SoSe 2020
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Optimization and Data Science
7. Homework exercises

Theoretical exercise 1:
Show that the BFGS update satisfies the secant equation.

Theoretical exercise 2:
Show that the BFGS update produces a positive definite matrix H_k if H_{k-1} was already positive definite and $y_k^T s_k > 0$.

Hint:

$$< \cdot, \cdot >: \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}, (x, y) \mapsto x^T H_{k-1} y$$

is a scalar product, because H_{k-1} is positive definite. (You do not have to prove this.) It might be useful to apply the Cauchy–Schwarz inequality to this scalar product.

Theoretical exercise 3:
Prove the Sherman-Morrison-Woodbury formula which states that for $A \in \mathbb{R}^{n \times n}$ invertible and $x, y \in \mathbb{R}^n$:

$$A + xy^T \text{ is invertible if and only if } y^T A^{-1} x \neq -1 \text{ and that in this case}$$

$$(A + xy^T)^{-1} = A^{-1} - \frac{A^{-1} x y^T A^{-1}}{1 + y^T A^{-1} x}$$

Programming exercise 1:
Extend your general descent method implementation such that the globalized Quasi-Newton method with inverse update can be used as well.

Try your implementation with the Roosenbrock function and the Bazaraa-Shetty function. Compare the behavior of this method with the previous methods using these two examples.

The solutions of the theoretical exercises will be discussed on 25. Mai 2020.