

Optimization and Data Science

10. Homework exercises

Programming exercise 1:

Implement the stochastic gradient method and the minibatch gradient method with

- a) 2^{-k}*
- b) k^{-1}*
- c) 1*

as step size at the k -th iteration.

Programming exercise 2:

We would like to determine the amplitude and the period length of a sine oscillation. The oscillation with the parameters s , where s_1 is the amplitude and s_2 is the period length, at a time point x is given by

$$F(s, x) := s_1 \sin\left(\frac{2\pi x}{s_2}\right).$$

For three different oscillations these values were measured at several time points (including some measurement noise). They are available in the `sine1.txt`, `sine2.txt` and `sine3.txt` files where the first column contains the time points x and the second column contains the corresponding values $F(s, x)$. Determine the parameters s with the stochastic gradient method and the loss function

$$L(y, z) := \frac{1}{2} \|y - z\|_2^2.$$

Start always with an initial guess $s_1 = 2$ and $s_2 = 2$. Try to minimize the total computational effort of evaluating $F(s, x)$ and $\nabla_s F(s, x)$ using minibatches of an appropriate size.

Programming exercise 3:

Implements the $(\mu + \lambda)$ -ES with Mutation Strategy Parameter Control (MSC).

Programming exercise 4:

Minimize the Roosenbrock function using

- a) the $(\mu + \lambda)$ -ES with MSC implemented in the previous exercise and different values for μ and λ and different initial mutabilities.*
- b) the CMA-ES implementation provided by the Python package `pycma` (<https://pypi.org/project/cma> and <https://github.com/CMA-ES/pycma>).*

Compare the total effort of these minimizations with the effort of your minimizations of the Roosenbrock function with previous optimization algorithms.

Programming exercise 5:

Visualize the two dimensional Griewank function

$$f : \mathbb{R}^2 \rightarrow \mathbb{R}, x \mapsto \sum_{i=1}^n \frac{x_i^2}{4000} - \prod_{i=1}^n \cos\left(\frac{x_i}{\sqrt{i}}\right) + 1$$

and minimize this function using your implemented

a) $(\mu + \lambda)$ -ES with MSC.

b) BFGS-method.

Compare the obtained results. Which of these algorithms is better suited for this minimization?

Hint: 0 is the minimizer of the Griewank function.

The solutions of the theoretical exercises will be discussed on 15. June 2020.