DAGs and the backdoor criterion (again)

ISTA 410 / INFO 510: Bayesian Modeling and Inference

U. of Arizona School of Information

September 16, 2020

Outline

- DAGs as probabilistic models
- How associations "flow" through DAGs
- Causal inference and paradoxes
    - Aside: multicollinearity

DAGs as probabilistic models

Recap: what is a DAG?

What is a DAG?

- Directed acyclic graph
- Nodes are variables
- Directed arrows are

What are we using DAGs for? Probabilistic models, on two levels:

- probabilistic model for causal associations between variables
- metadata that guides choice of variables for inference

Reference

BDA mentions causal inference and gives some details, but doesn't use DAGs

Main reference: Judea Pearl, *Causality* (available online through UofA library)
Chapter/section references:

- DAGs as probabilistic models: Chapter 1
- The backdoor criterion: Section 3.3
- Simpson's paradox and confounding: Chapter 6

Four technical slides

Probabilistic model of a DAG

Say we have n variables $X_1, \ldots, X_n$. We can always write

$$p(x_1, \ldots, x_n) = \prod_i p(x_i | x_1, x_2, \ldots, x_{i-1})$$

(the chain rule). We are interested in the case where each x_j is dependent on only some of the other variables:

$$p(x_i | x_1, \ldots, x_{i-1}) = p(x_i | pa_i)$$

where PA_i is a subset of the remaining variables, called the (Markovian) parents of X_i.

Probabilistic model of a DAG

Why Markovian? Because the restriction that

$$p(x_i|x_1, \ldots, x_{i-1}) = p(x_i|pa_i)$$

is a sort of Markov property: the distribution of X_i depends only on its immediate parents.

If the joint probability distribution of all the variables obeys this Markov property with respect to the parent relationships described by the graph:

$$p(x_1, x_2, \ldots, x_n) = \prod_i p(x_i|pa_i)$$

then the probability distribution is said to be *compatible* with the graph.

