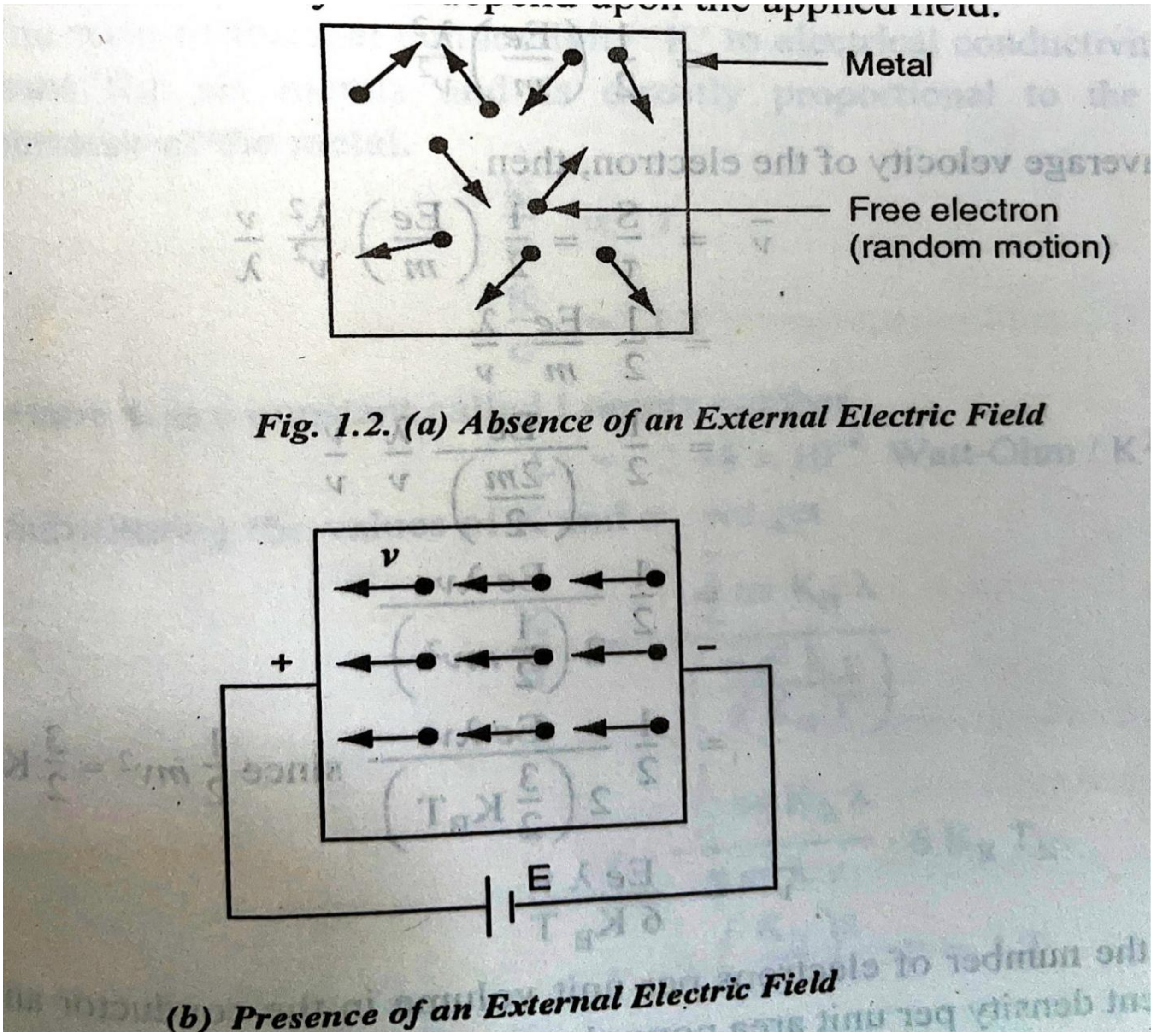


Fig.2.3. Motion of electrons in the absence of external electric field and in the presence of external electric field.

If 'n' is the free electron density and 'e' is the charge of electron then the current density (i.e) the current flowing through unit area is given by

$$J = nv_d(-e) \text{----------------------------}(1)$$

The –ve sign implies that the charge of the electron is negative and it also indicates that the conventional direction of current is in the opposite direction to the electron movement.

Due to the applied electric field, the electron gains the acceleration 'a'

$$\text{(i.e.,)Acceleration (a)} = \frac{\text{Drift velocity } (V_d)}{\text{Relaxation time } (\tau)}$$

$$v_d = a\tau \text{-------------------------}(2)$$

If E is the electric field intensity and 'm' is the mass of the electron. Then, the force experienced by the electron is $F = -eE$ ----------(3)

From Newton's Second law of motion,

The force experienced by the electron is $F = ma$ ---------- (4)

Using equations (3) and (4) we can have

$$F = -eE = ma \qquad \text{(or)}$$

$$a = \frac{-eE}{m} \text{ -------------- } (5)$$

Substituting equation (5) in equation (2) we have

$$\text{Drift Velocity } v_d = \frac{-eE}{m}\tau \text{----------- } (6)$$

Substituting equation (6) in equation (1) we have

$$\text{Current density } J = n\,(-e)\frac{-eE}{m}\tau \qquad \text{(or)}$$

$$J = \frac{ne^2\tau E}{m} \text{ ------------ } (7)$$

Here the number of electrons flowing per second through unit area (i.,e)., the current density depends on the applied electric field. Hence, if the applied field (E) is more, the current density (J) will also be more. Therefore, we can write $J \propto E$

(or) $\qquad J = \sigma E$ ------------ (8)

Comparing equations (7) and (8) we can write.

$$\sigma E = \frac{ne^2\tau E}{m} \qquad \text{(or)}$$

$$\boxed{\sigma = \frac{ne^2\tau}{m}}$$

2.1.8. Wiedemann –Franz law and Lorentz number

Statement: The ratio of coefficient of thermal conductivity (K) to the coefficient of electrical conductivity (σ) of a given metal is directly proportional to the absolute temperature (T) of the metal.

$$\text{(i.e)} \ \frac{K}{\sigma} \propto T$$

$$\frac{K}{\sigma} = LT$$

where L is a constant called as Lorentz number whose value is $2.44\text{x}10^{-8}$ (Quantum mechanical value) at temperature T=293 k.

Proof: By Classical theory

We know electrical conductivity (from classical theory) $\sigma = \frac{ne^2\tau}{m}$

Thermal conductivity (From classical theory) $K = \frac{1}{2} nv^2 k_B \tau$

$$\frac{K}{\sigma} = \frac{\frac{1}{2} nv^2 k_B \tau}{\frac{ne^2\tau}{m}}$$

$$\frac{K}{\sigma} = \frac{v^2 k_B m}{2e^2} = \frac{mv^2 k_B}{2e^2}$$

We know kinetic energy of an electron $= \frac{1}{2}mv^2 = 3/2 \ k_B T$

Substituting this in equation (1) we can write

$$\frac{K}{\sigma} = \frac{3/2 \ k_B T k_B}{e^2}$$

$$\frac{K}{\sigma} = \frac{3}{2}\left(\frac{k_B}{e}\right)^2 T \quad = LT \text{ where } L = \frac{3}{2}\left(\frac{k_B}{e}\right)^2$$

Substituting the value of Boltzmann constant $k_B = 1.38 \times 10^{-23} JK^{-1}$

Charge of electron e= 1.6021×10^{-19} Joules .we get

$L = 1.12 \times 10^{-8} \ W\Omega K^{-2}$

It is found that the classical value of Lorentz number, is only half of the experimental value (i.e) $2.44\text{x}10^{-8} \ W\Omega K^{-2}$

This Discrepancy in the experimental and theoretical value of 'L' is the failure of classical theory. This discrepancy can be rectified by quantum theory.

By Quantum theory

In Quantum theory the mass of the electron (m) is replaced by the effective mass (m*). Therefore, the electrical conductivity $\sigma = \frac{ne^2\tau}{m^*}$

Rearranging the expression for thermal conductivity and substituting the electronic specific heat, the thermal conductivity can be written as $K = \frac{\pi^2}{3}[\frac{nK^2{}_B}{m^*}\tau]T$

$$\frac{K}{\sigma} = \frac{\frac{\pi^2}{3}[\frac{nK^2{}_B}{m^*}\tau]T}{\frac{ne^2\tau}{m^*}}$$

$$\frac{K}{\sigma} = \frac{\pi^2}{3}[\frac{K^2{}_B}{e^2}]\ T$$

$\frac{K}{\sigma}$=LT Where $L = \frac{\pi^2}{3}[\frac{K^2{}_B}{e^2}]$

Substituting the values for Boltzmann constant (K) and the charge of the electron (e) we get, $L = 2.44 \times 10^{-8}\ W\Omega K^{-2}$

Thus quantum theory verifies Wiedeman- Franz law and has good agreement with the experimental value of Lorentz number.

<u>Verification of Ohm's law</u>

According to classical free electron theory

Current density $J = \sigma E$

Let us consider a conductor of length l and area of cross-section A

If dV is the voltage passing through the small length *dx*, then electric field $E = \frac{dV}{dx}$

Current density J is the current passing through the conductor per unit area (A)

$$J = \frac{I}{A} = \sigma\frac{dV}{dx}$$

$$I\,dx = \sigma A\, dV$$

Integrating the above equation $I\int_0^l dx = \sigma A\int_0^V dV$

$Il = \sigma AV$

$V = \frac{l}{\sigma A} I$

Conductivity (σ) $= \frac{1}{Resistivity(\rho)}$

Therefore $V = I \frac{\rho l}{A}$

$\rho l A$ are constant for any particular conducting material . $\frac{\rho l}{A} = R$

Thus V=IR, Ohms law is verified.

