SoSe 2020 Kiel University
Dr. Joscha Reimer Department of Computer Science

Solution homework exercise 6.2:

Statement: Consider the globalized Newton-Method with an efficient step-size method applied to

$$f : \mathbb{R} \to \mathbb{R}, x \mapsto x^2.$$

Let $(x_k)_{k\in\mathbb{N}_0}$ be the sequence of iterates. $(x_k)_{k\in\mathbb{N}_0}$ converges Q-superlinear to a global minimizer of f.

Proof: f is *differentiable* and *bounded from below* because

$$f(x) = x^2 \geq 0 \text{ for all } x \in \mathbb{R}.$$

The sequence of search directions generated by the globalized Newton-Method are always *gradient related* due to the definition of the globalized Newton-Method. The generated sequence of step-sizes is *efficient* as well because an efficient step-size method is used.

Thus due to the convergence theorem at slide 9 lecture 10,

$$\nabla f(x_k) = 0 \text{ for some } k \in \mathbb{N}_0 \text{ or } \lim_{k\to\infty} \nabla f(x_k) = 0.$$

If $\nabla f(x_k) = 0$, the globalized Newton-Method would (usually) terminate or

$$d_k = 0 \text{ and thus } x_l = x_k \text{ for all } l \in \mathbb{N}_0 \text{ with } l \geq k$$

due to the definition of the globalized Newton-Method and thus

$$\lim_{k\to\infty} \nabla f(x_k) = 0$$

as well.

Because

$$\nabla f(x) = 2x \text{ for all } x \in \mathbb{R},$$

this implies

$$\lim_{k\to\infty} x_k = 0.$$

$x^* := 0$ is the only minimizer of f because

$$\nabla^2 f(x^*) = 2 > 0$$

is positive definite.

Since f is *twice continuously differentiable* and $(x_k)_{k\in\mathbb{N}_0}$ *converges* to x^* and $\nabla^2 f(x^*)$ is *positive definite*, the convergence is Q-superlinear due to the convergence result at slide 24 of lecture 12. □