Graphical example

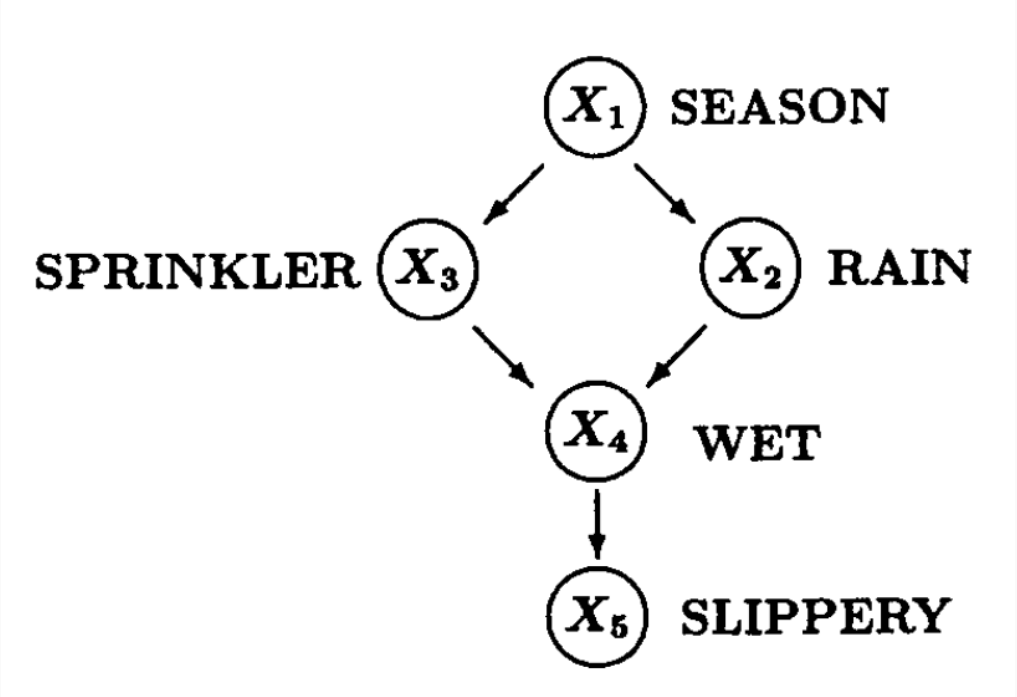


Figure from *Causality*

Graphical example

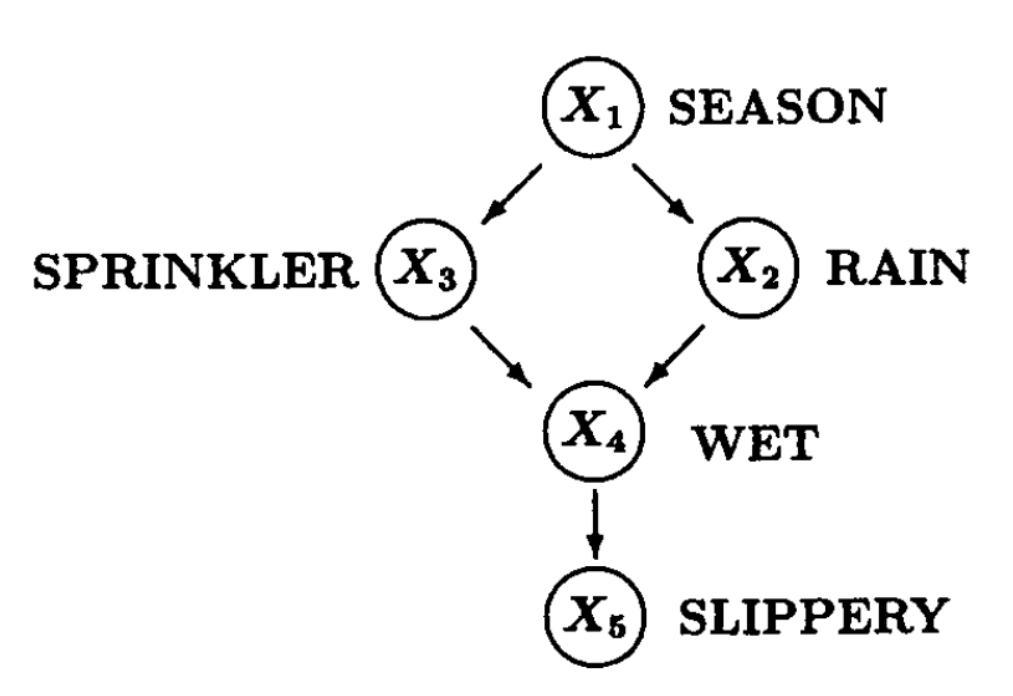


Figure from *Causality*

$$P(x_1, \ldots, x_5) = P(x_1)P(x_2|x_1)P(x_3|x_1)P(x_4|x_2, x_3)P(x_5|x_4)$$

Functional model of a DAG

Functional model: each variable X_i satisfies an equation in the graph:

$$x_i = f_i((pa_i), u_i)$$

where

- f_i is a function from w
- pa_i refers to the parent nodes of X_i in the graph
- u_i represents the unobserved and/or random components of the model

Special case: linear structural equation models:

$$x_i = \sum_j \alpha_j x_j + u_i$$

The hierarchical models we'll look at next week are restricted versions of this idea.

Three elemental paths

Three basic paths

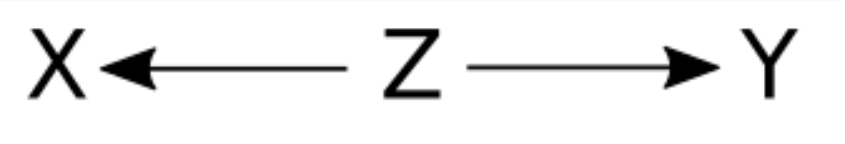
In a Bayesian network, information flows along paths (both with and against the arrows).

A path from X to Y can be a direct path – an arrow between X and Y. Or it can be an indirect path $X \leftrightarrow Z \leftrightarrow Y$ (or a concatenation of several of these).

Indirect paths lead to confounding / spurious associations; to deal with this, we need to classify the different types of indirect paths.

The "fork" path

The *fork* is the form most students learn as the sole definition of "confounding" in introductory classes: X and Y are confounded by their common cause, Z:

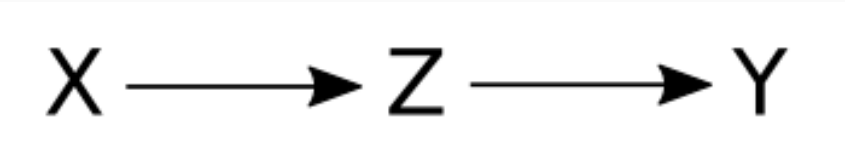


A statistical association exists between X and Y because they are both influenced by Z; if there is no arrow from X to Y, this association will be eliminated by controlling for Z. That is, controlling for Z blocks information flow along the path.

To estimate the causal effect of X on Y, control/stratify for Z.

The "chain" path

The *chain* is a similar-looking form, where Z sits in the middle of a causal path:



Typical case: Z is an effect of X that mediates the effect on Y

Example: X is pesticide application; Z is the pest population; Y is crop yield.

Controlling for Z blocks information flow along the path.

The "collider" path

The third form is the *collider* or inverted fork, and it behaves quite differently!

In contrast to the fork or chain, information flows through the collider only when it *is* observed / controlled; controlling *unblocks* the path.

Heuristic example

X: switch state on/off Z: light bulb on/off Y: power working/not working

The presence of power and the state of the switch are independent; but,

- turn on the switch and observe the light: it's off
- is the power working?

The explaining-away effect

This property of colliders is responsible for a sometimes counterintuitive effect:

- "explaining away": observing one of the common causes
- Berkson's paradox: conditioning on a variable can introduce a spurious association

They're really the same effect; explaining away common in AI/ML; Berkson's paradox in statistics

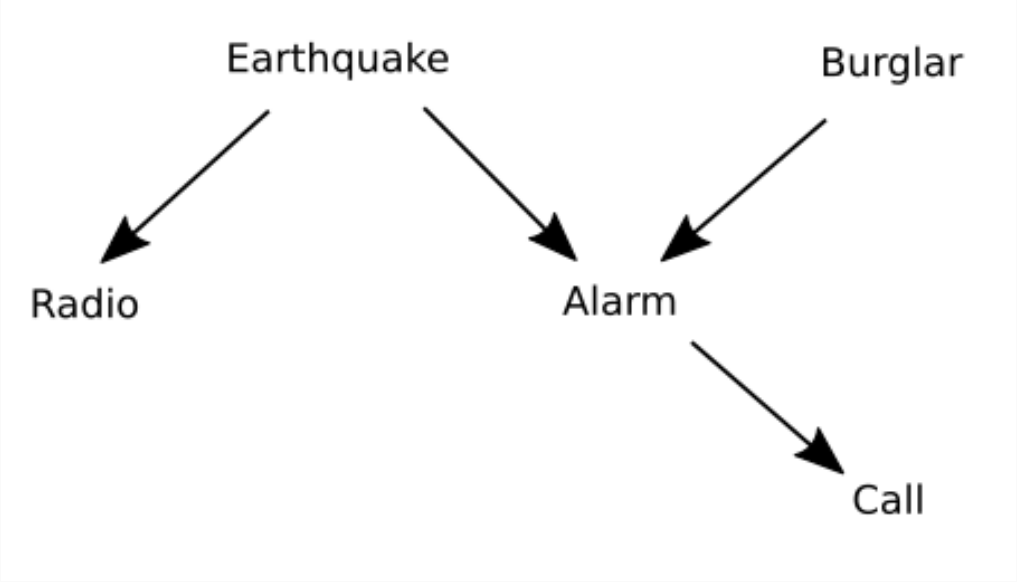
Explaining away: the burglar alarm

From Pearl by way of Mackay:

> *Fred lives in Los Angeles and commutes 60 miles to work. Whilst at work, he receives a phone-call from his neighbour saying that Fred's burglar alarm is ringing. What is the probability that there was a burglar in his house today? While driving home to investigate, Fred hears on the radio that there was a small earthquake that day near his home. 'Oh', he says, feeling relieved, 'it was probably the earthquake that set off the alarm'. What is the probability that there was a burglar in his house?*

Explaining away: the burglar alarm

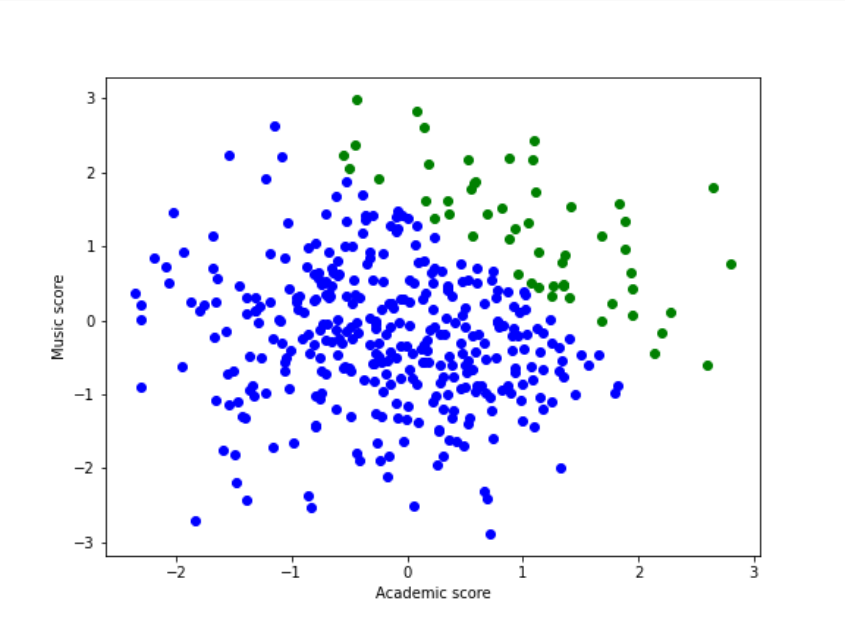
A DAG for the burglar alarm problem, showing the collider:



The alarm sits at a collider.

Conditioning on colliders creates confounding

The spurious-association effect of conditioning on a collider:



Berkson's paradox a.k.a. *selection bias*

The backdoor criterion

d-separation

A (possibly undirected) path p through a DAG G is said to be *d-separated* or *blocked* by a set of nodes Z if:

1. p contains a chain $X_i \to M \to X_j$ or fork $X_i \leftarrow M \to X_j$ such that $M \in Z$; or,
2. p contains a collider $X_i \to M \leftarrow X_j$ such that $M \notin Z$ and no descendent of M is in Z.

(Why the descendant property? Look back at the burglar alarm.)

The d-separation (blocking) definition for paths leads to another definition, for sets of variables.

The backdoor criterion

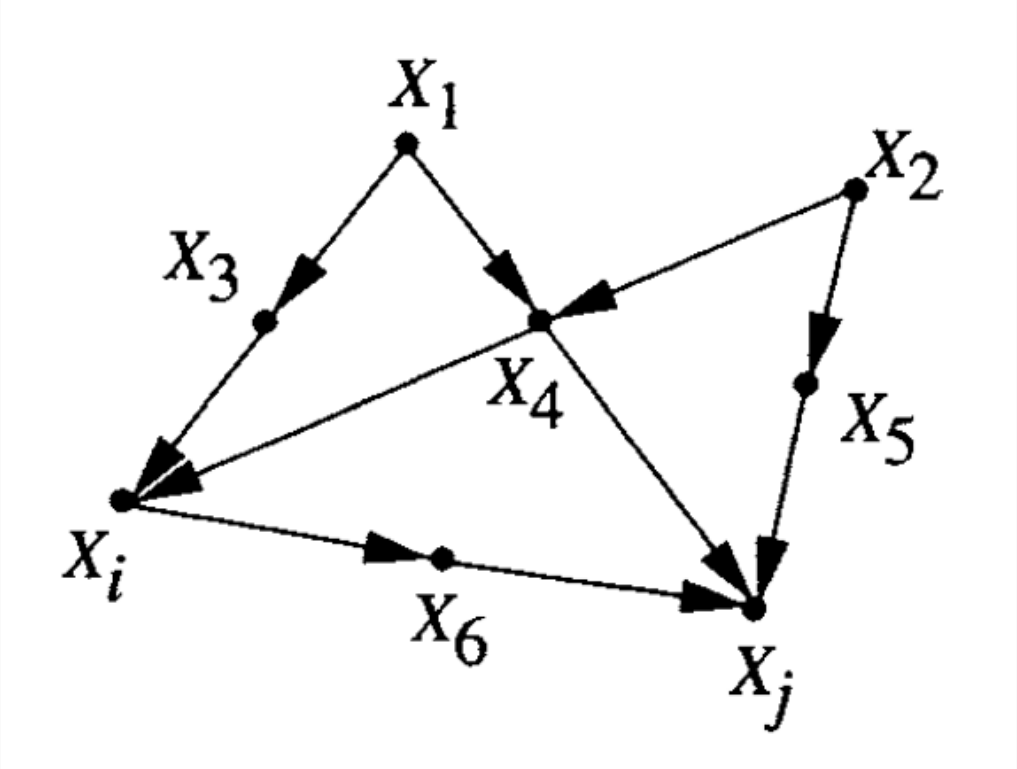
A related definition:

Definiton
A set of variables Z satisfies the backdoor criterion with respect to an ordered pair of variables (X_i, X_j) in G if:

1. *no node in Z is a descendent of X_i; and,*
2. *Z blocks every path from X_i to X_j that contains an arrow into X.*

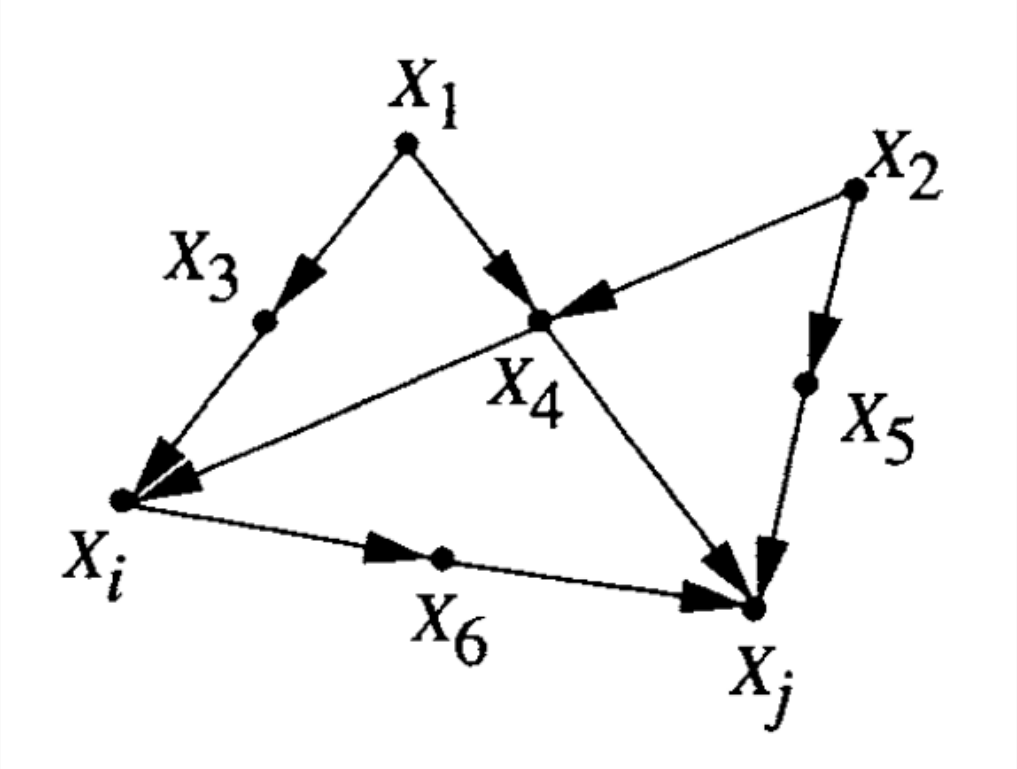
To estimate the causal effect of X on Y, condition on a set of variables satisfying the backdoor criterion with respect to (X, Y).

Example



Which variables satisfy the backdoor criterion?

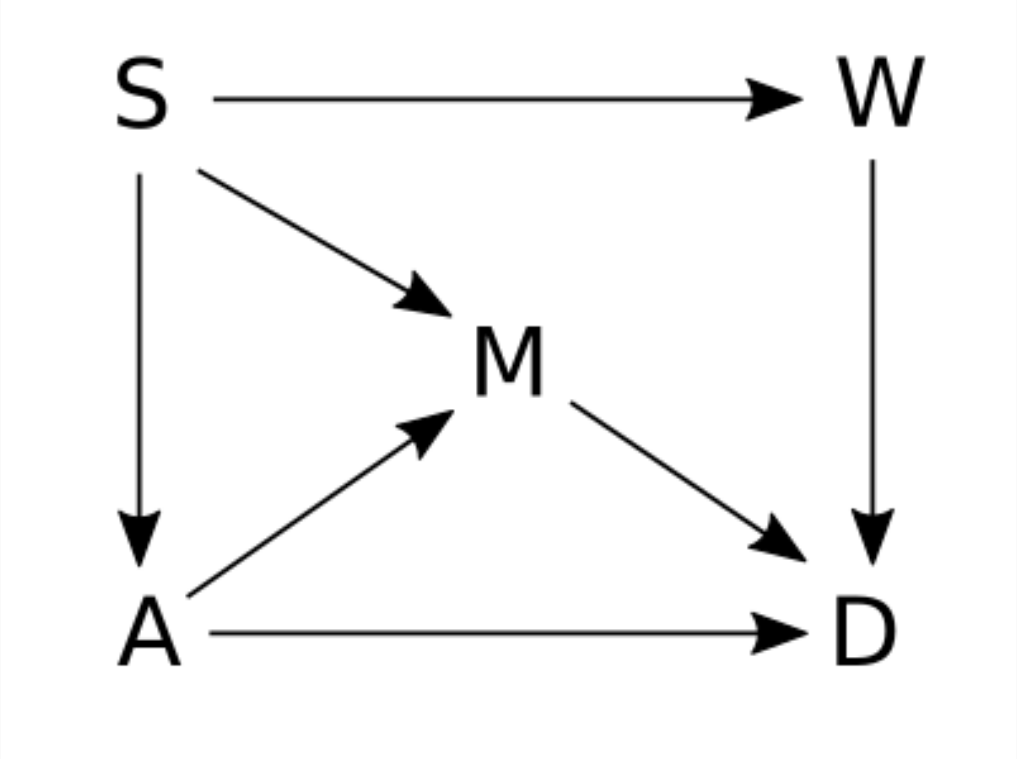
Example



Which variables satisfy the backdoor criterion?