2.1.9 Band theory of solids in brief

- When a number of atoms are brought close together to form a solid, the valance electrons are acted upon first by, the electrons and nuclei of the adjacent atoms. Hence the electronic wave functions overlap and the discrete energy levels of atoms get split into what are called as bands, which consist of electron energy states that are separated between each other by a very small energy difference of the order of 10^{-19}ev. This makes the band to be considered as though it contains almost a continuous range of energies.
- Electron energy level, whose orbital quantum number 'l', splits into (2l+1)N energy states. Therefore, s-band (l = 0) split into (2*0+1)N states, i.e. N states; p-band (l =1) split into (2*1+1)N states, i.e. into 3N states; each state has 2 electrons (spin up and down).Therefore, s-band consists of 2N electrons and p-band consists of 6N electrons. Here N represents the number of atoms.
- While occupying a band, electrons start from the lowest energy levels and fill the levels one after the other in the ascending order of energy. The energy of the highest filled electron energy state at zero Kelvin is called the **Fermi energy (E_F)** and the corresponding energy level is known as **Fermi Level.**
- The band that is occupied by the core electrons is called as the **core electronband** which is completely filled. (core electrons refer to the electrons which are more stable and occupy the energy states closer to the nucleus-closely bound to the nucleus). The band that contains the highest energy electrons or valance electrons (loosely bound to the nucleus) is termed as the **valance band**which is generally half-filled; the next higher energy band is called as the **conduction band**which is unfilled**. Forbidden energy**

gap is formed between the completely filled lower band and half-filled upper band. It should be remembered thatonly when there is movement of electrons in the conduction band, the material is said to be a conductor.

- Based on band theory, one can classify the given material as conducting, insulating and semi-conducting material.

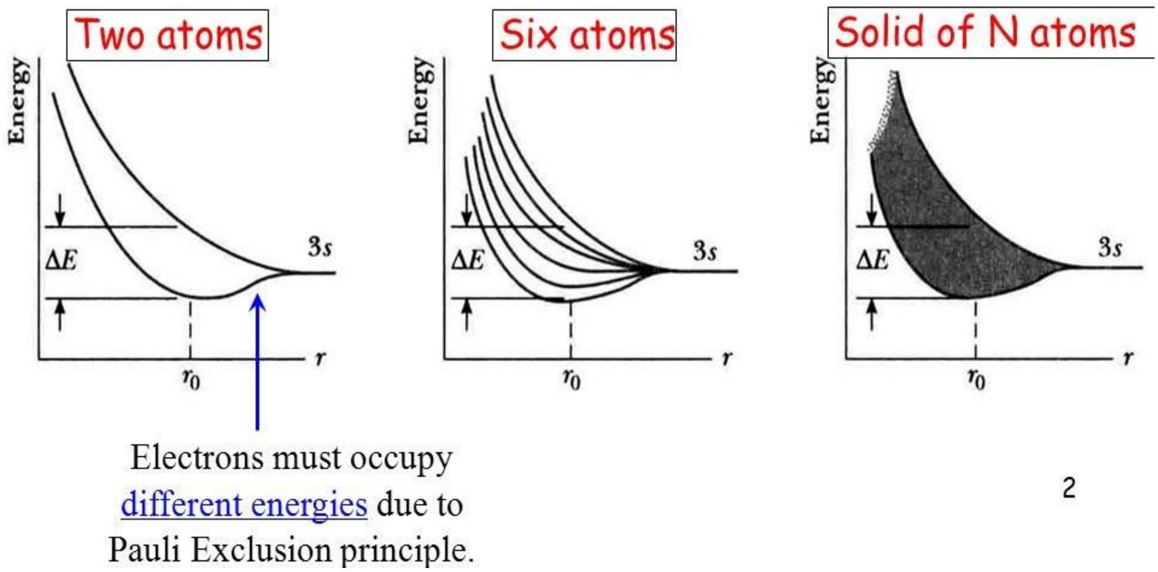


Fig. 2.4.Band Theory

2.1.10. Fermi Energy, Fermi Surface and its Importance

Top most filled electron energy state of a material at absolute zero (T=0 K) is called as the **Fermi energy (E_F) level**. It serves as the reference level that separates all the filled states from the vacant states at T=0 K. It separates the filledenergy level and empty energy levels.

Fermi distribution function F(E) represents the probability of an electron occupying a given energy state.

$$F(E) = \frac{1}{1 + \exp\left(\frac{E - E_F}{KT}\right)}$$

where E - energy of an electron, E_F - Fermi energy and K - Boltzmann constant.

Fermi sphere or Fermi surface is defined as a surface which is traced out by joining the lociof the end points of the wavevector $\vec{k}$ corresponding to the Fermi energy level.

Significance of Fermi Energy Level

- Fermi Energy is defined only at absolute zero whereas Fermi Energy level could be defined for any temperature of the given material.
- The concept of Fermi Energy is crucially important in understanding the electrical and thermal properties of solids.
- In other contexts, Fermi Energy Level is often referred as Electron Chemical Potential.
- In metals, the value of Fermi Energy gives an idea about the velocities of electrons that could participate in ordinary electrical conduction. This energy will be of the order of few micro eV, hence only those electrons that are very close to the Fermi Energy can take part in electrical conduction. i.e. density of conduction electrons can be implied from the Fermi Energy.
- Fermi Energy Level (E_F) plays an important role in band theory of solids. In p-type and n-type semiconductors, E_F is shifted by the impurities.
- In insulators, Fermi energy level lies at the centre of the forbidden energy gap. But in semiconductors, it lies in the small energy gap. In conductors, it occupies a location in the permitted band, since the valence and conduction band overlap without an energy gap.

2.1.11. Qualitative analysis of conductors, semiconductors & insulators based on band theory

The electrical conduction properties of different elements and compounds canbe explained in terms of the electrons having energies in the valence and conduction bands. The electrons lying in the lower energy bands, which are normally filled, play no part in the conduction process.

- **Insulator** is a solid which does not contain any free electrons in their conduction band. i.e. it has a filled valence band, empty conduction band and the width of the forbidden energy gap is maximum as shown in Fig.2.5(a) which is > 3eV. Therefore they do not conduct electricity.For conduction to take place, electrons must be given sufficient energy to jump from the valence band to the conduction band. Increase in temperature enables some electrons to go to the conduction band which in fact accounts for the negative resistance – temperature coefficient of insulators. In the case of insulators, only at very high temperature, the thermal energy will be sufficient to raise the electrons from valence band to conduction band. Therefore at high temperatures even insulators can conduct electric current. **Insulators have electrical resistivity of the order of 10^8 to 10^{18} Ωcm**

Semiconductor is a solid whose electrical properties lie in between those of insulators and good conductors. It contains fewer electrons in the conduction band. The width of the forbidden energy gap is small < 2 eV as shown in Fig.2.5(b).In terms of energy band, semiconductors can be defined as those materials which have almost an empty conduction band and almost filled valence band with a very narrow energy gap (of the order of 2 eV) separating the two as shown in Figure 2.5 (b). Hence, the electrons require small energy to jump from valence band to conduction band. This energy may be in the form of heat or light. Even at room temperature, the thermal energy is sufficient to transfer electrons from valence band to conduction band. But when the semiconductor is at zero Kelvin, the thermal energy is not sufficient to transfer the electrons from the valence band to conduction band. Semiconductors at 0K behave like insulators. In semiconductors the resistance decreases with increase in temperature. **Semiconductors have electrical resistivity of the order of 10^{-4} to 10^8 Ωcm.**

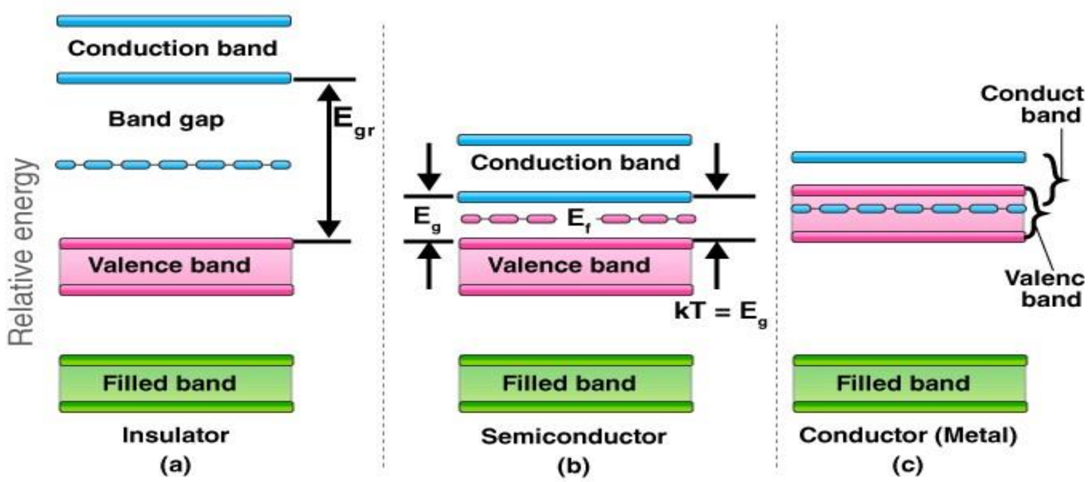


Fig.2.5. Classification of solids based on band structure

- **Conductor** is a solid which contains plenty of free electrons that are available for electrical conduction. in the conduction band. In terms of energy bands, it means that electrical conductors are those which have overlapping valence and conduction bands as shown in Figure 2.5(c) and hence the width of the forbidden energy gap is zero. In fact, there is no physical distinction between the two bands, hence, the availability of a large number of conduction electrons. Therefore the free electrons can easily move from the valence band to conduction band, and are available for electrical conduction under the action of an electric field. **Conductors have electrical resistivity on the order of 10^{-8} to 10^{-4} Ωcm.**

2.1.12. Some Important Electrical Materials

Important conducting materials

Conductors are materials that have high electrical conductivity and allow the flow of electrical current with low resistance. They are essential components in the electronics industry and are used in a wide range of applications, including electrical wiring, transformers, and electronic components. Here are some important conductors:

Copper: Copper is the most widely used conductor material and is used in electrical wiring, motors, transformers, and many other applications. It has high electrical conductivity, excellent thermal conductivity, and is relatively inexpensive.

