

Solution homework exercise 7.1:

Statement: Let $H \in \mathbb{R}^{n \times n}$ be symmetric and $y, s \in \mathbb{R}^n$ with $y^T s \neq 0$ and $s^T H s \neq 0$.
Define

$$U := \frac{y y^T}{y^T s} - \frac{H s (H s)^T}{s^T H s}.$$

Then

$$(H + U)s = y.$$

Proof: Due to the definition of U and since $H^T = H$,

$$\begin{aligned} U s &= \left(\frac{y y^T}{y^T s} - \frac{H s (H s)^T}{s^T H s} \right) s \\ &= y \frac{y^T s}{y^T s} - H s \frac{(H s)^T s}{s^T H s} \\ &= y \frac{y^T s}{y^T s} - H s \frac{s^T H s}{s^T H s} \\ &= y - H s. \end{aligned}$$

Thus

$$(H + U)s = H s + y - H s = y.$$

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