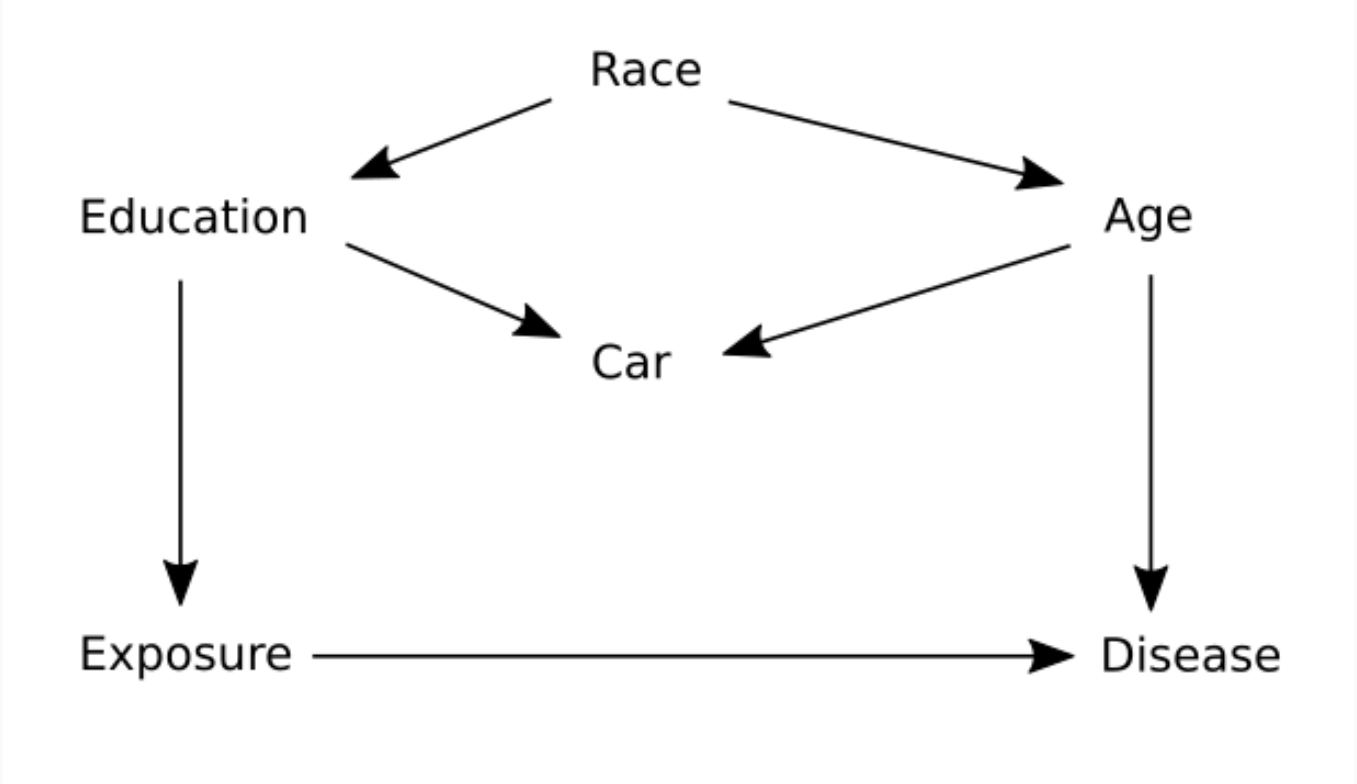
- $\{X_3, X_4\}$ or $\{X_4, X_5\}$
- Not $\{X_4\}$ (doesn't block every backdoor path), nor $\{X_6\}$ (descendent of X_i)

Return to the Waffle House



To estimate the direct effect of W on D, what do we condition on?

Example



What to condition on to estimate Exposure $\rightarrow$ Disease?

More on confounding and Simpson's paradox

Simpson's paradox

Very famous phenomenon: an observed association reverses direction after conditioning on another variable

Often framed as: population-wide association is reversed after stratification on every sub-population

- Kidney stones: treatment A succeeds more often than treatment B, but treatment B performs better on large stones and on small stones
- Graduate admissions: men admitted to graduate programs at a higher rate, but women more successful in admission to every individual department

Simpson's paradox: example

Fake data about a drug:

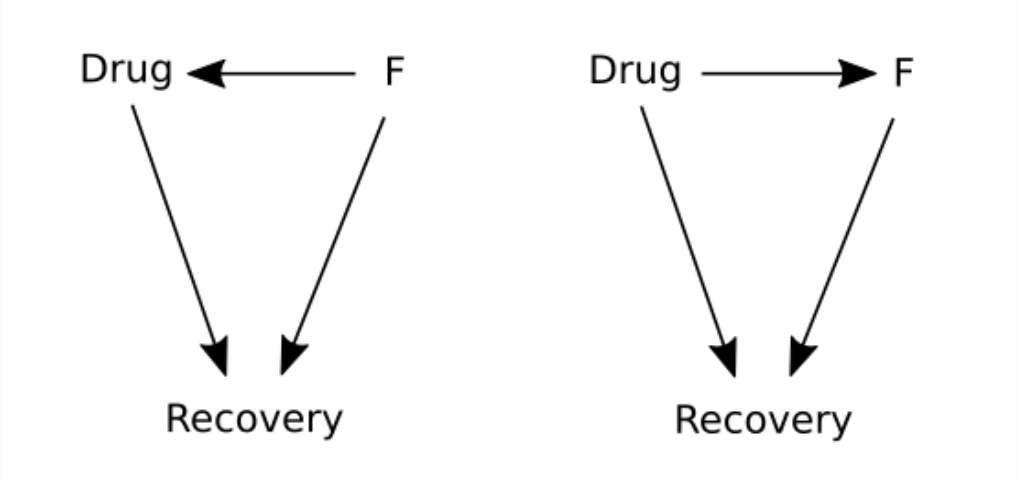
Combined	Recovered	Not recovered	% Recovery
Drug	20	20	50%
No drug	16	24	40%
$F = 1$	Recovered	Not recovered	% Recovery
Drug	18	12	60%
No drug	7	3	70%
$F = 0$	Recovered	Not recovered	% Recovery
Drug	2	8	20%
No drug	9	21	30%

The variable F is a potential confound; this data displays Simpson's paradox.