Aluminum: Aluminum is another commonly used conductor material and is used in electrical wiring, power transmission lines, and many other applications. It has a high electrical conductivity and is lighter in weight than copper, making it useful in applications where weight is a concern.

Silver: Silver is the most conductive metal and has the highest electrical conductivity of any element. It is used in applications where high conductivity is critical, including electrical contacts and high-performance audio cables. **Gold:** Gold is also highly conductive and is used in electrical contacts and connectors, especially in applications where corrosion resistance is important. **Platinum:** Platinum is a highly conductive metal and is used in electrical contacts, spark plugs, and other applications where high temperature resistance and corrosion resistance are required.

Overall, conductors play an essential role in the electronics industry, and ongoing research is focused on developing new materials with even better conductivity properties for use in advanced electronic and electrical applications.

Important Semiconducting materials

Semiconductors are materials that have electrical conductivity between conductors (metals) and non-conductors (insulators). They are essential components in the electronics industry and are used in a wide range of applications, including computer chips, solar cells, LEDs, and transistors.

Here are some important semiconductors:

Gallium arsenide (GaAs): Gallium arsenide is a compound semiconductor that is used in high-speed electronic devices, including microwave amplifiers, solar cells, and LEDs. It has a higher electron mobility than silicon and can operate at higher frequencies.

Indium phosphide (InP): Indium phosphide is another compound semiconductor that is used in high-speed electronic devices, including laser diodes, high-speed transistors, and solar cells.

Cadmium telluride (CdTe): Cadmium telluride is a semiconductor material that is used in thin-film solar cells. It has a high absorption coefficient, which makes it efficient at converting sunlight into electricity.

Gallium nitride (GaN): Gallium nitride is a wide-bandgap semiconductor that is used in high-power electronic devices, including LEDs, power amplifiers, and radio-frequency devices.

Silicon carbide (SiC): Silicon carbide is a wide-bandgap semiconductor that is used in high-temperature and high-voltage applications, including power electronics, LEDs, and solar cells. It has a higher thermal conductivity than silicon, which makes it suitable for high-temperature applications.

Important insulating materials

Insulators are materials that do not conduct electricity easily, and they are used to prevent the flow of electrical current between conductors. Insulators are essential components in the electronics industry and are used in a wide range of applications, including electrical wiring, transformers, and electronic components. Here are some important insulators:

Glass: Glass is an excellent insulator and is used in the construction of electronic components, such as vacuum tubes and CRT displays..

Teflon: Teflon is a polymer that is widely used as an insulator in electrical wiring, coaxial cables, and other electronic components. It has excellent electrical insulation properties and can withstand high temperatures and harsh environments.

Rubber: Rubber is an excellent insulator and is used in electrical wiring, power cables, and other applications that require insulation from electrical current.

Mica: Mica is a mineral that is widely used as an insulator in electronic components. It has excellent electrical and thermal insulation properties, making it ideal for use in high-temperature and high-voltage applications.

PVC: Polyvinyl chloride (PVC) is a widely used polymer that is used as an insulator in electrical wiring and cables. It has excellent electrical insulation properties and is resistant to moisture and chemicals.

Overall, insulators play an essential role in the electronics industry, and ongoing research is focused on developing new materials with even better insulation properties for use in advanced electronic and electrical applications.

Important magnetic materials

Magnetic materials are materials that exhibit magnetic properties such as attraction or repulsion in the presence of an external magnetic field. They have a wide range of applications, including data storage, motors, generators, and sensors. Here are some important magnetic materials:

Iron: Iron is one of the most commonly used magnetic materials and is used in the construction of motors, generators, and transformers. It has a strong magnetic field and can be easily magnetized.

Cobalt: Cobalt is another commonly used magnetic material and is used in the production of high-performance magnets. It has a high magnetic anisotropy, which means it has a preferred direction of magnetization.

Nickel: Nickel is a magnetic material that is used in the production of alloys, such as alnico magnets, which have a high magnetic strength and temperature stability.

Neodymium: Neodymium is a rare-earth metal that is used in the production of strong magnets, such as those used in hard disk drives, electric motors, and wind turbines.

Ferrite: Ferrite is a ceramic magnetic material that is used in the production of permanent magnets, as well as in applications such as microwave devices and transformers.

Samarium-cobalt: Samarium-cobalt is another rare-earth magnetic material that is used in high-temperature applications, such as in motors and generators that operate at high temperatures.

Permalloy: Permalloy is a nickel-iron alloy that exhibits strong magnetic properties and is used in applications such as magnetic shielding and transformers.

Overall, magnetic materials play a crucial role in many technologies, and ongoing research is focused on developing new materials with even better magnetic properties for use in advanced electronic and electrical applications.

Important superconducting materials

Superconducting materials are materials that exhibit zero electrical resistance and perfect diamagnetism below a certain critical temperature. They have a wide range of applications, including magnetic levitation, high-speed computing, and energy transmission. Here are some important superconducting materials:

Niobium-titanium (NbTi): NbTi is a superconducting material that is widely used in the production of superconducting magnets for applications such as magnetic resonance imaging (MRI) machines and particle accelerators. It has a critical temperature of around 10 K.

Niobium-tin (Nb3Sn): Nb3Sn is a superconducting material that has a higher critical temperature than NbTi, around 18 K. It is used in the production of high-field superconducting magnets for applications such as fusion reactors and particle accelerators.

High-temperature superconductors (HTS): HTS are a class of superconducting materials that exhibit superconductivity at temperatures above 30 K. They are used in a variety of applications, including power transmission, magnetic levitation, and energy storage. Examples of HTS materials include Yttrium barium copper oxide (YBCO) and Bismuth strontium calcium copper oxide (BSCCO).

Magnesium diboride (MgB2): MgB2 is a relatively new superconducting material that has a critical temperature of around 39 K. It is used in applications such as superconducting cables and motors.

Iron-based superconductors: Iron-based superconductors are a relatively new class of superconducting materials that exhibit high critical temperatures (up to 56 K) and could have potential applications in a range of fields, including energy transmission and computingOverall, superconducting materials are an important area of research, and ongoing research is focused on developing new materials with even higher critical temperatures and better performance for use in advanced electronic and electrical applications.

Yttrium barium copper oxide (YBCO): YBCO is one of the most widely studied and used high Tc superconductors. It has a critical temperature of around 90 K (-183 °C) and is used in a variety of applications, including power transmission, magnetic levitation, and energy storage.

Bismuth strontium calcium copper oxide (BSCCO): BSCCO is another well-known high Tc superconductor. It has a critical temperature of around 107 K (-166 °C) and is used in a range of applications, including superconducting magnets and high-speed computing.

Mercury-based cuprates: Mercury-based cuprates are a class of high Tc superconductors that exhibit critical temperatures as high as 135 K (-138 °C). They are currently being studied for potential applications in energy transmission and computing.

Iron-based superconductors: Iron-based superconductors are a relatively new class of high Tc superconductors that exhibit critical temperatures as high as 56 K (-217 °C). They could have potential applications in a range of fields, including energy transmission and computing.

Overall, high Tc superconductors have the potential to revolutionize a range of fields, from energy transmission to computing. Ongoing research is focused on developing new high Tc superconducting materials with even higher critical temperatures and better performance for use in advanced electronic and electrical applications.

Important dielectric materials

Dielectric materials are materials that do not conduct electricity easily and are used in a variety of applications, including capacitors, insulators, and electronic components. Here are some important dielectric materials:

Ceramic materials: Ceramic materials are a type of dielectric material that are widely used in electronic components, such as capacitors and resonators. They are highly stable and have a low loss factor, making them suitable for high-frequency applications.

Polymers: Polymers are another important class of dielectric materials that are used in a wide range of applications, including insulation for electrical wires and cables. They are highly flexible and have excellent electrical insulating properties.

Glass: Glass is a commonly used dielectric material in electronics, particularly in display technologies such as LCD screens. It has a high dielectric constant, making it suitable for use in capacitors.

Teflon (polytetrafluoroethylene): Teflon is a high-performance polymer that has excellent dielectric properties, including a low dielectric constant and high electrical resistance. It is commonly used in high-frequency applications, such as coaxial cables and microwave components.

Silicon dioxide (SiO2): Silicon dioxide is a widely used dielectric material in the semiconductor industry. It is used as an insulator in transistors and other electronic components due to its high dielectric strength and low thermal expansion coefficient.

Overall, dielectric materials are an important component of many electronic and electrical systems, and ongoing research is focused on developing new materials with even better dielectric properties for use in advanced applications.

2. 2.SUPERCONDUCTORS

2.2.1. Introduction

Consider the Fig. 2.6 that shows the variation of electrical resistivity with temperature for normal and super conductor. From this graph, one can find that the electrical resistivity of normal conductor decreases gradually with decrease in temperature and reaches a low but measurable resistivity value even at zero Kelvin; whereas there are few materials for which, initially there is decrease in electrical resistivity with decrease in temperature but however at one finite temperature, there is abruptdrop in their electrical resistivity to zero.

