SoSe 2020 Kiel University
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Optimization and Data Science
6. Homework exercises

Theoretical exercise 1:
Define

$$x_k := \begin{cases} \left(\frac{1}{3}\right)^k & \text{if } k \text{ is odd} \\ \left(\frac{1}{4}\right)^k & \text{if } k \text{ is even} \end{cases} \qquad \text{for all } k \in \mathbb{N} \text{ and } x_0 = \tfrac{1}{3}.$$

Examine the sequence $(x_k)_{k\in\mathbb{N}}$ *regarding convergence rate with the Q-factor and the R-factor.*

Theoretical exercise 2:
Consider the globalized Newton-Method with an efficient step-size method applied to

$$f : \mathbb{R} \to \mathbb{R}, x \mapsto x^2.$$

Let $(x_k)_{k\in\mathbb{N}_0}$ *be the sequence of iterates. Show that* $(x_k)_{k\in\mathbb{N}_0}$ *converges Q-superlinear to a global minimizer of* f*.*

Hint: Apply the convergence results from the lecture.

Theoretical exercise 3:
Consider the globalized Newton-Method with $c \in (0, 1]$ *applied to*

$$f : \mathbb{R} \to \mathbb{R}, x \mapsto |x|^p$$

with $p > 2$*. Let* $(x_k)_{k\in\mathbb{N}_0}$ *be the sequence of iterates. Show that*

a) *If* $x_k > 0$*, then* $d_k = -\frac{1}{p-1}x_k$*, for all* $k \in \mathbb{N}_0$*.*

b) *If* $x_0 > 0$ *and* $\rho_k = 1$ *for all* $k \in \mathbb{N}_0$*, then* $x_k > x_{k+1} > 0$ *for all* $k \in \mathbb{N}_0$*.*

c) *If* $x_0 > 0$ *and* $\rho_k = 1$ *for all* $k \in \mathbb{N}_0$*, then* $(\rho_k)_{k\in\mathbb{N}_0}$ *is an efficient step-sizes sequence.*

d) *If* $x_0 > 0$ *and* $\rho_k = 1$ *for all* $k \in \mathbb{N}_0$*, then* $(x_k)_{k\in\mathbb{N}_0}$ *converges Q-linear but not Q-superlinear to a global minimizer of* f*.*

e) *If* $x_0 > 0$ *and* $\rho_k = 1$ *for all* $k \in \mathbb{N}_0$*, can we apply the convergence result for the globalized Newton method?*

Hint: Use the definition of gradient-related and efficient step-sizes to prove a) and c), respectively. A mathematical induction might be useful to prove part b). Calculate the resulting Q-factors to prove d).

Programming exercise 1:
Implement a general descent method using the armijo step-size method and supporting the gradient method and the globalized Newton method. The following inputs should be supported:

- *the function (including first and second derivative)*
- *whether the gradient method or the globalized Newton method should be used*
- *a stopping bound for the stopping criterion*
- *a maximal number of iterations*
- *parameters of the step-size method and the optimization method*
- *a callback function which is called during the optimization in each iteration step with the current iterate as argument (and thus allows for example printing the progress)*

as well as the following outputs:

- *the minimizer and the minimum*
- *the number of needed iterations, function evaluations, gradient evaluations and Hessian matrix evaluations.*

If the first derivative of the function is not supplied, your method should approximate it using finite differences.

Try your implementation with the Roosenbrock function and the Bazaraa-Shetty function. Compare the behavior of both methods with different configurations using these two examples.

The solutions of the theoretical exercises will be discussed on 18. Mai 2020.