Question: does the drug help people recover?

Two DAGs

The data from the previous slide could be generated by processes represented by either of the following causal DAGs:



But the inference we should make about the effectiveness of the drug is very different in each case!

Situation 1: gender and compliance

Situation 1: F is a fork variable, influencing both recovery and treatment

Example:

- F is gender
- the drug negatively influences recovery
- men are both less likely to recover *and* less likely to take the treatment, so a positive association between treatment and recovery is observed in the pooled data

Action: to estimate causal effect of treatment, condition on the fork; conclude the treatment is bad

Situation 2: post-treatment effect

Situation 2: F is a treatment effect that mediates the recovery (a chain)

Example:

- F is blood pressure (high or low)
- One mechanism by which the drug works is by reducing blood pressure
- Controlling for post-treatment effect masks influence of the drug

Action: to estimate causal effect of treatment, don't condition on the post-treatment effect; conclude the treatment is good

Example: incumbency effect in US Congressional elections

Overview

Following section 14.3 in BDA:

- US congressional elections: every 2 years, all seats in the House of Representatives (currently 435) are open for re-election
- It is generally believed that the current officeholder (incumbent) has an advantage in these elections if they choose to run
- Goal: estimate the causal effect of incumbency on vote share

Data available

Data from BDA book website: vote totals and incumbency codes for congressional elections, 1896-1992.

Each year has:

- State and district ID codes
- Incumbency (coded -1, 0, 1 based on party and whether the incumbent ran)
- Democratic and Republican vote total

What we exclude:

- Races where a minor party member won
- Uncontested races (only one candidate)

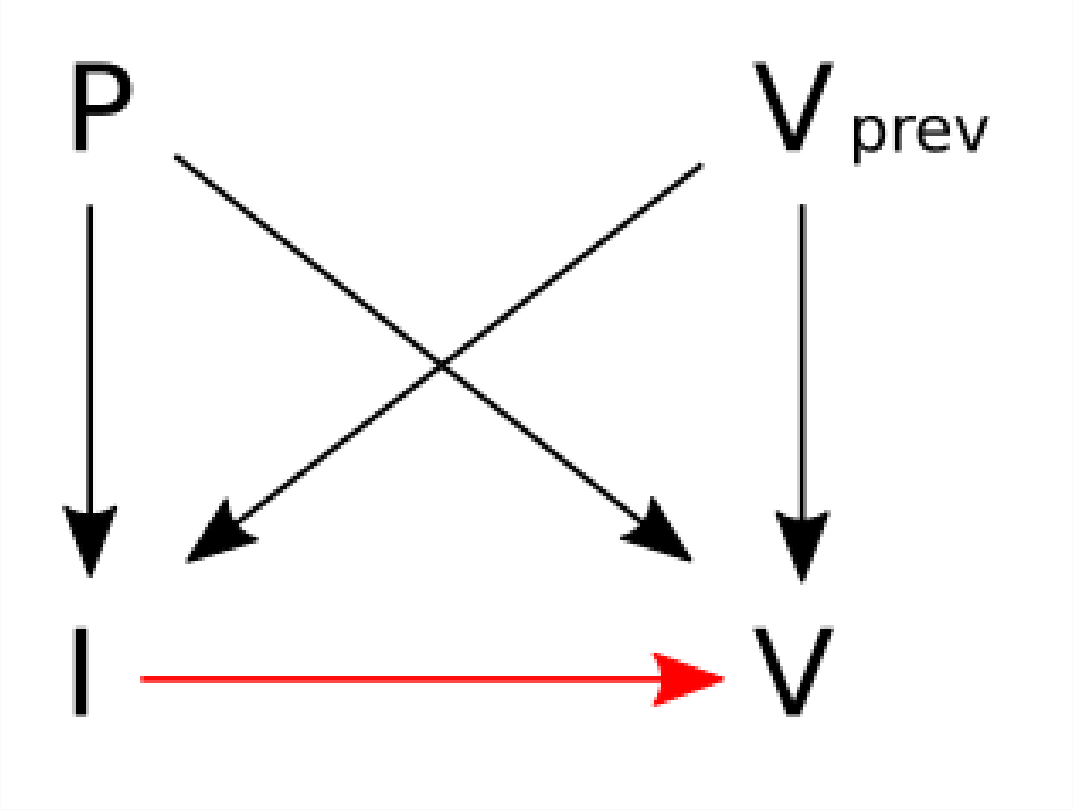
Two potential confounds

Two potential confounds:

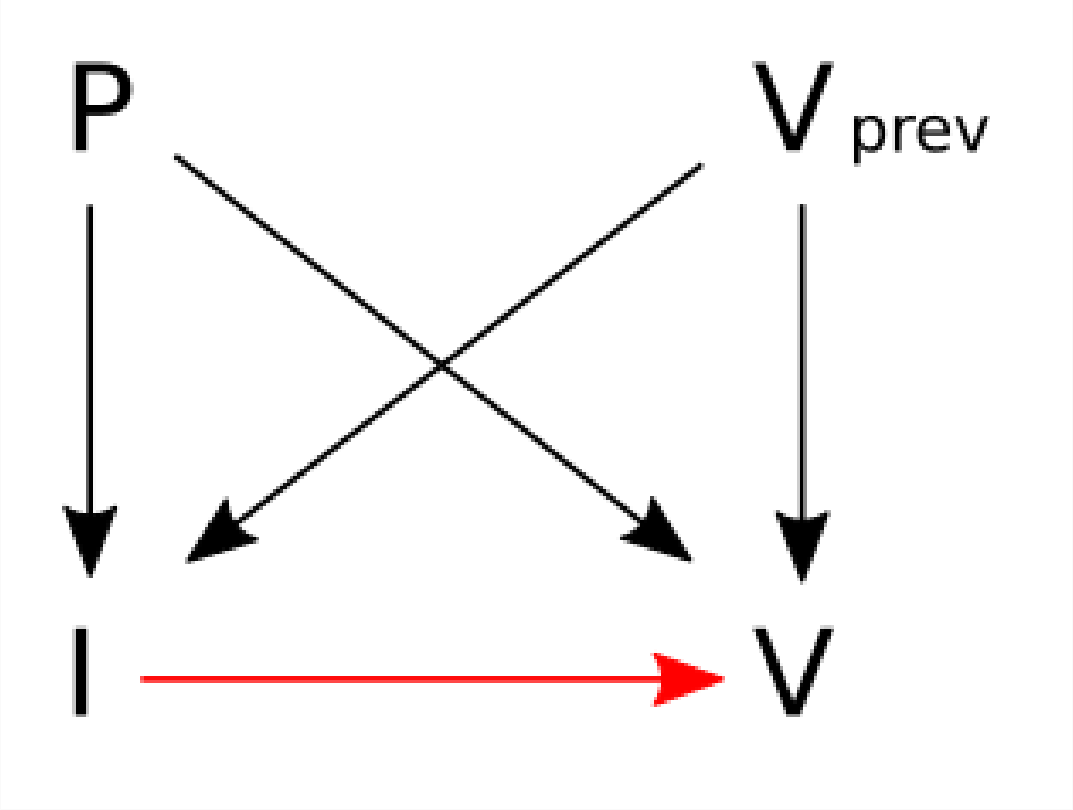
- Results of the previous election: if the incumbent thinks it may be a tight race, they may be less likely to run. Previous election results may be a proxy for the incumbent's knowledge of how safe their seat is.
- Political party: In some years, nationwide partisan shifts may influence both decisions to run and election results.

Let's draw a DAG.

Election DAG



Election DAG



Paths through P and V_{prev} are both forks, so we control for those variables. (Backdoor criterion)

The model

Let's write down the model:

$$
\begin{aligned}
V_i &\sim \text{Normal}(\mu_i, \sigma) \\
\mu_i &= \alpha + \beta_I I_i + \beta_P P_i + \beta_V V_i^{(t-2)} \\
\alpha &\sim \text{Normal}(0, 0.3) \\
\beta_I &\sim \text{Normal}(0, 0.3) \\
\beta_P &\sim \text{Normal}(0, 0.3) \\
\beta_V &\sim \text{Normal}(0, 0.3) \\
\sigma &\sim \text{HalfCauchy}(1)
\end{aligned}
$$

Adding another variable: campaign spending

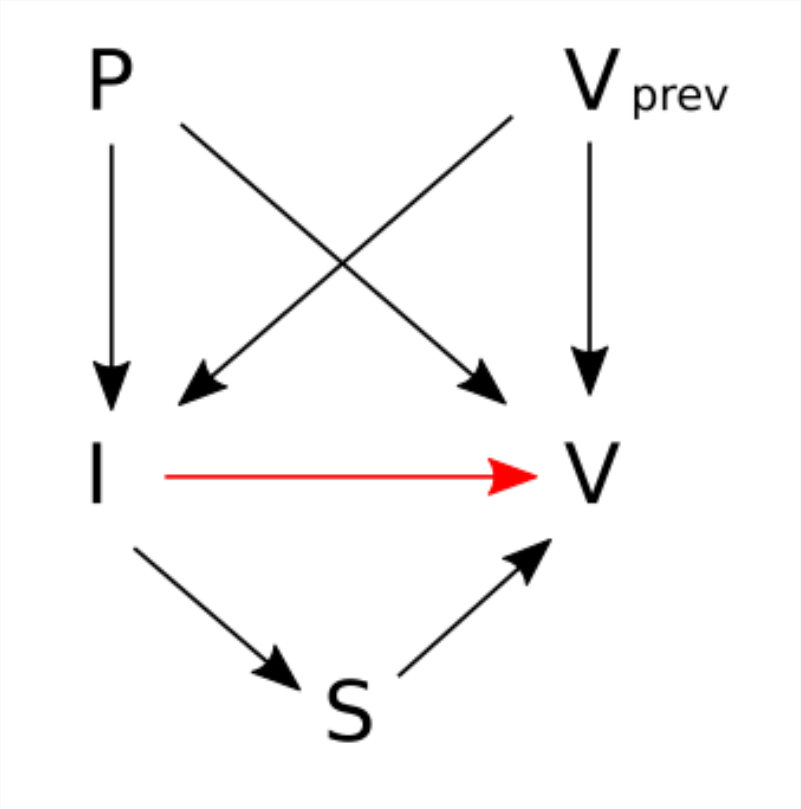
One way we can extend this model is to add some more variables.

A natural variable to add into this model for predictive purposes: campaign spending. But should we use it for this inference?

Let's add it to the DAG.

Campaign spending

Spending likely increases after the incumbent makes the decision to seek reelection, so we'll add the variable to the DAG as follows:



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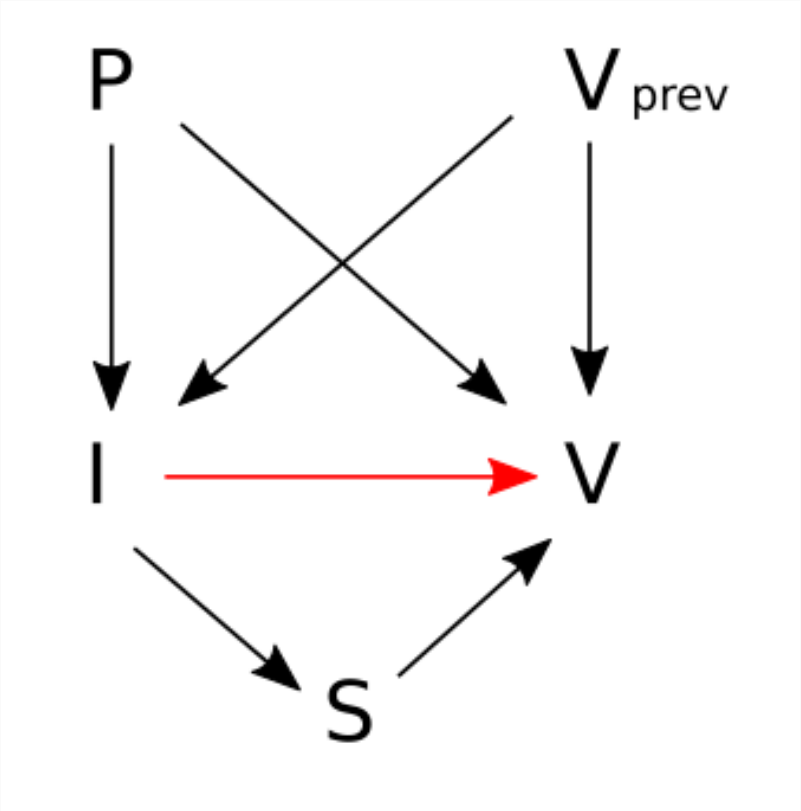

Backdoor criterion: don't include descendants of I.

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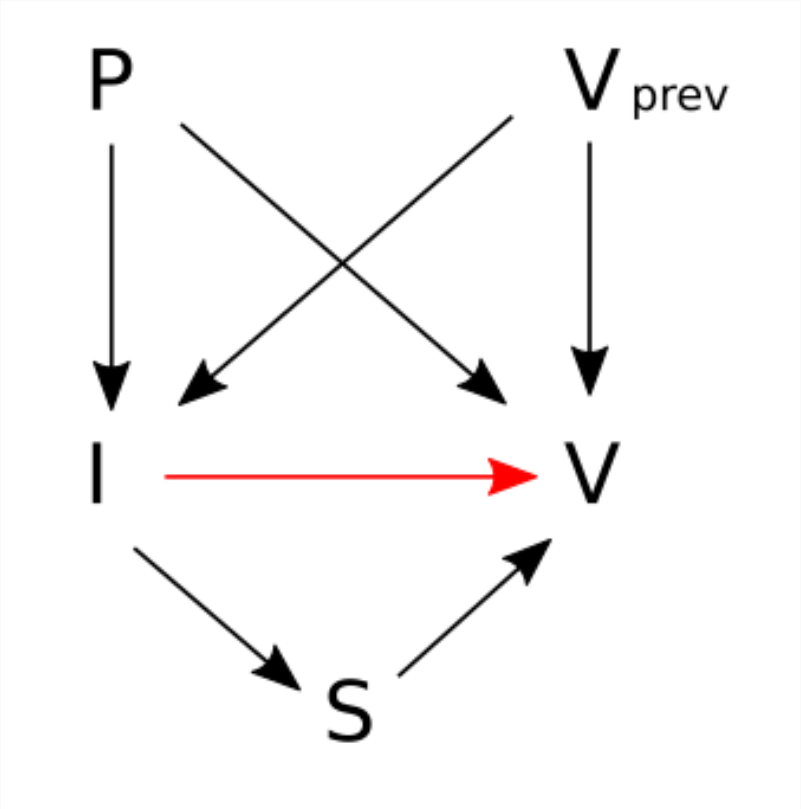

Post-treatment bias – some causal effect is blocked. Can include this if we want to predict election outcomes; don't include it to

Other improvements

What else could we do to this model?

- Other variables (spending, demographic data, urbanization)
- State level data $\rightarrow$ multi-level (hierarchical) model
- Add data from the past 30 years

Summary

Summary:

- DAGs are probabilistic and/or functional models of dependency in multi-variable systems
- Confounding and statistical "paradoxes" can be modeled by information flow through the graph
- DAGs can help us decide which variables to include in a model, based on what we want to infer/predict

Next week:

- Hierarchical models
- Examples in PyMC3 with sampling

HW3 available soon, due 9/25: fun with DAGs

Aside: multicollinearity and variable selection

Multicollinearity in regression

Multicollinearity is a problem that appears in linear regression when two or more predictors are not linearly independent – when they are tightly correlated with one another.

Multicollinearity is bad:

- models overly complex
- numerical problems in estimation
- uninformative inferences

Heuristic example

Problem: predict human height from leg length

- We could reasonably expect leg length to be a strong, but not perfect predictor
- Total height $\approx$ leg length $+$ torso length $+$ const.

What if we have more granular data: measurements on both left and right legs?

Heuristic example

Model based on artificial data:

Left leg only:

	mean	sd	hpd_3%	hpd_97%
betal	2.151	0.049	2.065	2.240
alpha	5.909	3.593	-0.921	12.224

Right leg only:

	mean	sd	hpd_3%	hpd_97%
betar	2.158	0.048	2.070	2.251
alpha	5.516	3.576	-1.820	11.498

Both legs:

	mean	sd	hpd_3%	hpd_97%
betal	-0.627	1.467	-3.279	2.232
betar	2.785	1.470	-0.007	5.509

Heuristic example

- We can precisely estimate the effect of left leg length on height
- We can precisely estimate the effect of left leg length on height
- If we try to estimate both at once, we lose all precision

The problem: once we control for one leg length, all that's left in the other observation is noise

Variable selection

To deal with multicollinearity, there are several tools for *model comparison*, to choose which variables should go in a model

- e.g., stepwise selection with information criteria (we'll cover information criteria later)
- variance inflation factor

While DAGs can be seen partially as a tool for fighting multicollinearity, it's not exactly the same:

- Variable selection: numerical tools based on fit of predictions to data. Attempt to maximize out-of-sample prediction accuracy.
- DAGs: external causal models, related to but distinct from the data. Attempt to estimate causal effects.