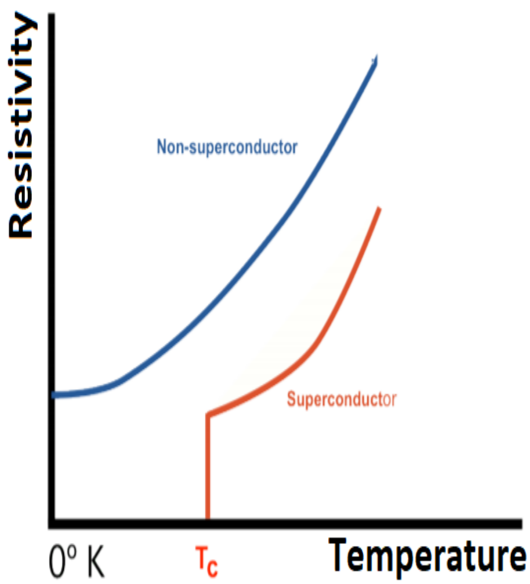


Fig. 2.6. Variation of electrical resistivity with temperature for normal and super conductor

Hence one can define, that there are certain materials where upon cooling, there is abrupt drop in their electrical resistivity from a finite value to zero and remains there, upon further cooling. Such materials are called as **superconductors.** The phenomenon of losing the electrical resistivity to zero from a finite value when cooled to a sufficiently low temperature is called as **superconductivity.** The temperature at which there is abrupt drop in electrical resistivity to zero value is called as superconducting **transition temperature or critical temperature(T_c).** At Tc, the material gets transformed from a normal state to the superconducting state. Therefore the given superconducting material exhibits infinite electrical conductivity for all temperatures that are less than its transition temperature. Some examples of superconducting materials are shown in the below table.

Element/Compound	Transition Temperature (T_c) in Kelvin
Mercury (Hg)	4.2
Lead (Pb)	7
Aluminium	1.175
Zinc	0.85
Niobium Nitride (NbN)	16

2.2.2. First occurrence of Superconductivity

The superconducting phenomenon was first observed by KammerlinghOnnes in 1911, when he tried to study the electrical properties of solid mercury at very low temperature, say at the boiling point of liquid helium (4.2 K). At this temperature, he found that the electrical resistivity of mercury drops to 10^5 times less than the initial value i.e. at this temperature mercury behaves as a superconductor.

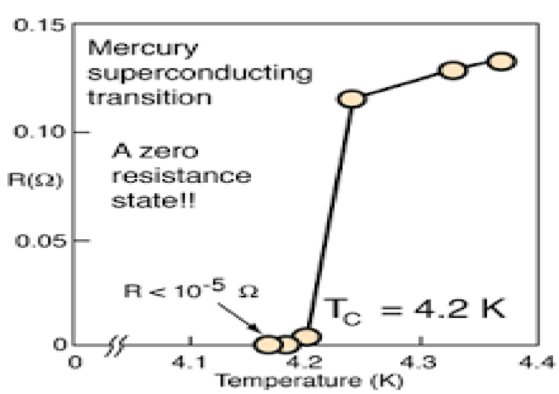


Fig.2.7.Variation of electrical resistivity with temperature for mercury

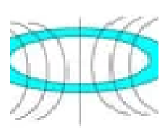
2.2.3. Properties of superconducting materials

(i) Zero electrical resistivity (ie) infinite conductivity ($\rho=0$ for all $T < Tc$)

The dc electrical resistance of a superconductor at all temperatures below a critical temperature (Tc) is practically zero. The transition from normal state to superconducting state occurs sharply in pure metals whereas it is not so in impure, deformed HTSC oxides(cuprates). In HTSC oxides there exist several superconducting phase

(ii) Persistent current

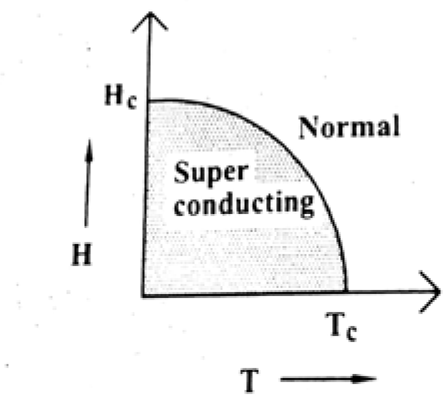
Consider a small amount of current is applied to a superconducting ring. The superconducting current will keep on flowing through the ring without any change in its value. Such a current is said to be a persistent current (eg) Dr. Collins experimentally observed that the current in the superconducting ring does not change for more than $2\frac{1}{2}$years and the ratio of the resistance of the material in the superconducting state (Rs) to the resistance of the same material in the normal state (R_n) is less than 10^{-5}(ie) $\frac{R_s}{R_n}<10^{-5}$.



(iii) Effect of external magnetic field

When super conducting materials are subjected to very large value of magnetic field, the super conducting property is destroyed. The field required to destroy the super conducting property is called as the **critical magnetic field (H_c)** given as

$H_c(T) = H_0\ (1 - \frac{T^2}{Tc^2})$, where $H_0$and $H_c(T)$represents the critical magnetic field at 0 Kelvin and at T Kelvin respectively; T_c the transition temperature.

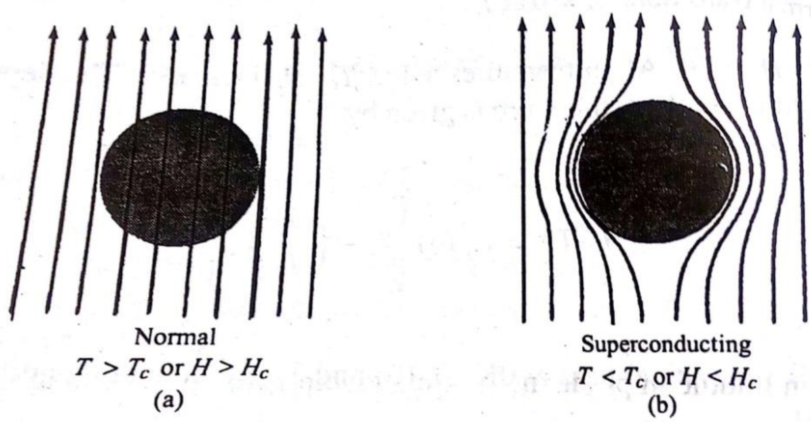


From the above figure, we can find that when the temperature of the material increases, the value of the critical magnetic field decreases.

(iv) Meissner Effect(B=0 inside the superconducting Specimen)

When a magnetic field is applied to a normal conducting material, the magnetic lines of force penetrate through the material

When a superconducting material is kept in an external magnetic field under the condition when T<Tc and H<Hc, the magnetic flux lines are completely excluded from the material and this phenomenon is known as Meissner effect.



Superconductors exhibit perfect diamagnetism

We know that a diamagnetic material have the tendency to expel the magnetic lines of forces. Since the superconductor also expels the magnetic lines of forces, it behaves as a perfect diamagnet.

Proof

In general, the magnetic flux density is given by, $B=\mu_0(M+H)$, where, μ_0- magnetic permeability of the material at free space, M-intensity of magnetization and H- Magnetizing field. For a superconductor, according to Meissiner effect, B=0 inside the specimen. Therefore substituting B = 0 in the above equation, we get,

$B = \mu_0 (M+H)$ becomes

$0 =\mu_0 (M+H)$ (or)

$M = - H$ (since μ_0 cannot be equal to zero)

Therefore the magnetic susceptibility (χ) which is given by $\chi = M/H = -H/H = -1$.

$$\boxed{\chi = -1}$$

The negative value of magnetic susceptibility shows that a superconductor exhibits perfect diamagnetism.

(v).Critical current (or) Silsbee current (I_c)

It is known that a current carrying conductor has a magnetic field around it. Therefore, the critical magnetic field (H_c) can be obtained in terms of heavy current that is passed through the given superconducting coil. The heavy current required to destroy the superconducting property is called as the critical current (I_c) and is given by

$$\boxed{I_c=2\pi rHc}$$

This equation is called as silsbee's law, where 'r' is the radius of the superconducting coil.

(vi)Isotope Effect:

The transition temperature of the given superconducting material varies with its mass number and this effect is known as Isotope effect.

Maxwell showed that **$T_c.M^{\alpha}$ = constant,** where T_c is the transition temperature and Mis the atomic mass of the isotope of the given superconducting material;**α**is roughly equal to 0.5.

Example:

Mass number (M) of mercury varies from 199.5 to 203.4 and the corresponding transition temperature (Tc) varies from 4.146 kelvin to4.185 kelvin respectively.

Mass Number of Hg	T_c
199.5	4.146 K
203.4	4.185 K

$T_c.M^{0.5} = (4.146)\,(199.5)^{0.5} = 58.559$

$T_c.M^{0.5} = (4.185)\,(203.4)^{0.5} = 59.685$

$T_c.M^{x}$= constant.

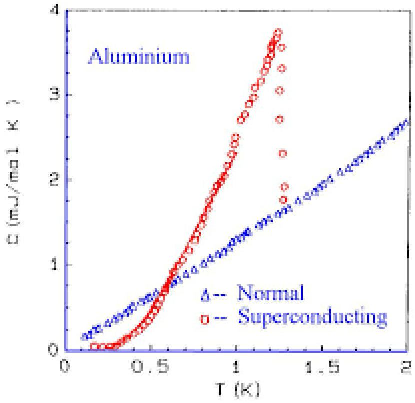
Obviously the existence of isotope effect indicates that although superconductivity is an electronic phenomenon it depends on the vibrations of the crystal lattice (i.e.) positive ions are involved in the electrical conductivity of the superconducting material.

(vii) Specific heat capacity

The specific heat capacity of a normal conductor (C_n) consists of two contributions, one from the conduction band electrons and the other from the lattice.$C_n= C_n^e+ C_n^l$where 'n' stands for normal conductor, 'e' for elections and I for lattice. (Or) $C_n = \gamma T+ \beta T^3$, where γT arises from electron contribution and βT^3arises from lattice contribution.Consider the specific heat capacity of a superconductor (C_s) which varies in a characteristic way at T_c. It is given by

$$\mathbf{C_s\ (T<Tc) = A.e^{-\Delta/K_B.T}}$$

The variation of specific heat with temperature for normal and superconductor is shown in the below figure.



(ie) there appears a discontinuity in specific heat at T_c, K_B-Boltzmann constant T-temperature of the specimen, Δ- a gap in the spectrum of allowed energy states separating the excited states from the ground state.

(viii). Entropy

Entropy is a measure of degree of disorderness of the given system. The entropy of a superconductor material is found to be lower than that of the normal conducting material which indicates that in the superconducting state the electrons are in the more ordered state than they are in the normal conducting state.

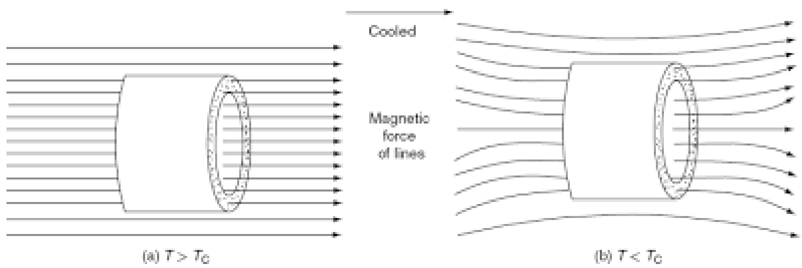
(ix).Flux Quantization

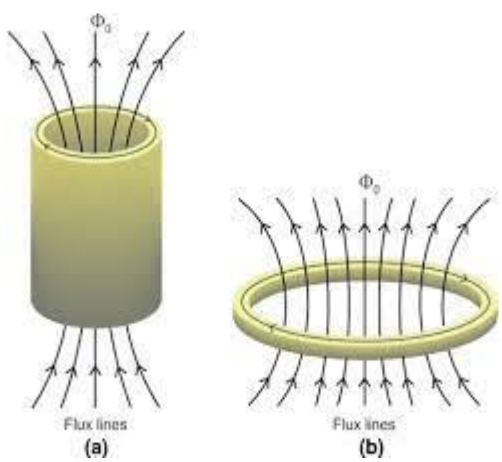
The magnetic flux that threading a superconducting (or) a hollow superconducting cylinder cannot have an arbitrary value but it should be quantized in terms of h/2e. This phenomenon is said to be **flux quantization.** In 1957, A.A.Abrikosov, predicted the existence of magnetic flux quanta. This has been confirmed experimentally by Dearer and Fairbank in 1961. The value of flux quantum (or) fluxoid (Φ) is equal to $\Phi_0 = 2.0678\ X10^{-15}$ weber.

One flux quantum / one fluxoid / one fluxon is given by

$\Phi_0=h/2e$, where
h –Planck's constant,
2e – charge of the cooper pair,
Therefore, $\Phi_0 = 6.6x\ 10^{-34}/\ (2x\ 1.6x10^{-19}) = 2.0678x\ 10^{-15}$ Weber.

It is known that the superconducting ring has persistent current. Hence it can be compared with Bohr's non-radiative orbit. According to Bohr atom model, the electrons orbits' are quantised. Therefore, the supercurrent or persistent current in a ring may also be considered as a macroscopic quantized orbit. The quantized orbit naturally produces a quantized magnetic field. Magnetic flux lines produced in ordinary transformer or solenoid coil is not quantized. Hence quantization of magnetic flux lines is a special property of superconductors. This property is applied in SQUID. Super conducting devices can measure this tiny variation of magnetic flux which is exceedingly important in metrology and advanced instrumentation.





Flux lines are trapped when T <Tc and H=o.

(x).Effect of pressure.

Some materials which behave as normal conductors at normal pressure, can undergo phase transformation and be have as superconductor under the application of external pressure.

Examples:

a. cesium becomes a superconductor under a pressure of 110 Kilo bar and at Tc=1.5kelvin
b. Silicon becomes a superconductor at 165 kilo bar pressure and its Tc =8.3kelvin

(xi)Others

- Generally good electrical conductors are not good superconductors (e.g) copper, gold
- The superconducting property of a material is a function of its crystal structure:
  White tin which is in tetragonal crystal structure is a superconductor, whereas the grey cubic tin is not a superconductor.Beryllium will become superconductor when it is in the form of thin films. But Bismuth will become superconductor when it is in bulk form and simultaneously pressure needs to be applied on it.

2.2.4. Types of superconductors

There are two types of superconductors namely Type I and Type II.

<u>Type I superconductor</u> :

When a superconductor is placed in an external magnetic field (H)whose strength is getting increased andif the superconductor becomes a normal conductor **abruptly** at one external magnetic field (called as critical magnetic field H_c), then such type of superconductors are called as **Type I or soft superconductors**.i.e. above H_c, the specimen is a normal conductor and below $H_c$the specimen excludes all magnetic lines of force inside the specimen (it becomes **a perfect diamagnetic material)**, hence exhibiting a complete Meissner Effect.Once $H_c$is removed, they regain the superconducting nature (reversible).These superconductors are called as **soft superconductors** as the required critical magnetic field H_c, is very low of the order of **0.1 Tesla**In the below graph, applied magnetic field (H) is taken on the X-axis and the induced field (I), the intensity of magnetization is taken on the Y-axis. . (e.g) Al, Zn, Ga, Pb, Sn, Hg.

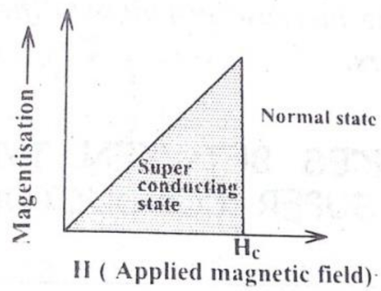


Fig. Type I Superconductors

Type II superconductors**:**

When a superconductor is placed in an external magnetic field and if the superconductor becomes a normal conductor **gradually** with respect to various critical magnetic fields, then it is known as **type II or Hard superconductors.**Here, the magnetic flux lines start to penetrate the superconducting material at Hc_1 which is below Hc and the penetration of flux lines is completed at $Hc_{2,}$ which is above Hc. Hence Type II superconductors do not exhibit complete Meissner effect or they **do not exhibit perfect diamagnetism**. Therefore the specimen is in a sort of mixed state between Hc_1 and $Hc_{2.}$ Above $Hc_{2,}$ the specimen is a normal conductor. They are called as **hard superconductors**, since Hc is 100 times more than that in Type I superconductors (**10 Tesla**). (e.g). Zr, Nb, Vanadium. Type II superconductors can carry high super current densities in high magnetic fields which are of great commercial importance.

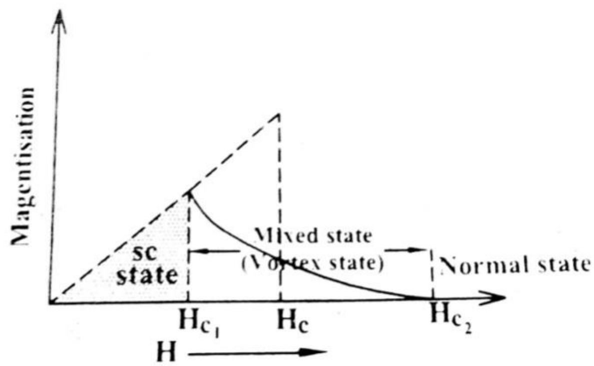


Fig. Type II Superconductors

2.2.5. BCS Theory

Phenomenon of Superconductivity was discovered in the year 1911 and it took almost 4 decades to develop an acceptable theory to explain the superconducting behavior, which was due to Bardeen, Cooper and Schrieffer (BCS) in the year 1957. It was then realized that it was successful only for low temperature or the so called conventional superconductors.

From the Isotope effect (one of the properties of superconductors), one can infer that mass of the atom-positive ions are also involved in the phenomenon of superconductivity. Besides, by analysing the various properties of superconductors, these three scientists put forward a model or a theory that explained the reason for infinite electrical conductivity exhibited by superconducting materials at ultracold temperature. In order to understand this theory, let us recall the concept of lattice of a solid along with quantum concepts namely wave-particle dual nature, superposition and entanglement (Entanglement refers to the correlation between two or more atoms whatever may be the distance between them). Bardeen, Cooper and Schrieffer explained the model through a special type of interaction called '**Electron-lattice-electron interaction**'.

Electron-lattice-electron interaction

Consider the positive ion lattice of the given superconducting material where the valence electrons move around the lattice.Now consider an electron-1 with wave vector K1(with upspin↑), is passing closer to a positive ion in the lattice due to electrostatic attractive force between them.The Electron-1 interacts with the positive ion and transfers some of its energy to the positive ion, thereby the lattice gets distorted and Electron-1 gets scattered with (K1-k) and ↑.Here it should be noted that the lattice vibrations are quantized in terms of phonons; phonons are charge carriers of lattice vibration.The distortion in the lattice is in such a way that it enhances the positive ion density around the distorted place.Hence it is natural that, to this positive ion cloud, another Electron-2 with wave vector K2 (with downspin↓) comes closer, interacts with the positive ion and gets scattered with (K2+k).Energy of Electron2 is found to be lesser than that of Electron-1. This is possible only in the case of attractive type of interaction. Therefore, it could be stated that the two Electrons -1 and -2 are interacted attractively through phonons (electron-lattice-electron interaction). Attractive interaction between the two electrons is not possible without phonons as we know there is repulsive force between two electrons in vacuum. Hence here in the lattice the two electrons are interacted attractively through the lattice vibrations and they form a bound electron pair called as Cooper pair.In the electron-lattice-electron interaction, the electrons will not be fixed and they try to move in two opposite directions but their correlation/coupling persist over a maximum distance of 10^{-6} m .This distance or length is called as **coherence length**(ξ).

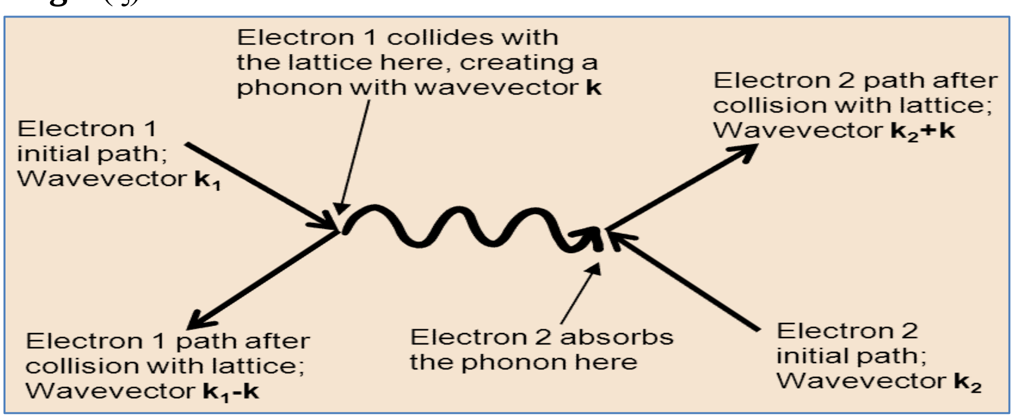
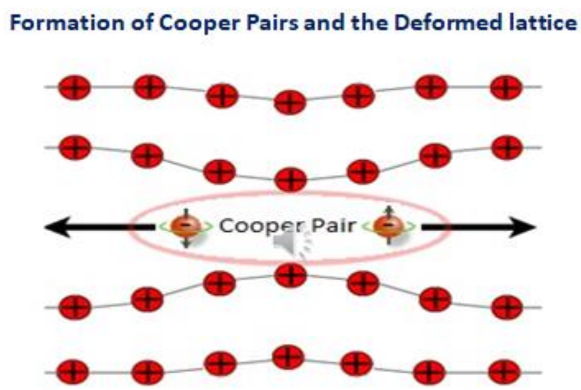


Fig. Electron-lattice-electron interaction

Cooper Pair: Two electrons with equal momentum and opposite spin pair up with each other due to a special type of attractive interaction through phonons. This pair of electrons is known as Cooper pair. The charge on the cooper pairs is 2e, where 'e' is the charge of the electron. The electrons in a cooper pair are bound or correlated with each other.



How the cooper pairs contribute to zero resistivity or infinite conductivity?

Generally, we know in normal conductors, movement of electrons contribute to electric current. In a typical superconductor, the volume of a given cooper pair can encompass as many as 10^6 other cooper pairs. This dense cloud of cooper pairs drift cooperatively throughout the lattice of the material. i.e. the motion of all cooper pairs are identical-either they are at rest or if the superconductor carries a current, they drift with identical velocity. Thus superconducting state is a more ordered state of the conduction electrons (we have seen in one of the properties of the superconductor that $\text{entropy}_{\text{supercond.}} < \text{entropy}_{\text{normal cond.}}$). Since the density of cooper pairs is quite high, for a large current even a small velocity is suffice;this small identical velocity combined with their precise ordering reduces the collision of the cooper pairs with the lattice.

Since there is no collision between them or with the lattice there is no loss of energy or no change in resistance. Hence the coherence nature of cooper pairs and their orderly identical movement throughout the lattice leads to vanishing resistivity, hence zero resistivity or infinite conductivity in the lattice of the superconducting material. Current due to movement of cooper pairs is quite often called as **super current**.

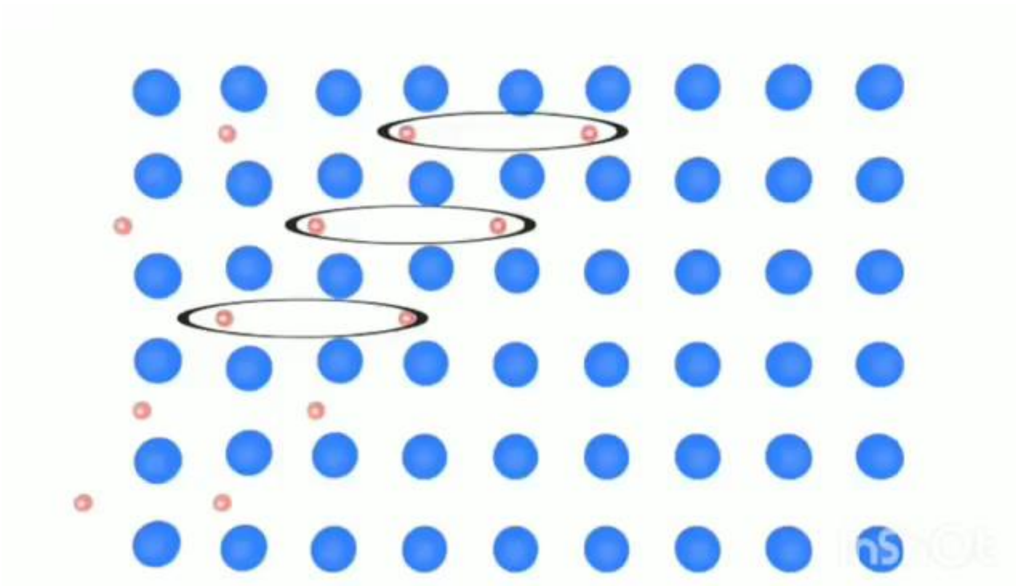


Fig. Movement of cooper pairs through the lattice

At what temperature, formation of cooper pairs is favourable?

The energy of the two electrons in the paired state is less than that when they are in the unpaired state. This difference in energy (called as binding energy of the cooper pair) appears as a band gap on the Fermi surface.

At T=Tc, cooper pairs are scattered, because there is sufficient energy-thermal energy to break up the pairs-hence the electrons are in the normal conducting state and they lie above the energy gap in the band structure of superconducting material; When T approaches zero Kelvin (T<Tc), the energy is not sufficient to break up the pairs, hence the energy gap is maximum and the cooper pairs are in the superconducting state and they lie below the energy gap. Therefore the temperatures that are nearing zero Kelvin are favourable for the formation of cooper pairs.

Major accomplishment of BCS theory:

- BCS theory explains Meissner effect, coherence length, penetration depth, flux quatisation, zero resistivity and energy gap parameter.
- It has solved the problem of electron energy, when there is attractive interaction and it gives the expression for transition temperature, Tc and the expression for Tc is known as McMillan's formula.

2.2.6. High Temperature super conductors

Conventional super conductors have their transition temperature less than 25 Kelvin. Those superconductors having the transition temperature higher than 25 Kelvin (conventional superconductors) are said to be High Temperature SuperConductors (HTSC).

From 1911(Tc of Hg4.2K) to 1986 (Tc of Nb_3Ge 23K) only few super conductors were discovered and this was accomplished with liquid Hellum ofBoiling Point 4.2 Kelvin.A major breakthrough occurs in 1987 when ceramic Yttrium-Banum - Copper oxide (**$YBa_2Cu_3O_7$**) with Tc of 92 Kelvin was synthesized, which is greater than the Boiling Point of liquid Nitogen (ie) 77 Kelvin. The superconducting materials who's Tc cross 77kelvin is important since here the cooling is accomplished with liquid N_2 instead of liquid Helium. This trend confirms that it might be possible to develop the superconductors at room temperature. Moreover nitrogen is much more abundant than helium and liquid nitrogen cryogenic systems are cheaper & faster.

*** Some of the Important HTSCthat have CuO2 layers**

(i) Rare earth based copper oxide compounds: The chemical formula for the rare-earth modified compound in MBa_2Cu3O7-s where M is a earth element such as La, y, Nd, Sm, etc. :$YBa_2Ca_3O_{7-\delta}$with Tc=92 Kelvin, notation: Y- 123

La $Ba_2Cu_30_{7-\delta}$ withTc = 95 Kelvin, notation La-123

The HTSC compounds are normally represented by 1212, 1234, etc. where these notations are based on the number of atoms on each metal element.

(ii) Metallic oxides: Some HTSC cuprates exhibit metallic behavior (e.g)

$La_{2-x}Sr_xCuO_4$ with a maximum Tc=40 Kelvin at x=0.17.

$Nd_{2-x}Ce_x$ Cu O_{4-y} with a maximum Tc=25 Kelvin at x=0.15 and y=0.02

<u>(iii) Bi-based, Tl based, Hg -based compounds</u>: The Bi, Tl and Hg based copper oxide compound are showing high temperature superconducting property.

eg: $Bi_2Sr_2Ca_2Cu_3O_{10}$ with Tc = 107 Kelvin; Notation: B1-2223,

$Tl_2Ba_2Ca_2Cu_3O_{10}$;Tc= 127Kelvin; Notation: Tl-2223,

$HgBa_2Ca_2Cu_20_8$; Tc= 133 K; Notation: Hg-1223

The super conducting property of the above said compounds are controlled by the number of CuO_2layers.

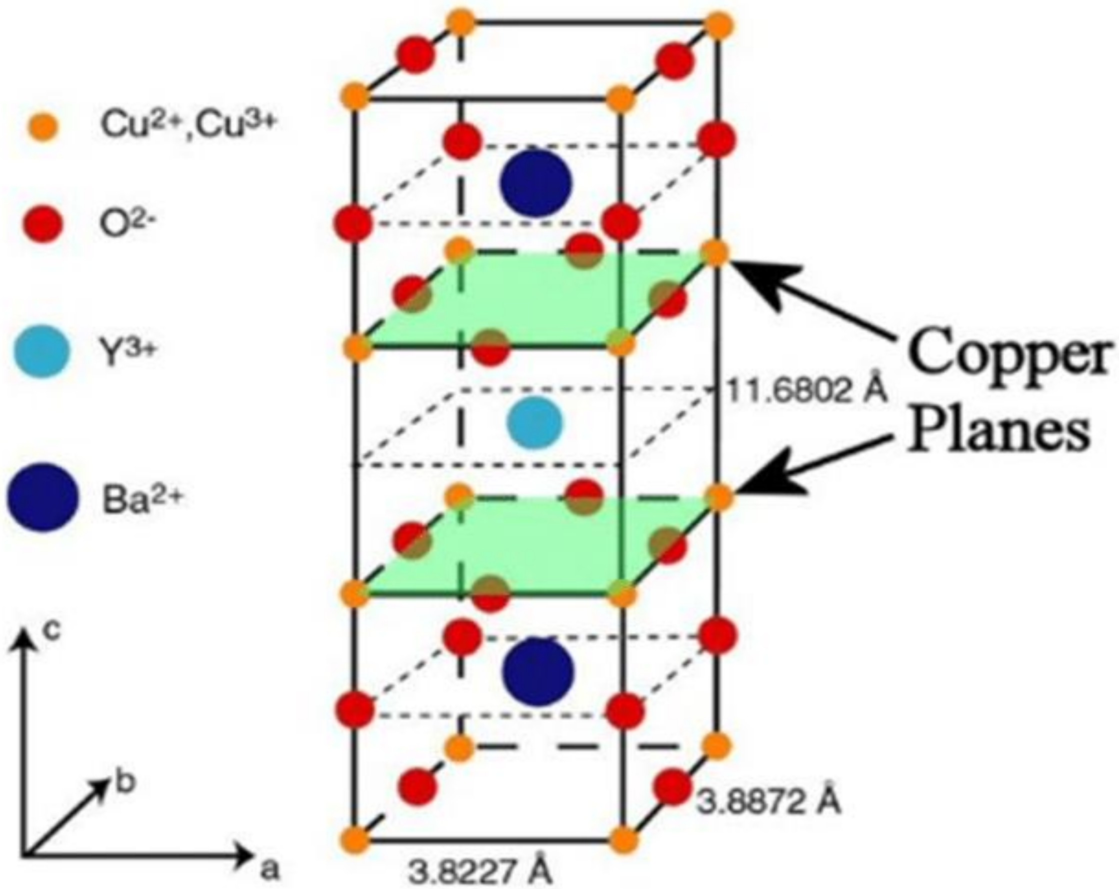
***High Tc is also observed in some new materials that do not have CuO_2layers.**

(i) MgB_2 with Tc = 40 Kelvin

(ii) Fullerene based compounds

e.g .K_3C_{60} with Tc 19 kelvin; Rb_3C_{60} with Tc 33 Kelvin

(iii) Iron pnictides with Tc of 34 K



***Significant properties of HTSC**

- Highly Anisotropic
- Short coherence length
- Anomalous magnetoresistance
- Presence of pseudo gap in its band structure.
- Mostly they are oxides of copper in combination of other elements
- They are reactive, brittle, easy to form tapes and wires which provide transmission of electrical power over a long distance without any radiative loss.

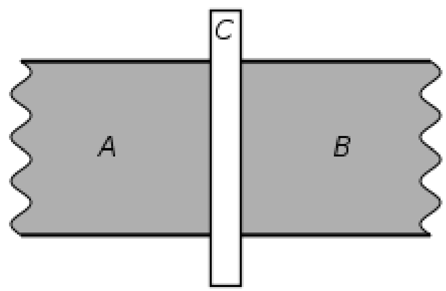
2.2.7. Josephson Effect

In 1962, B.D.Josephson predicted theoretically that **cooper pair tunneling** is possible through a very thin insulating layer that is sandwiched between two superconducting films of different electron densities. This tunneling may be made to result either with a d.c potential (DC Josephson effect) or with an a.c.potential

(AC Josephson effect) across the tunnel barrier. This phenomenon is known as **Josephson Effect**.

Josephson device and Josephson junction

Josephson device is a sophisticated sandwich structures of superconducting films (A and B in the below diagram) usually of Niobium, separated by extremely thin insulating oxide layer of thickness 1 nano meter (C in the diagram). The insulating layer is the **Josephson junction** or often called as weak link or a barrier. Then a super current flows across the junction without developing a voltage. Note the surprise here that without applying any sort of input voltage, one can have super current.



DC Josephson Effect

In a Josephson device, direct current flows across the junction even when no voltage is applied across it. This is known as DC Josephson effect. Here, the super current (Is) flowing across the junction is given by Is = Ic.Sinφ, where Ic is the maximum current that the junction can sustain called as critical current and $\varphi = \varphi_1 - \varphi_2$, the phase difference of wave functions of cooper pairs on both the sides of the junction.

AC Josephson Effect

In a Josephson device, when a constant non zero dc potential (V) is maintained across the barrier of the Josephson device, radio frequency current oscillations are set up in the device. i.e. An alternating super current flows through the barrier in addition to the dc current. This is known as AC Josephson Effect.

Since the application of external potential (V) makes the Josephson current to be time dependent, φ the phase difference changes with time according to the relation,

$\frac{d\varphi}{dt} = \frac{2eV}{\hbar}$, where $\hbar$ = h/2Π

$d\varphi = \frac{2eV}{\hbar}.dt$

Integrating on both sides, we get, $\varphi = \frac{2eV}{\hbar}.t + C$, ------(1), where C is the constant of integration. When t=0, $\varphi = \varphi_0$, therefore, equ. (1) becomes

$\varphi = \frac{2eV}{\hbar}.t + \varphi_0$ --------(2).

Hence super current Is = Ic.Sin φ, takes the form,

$$I_s = I_c.\ \mathrm{Sin}\left(\frac{2eV}{\hbar}.t + \varphi_0\right) \text{ -------(3)}$$

Comparing equ (3) with $I = I_0 \sin \omega t$, we get,

$\omega = \frac{2eV}{\hbar}$ ------------**(4)** is called as Josephson frequency**.**

We know angular frequency ω = 2Πν, hence equation (4) becomes

$2\Pi\nu = \frac{2eV}{\hbar}.t$

(or) $\boxed{\gamma = \frac{2e}{h}V = 483.55 \times 10^{12}\ V \text{ Hertz -------------------(5)}}$

Where V is the applied potential.

Case: V = 1 miili volt,

Then $\gamma = 483.55 \times 10^{12} \times 10^{-3}$

$= 0.483 \times 10^{12}$ Hz ------frequency of micro waves.

By measuring the frequency, the value of e/h and photon energy 2eV could be accurately measured.

I-V Characteristics of Josephson Current

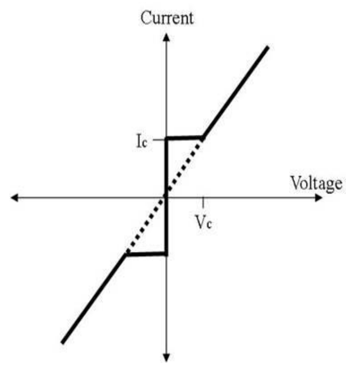


Fig. I-V Characteristics of Josephson junction

Let V_0 be the applied dc potential and Vc be the minimum dc potential required to produce the AC Josephson effect. Now consider the below graph drawn between current and voltage across the Josephson device.

(i) When $V_0 = 0$, there is a constant dc current (Ic) flows through the junction (DC Josephson effect)

(ii) When $V_0 < Vc$, again we have only the constant dc current.

(iii) When $V_0 > Vc$, the junction has a finite resistance and the current oscillates (AC Josephson effect)

(iv) Hence one can have finite voltage drop (ON) and zero voltage drop (OFF) across the junction alternatively with an angular frequency of $\omega = \frac{2eV}{\hbar}$.

Hence Josephson junction can act as a fastest switching element in fastest computers.

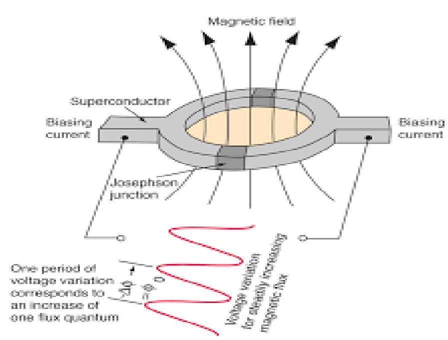
Applications of Josephson Effect

- ✓ Josephson junction is capable of switching signals from one circuit to another with a switching time of 1 pico second(10^{-12} second). Hence it acts as a fastest switching element.

- ✓ It is capable of storing information, smaller in size, lighter in weight; hence provide the basis for the architecture of the fastest computers.
- ✓ It is used to produce microwaves, when the applied potential is 1 millivolt.
- ✓ It is used in the precision determination of e/h.
- ✓ AC Josephson effect is used to measure very low temperatures.
- ✓ It is used in the construction of SQUIDs. SQUIDs are used in magnetocardiography (MCG), magnetoencephalography (MEG) and in magnetic resonance imaging (MRI).

2.2.8. SQUID (Superconducting Quantum Interference Device)

SQUID is basically a high sensitive magnetometer – a sensor or a gadget that can detect minute changes in magnetic field/flux, say of the order of 10^{-14} Tesla. (Recall that the magnetic flux around the superconducting coil is quantized in terms of fluxoid Φ_0).



Construction and working

- It is a double junction quantum interferometer formed from two Josephson Junctions mounted on a superconducting ring.
- Magnetic field is applied normal to the plane of the ring and it induces current through the arm T1.
- The current splits into two components and they flow through the Josephson devices 1 and 2 as shown in the below diagram. This is similar to the splitting of light into two coherent sources in Young's double slit experiment.

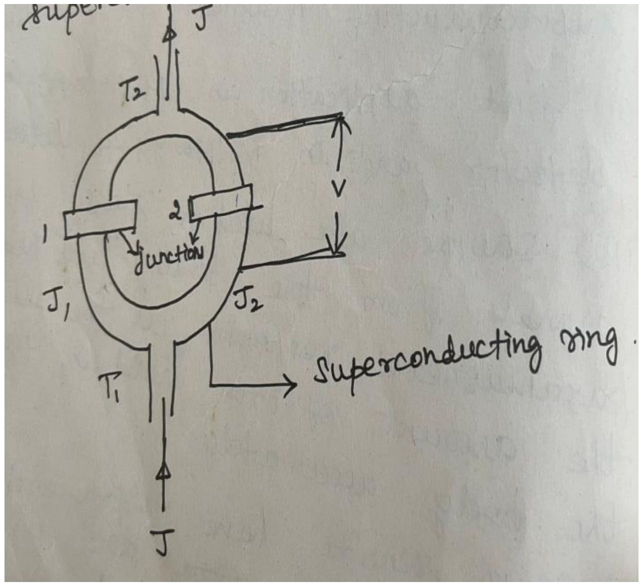


Fig. SQUID

- The current leaving the two junctions 1 and 2 are combined together at the common arm T_2.This is similar to the two splitted beams of light are allowed to interfere with each other in Youn's Double slit experiment.
- The current flowing out through the arm T2 is given by,
  $J= J1+J2= 2\ J_o.\cos\ (\frac{e\phi}{\hbar})$ where J_0 is the maximum current and ϕ the total magnetic flux.

- When a given sample is moved through the superconducting coil, it induces an electric current and hence there is a change in the magnetic flux. The voltage that we measure across the device is strongly correlated to the change in magnetic flux minute magnetic signals are detected by these squid sensors.

Applications of SQUID

- ✓ SQUID can be used to detect the variation of very minute magnetic signals in terms of quantum flux.
- ✓ It is used as a storage device for magnetic flux.
- ✓ It is used to study earthquakes and to remove paramagnetic impurities.
- ✓ The main application of squid is to detect geological layers in different minerals, to detect NMR signals at low temperature.

- ✓ A novel application is the magnetic marker monitoring method, used to trace the path of orally applied drugs.
- ✓ Used to study tiny magnetic signals from the brain and heart, can detect paramagnetic response in the liver and give amount of iron held in the liver accurately.

2.2.9. Magnetic Levitation

Levitation is the process by which an object is suspended by a force against gravity, in a stable position without solid physical contact. This principle is involved in magnetic levitation where a superconductor repels a magnetic field, so a magnet will float above a superconductor. This is employed in frictionless bearings and superfast electric trains, etc. Magnetic levitated trains are operated in Japan and are called as Maglev Trains whose speed is nearly 500-700 Km/hour.

Maglev technology

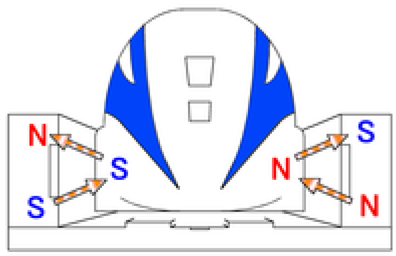
Maglev is a magnetic levitated train and it works under the principal of **Electromagnetic induction**. There are two primary types of maglev technology.

(i) Electromagnetic suspension (EMS) systems where the train levitates above a steel rail. Here, the electromagnets are attached to the train and are oriented towards the rail. Hence the system makes use of the attractive magnetic force.

(ii) Electrodynamic Suspension (EDS) where both rail and train exert a magnetic field and the train is levitated by the repulsive force between these magnetic fields. The repulsive force in the track is created by induced magnetic field in wires (or) other conducting strips in the track.

Construction and working

It has two superconducting magnets on each side of the train and there is a guiding system consisting of ‘S” shaped coils on each side. Initially when the train starts, they slide on the rails. Now, when the train moves faster, the superconducting magnets on each side of the train will induce a current in the ‘S’ shaped coils kept in the guiding system. This induced current generates a magnetic force in the coils in such a way that the lower half of ‘S’ shaped coil has the same magnetic pole as that of the superconducting magnet in the train, while the upper half has the opposite magnetic pole. Therefore, the total upward magnetic force acts on the train and the train is levitated or raised above the rails and floats in the air. Now, by alternatively changing the poles of the superconducting magnet in the train, alternating currents can be induced in ‘S’ shaped coils. Thus, alternating series of north and south magnetic poles are produced in the coils, which pulls and pushes the superconducting magnets in the train and hence the train is further moved. This train cannot move over the rail. Instead it floats above the rails, so that it moves faster with speed of 500 Km/hr to 700 Km/hour without any frictional loss.



2.2.10. General applications superconductors

- ✓ Since there is no loss in power in superconductors, it is used for the transmission of power over very large distance
- ✓ They act as magnetic shield due to their diamagnetic property.
- ✓ Cosmic rays are deflected away from the space vehicle by a superconducting magnet.
- ✓ Josephson device provides a number of wide applications such as SQUIDS, determination of h/e, fastest switching element, storage element and is used to measure very low temperature.
- ✓ It is used to manufacture electrical generators and transformers in small sizes with high efficiency.
- ✓ It is used to design cryotron, Josephson devices, SQUID, magnetic levitated trains (MAGLEV), modulators, rectifiers, magnetic separators, commutators, etc.
- ✓ Finds application in metrology, radiation detector and in military to detect submarines.

Note:

- **Skin effect:** According to Meissner Effect, the magnetic flux lines are excluded from the given superconducting specimen at very low temperatures ($T < T_c$) in a weak external magnetic field ($H<H_c$). But experiments reveal that the applied magnetic field does not drop suddenly to zero, at the surface the specimen, but it decays exponentially to zero over a distance. Therefore there exist some flux lines over the surface of the superconducting specimen and this is referred as **Skin effect**. $H=H_o.e^{-x/\lambda}$, where $H_o$represents the magnetic field at the boundary of the specimen, λ represents the penetration depth and is defined as the distance in which the magnetic field (H) decreases by a factor of 1/e . Hence, it is interred that B penetrates up to thin surface layer of the superconducting slab.
- **Penetration depth (λ)** is defined as the distance over which the magnetic flux lines decreases to zero by a factor of 1/e, when the external magnetic field is cut-off for the given superconducting specimen.

$$\boldsymbol{\lambda=\lambda_0 \left(1 - T^4 / T_C^4\right)^{-1/2}}$$

where λ_0 represents penetration depth at 0 Kelvin, $\lambda_T$represents the penetration depth at T Kelvin, T_c – the transition temperature.λ will be of the order of 500 A^o .

- **Ginsberg-Landou parameter (K):** The ratio of the penetration depth(λ) to the coherence length(ξ) of the given superconductor is called as Ginsberg-landov parameter (K).$K= \lambda / \xi$. If K is less than 0.7, the material is type I superconductor and if it is more than 0.7, it is type II.
- **Energy gap (Δ):**

Specific heat capacity measurement provided the first indication of such an energy gap (Δ) in superconductor. The prediction of such an energy gap in super conductor is one of the key features of BCS theory.

Expression for energy gap is given by $\mathbf{\Delta(T) = 3.2\ K_B T_c (1\text{-}T/T_c)^2}$. From this expression, it is noted that (i) at T=Tc, $\Delta(T) = 0$, energy gap reduces to zero where cooper pairing is dissolved (normal state); (ii) when T_c approaches zero Kelvin, $\Delta(T)$ reaches a maximum value of $\Delta_0(T)$. (i.e) energy gap is maximum and cooper pairing increases (superconducting state).

- **Comparison of Energy gap of a superconductor with that of a semiconductor.**

Semiconductor	Superconductor
Here, the band energy gap is tied to the lattice.	Here, the energy gap is tied to the Fermi energy.
Here, it corresponds to the energy difference between the valence band and conduction band.	Here, the gap lies within the spectrum of allowed energy states that separates the excited state from the normal state.
Energy gap is about < 2 eV	For conventional SC, energy gap is app. 1meV, while for HTSC, it is app 1 to 10 meV.

- **Comparison of Low Tc and HignTc superconductors**

S.No	**Low temperature superconductors**	**High temperature superconductors**
1	Superconducting materials whose Tc is less than 25 kelvin are called low $T_c$superconductors.	Superconducting materials whose Tc is greater than 25 kelvin are called high T_c superconductors.
2	Exhibits complete Meissner effect, isotope effect, zero resistivity, persistent current, etc.	Does not exhibit complete Meissner effect. Along with the properties of conventional/low Tc superconductors, they also exhibit anomalous magneto resistance, anisotropic resistivity and pseudo gap.

3	BCS theory was proposed to explain the phenomenon	Resonating Valence bond theory was proposed to explain the High Tc phenomenon
4	Examples. Hg, Sn, Pb	Y-Ba-CuO, La-Ba-Cuo
5	This can be accomplished with liquid Helium (4.2 K) bath.	This can be accomplished with liquid Nitrogen (77 K) bath.
6	Coherence length 1600A^{o}	Coherence length 10A^{o}