

Asymptotic Analysis – measures the order of growth of an algorithm in terms of the input size

$$1 < \log \log n < \log n < n^{1/3} < n^{1/2} < n < n^2 < n^3 < n^4 < 2^n < n^n$$

Big O Notation – upper/exact bound

Theta Notation – exact bound

Omega Notation – lower/exact bound

```
for (int i = 0; i < n; i = i + c) {  
    //  $\theta(1)$  work  
}  
 $\theta(n)$ 
```

```
for (int i = 0; i < n; i = i - c) {  
    //  $\theta(1)$  work  
}  
 $\theta(n)$ 
```

```
for (int i = 0; i < n; i = i * c) {  
    //  $\theta(1)$  work  
}  
 $\theta(\log n)$ 
```

```
for (int i = 0; i < n; i = i / c) {  
    //  $\theta(1)$  work  
}  
 $\theta(\log n)$ 
```

```
for (int i = 0; i < n; i = pow(i, c)) {  
    //  $\theta(1)$  work  
}  
 $\theta(\log \log n)$ 
```

add subsequent loop values

multiply nested loop values

```

void fun (int n) {
    if (n <= 0)
        return;
    //  $\theta(1)$  work
    fun (n / 2);
    fun (n / 2);
}
 $T(n) = 2T(n/2) + \theta(1)$ 
 $T(0) = \theta(1)$ 

```

```

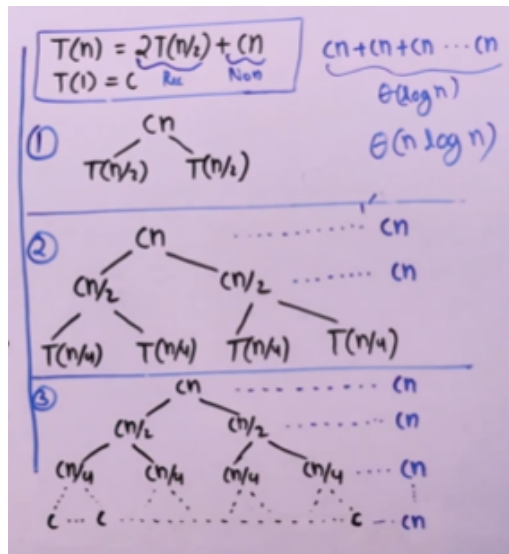
void fun (int n) {
    if (n <= 0)
        return;
    for (int i = 0; i < n; i = i++) {
        //  $\theta(1)$  work
    }
    fun (n / 2);
    fun (n / 3);
}
 $T(n) = T(n/2) + T(n/3) + \theta(n)$ 
 $T(0) = \theta(1)$ 

```

```

void fun (int n) {
    if (n <= 1)
        return;
    //  $\theta(1)$  work
    fun (n - 1);
}
 $T(n) = T(n - 1) + \theta(1)$ 
 $T(1) = \theta(1)$ 

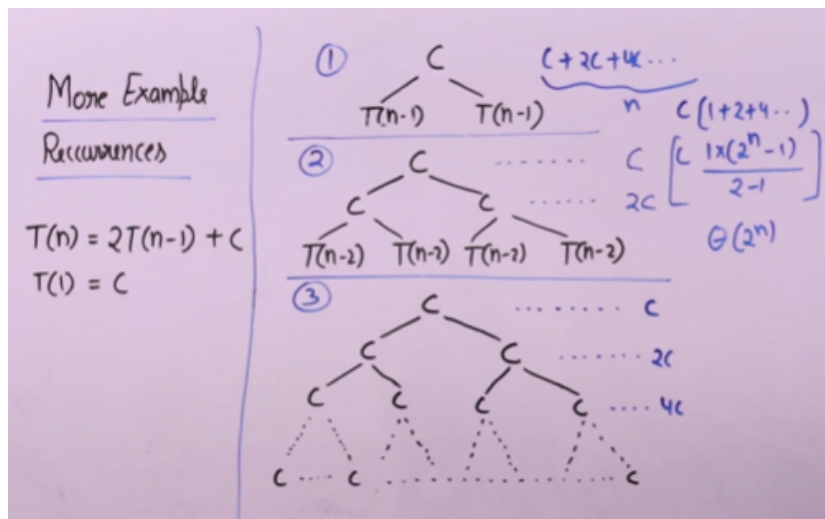
```



$$T(n) = 2T(n/2) + c_1n$$

$$T(1) = c_2$$

$$\Theta(n \log n)$$

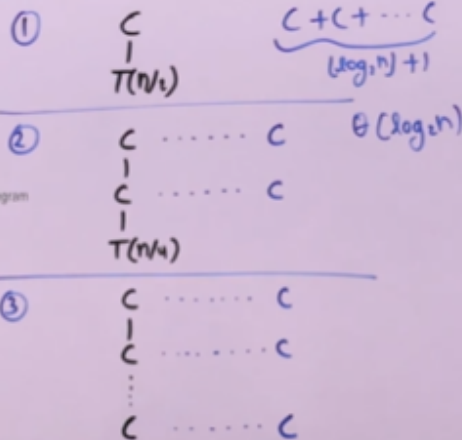


Morse Example

Recurrences

$$T(n) = T(n/2) + c$$

$$T(1) = c$$

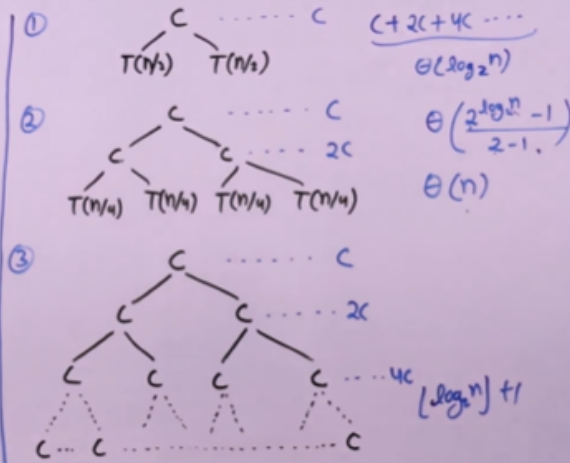


Morse Example

Recurrences

$$T(n) = 2T(n/2) + c$$

$$T(1) = c$$



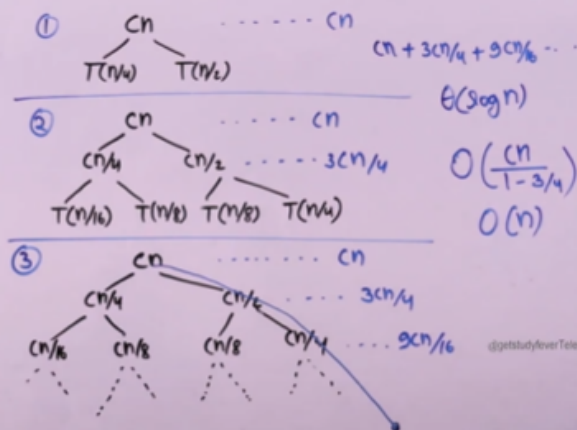
Upper Bounds

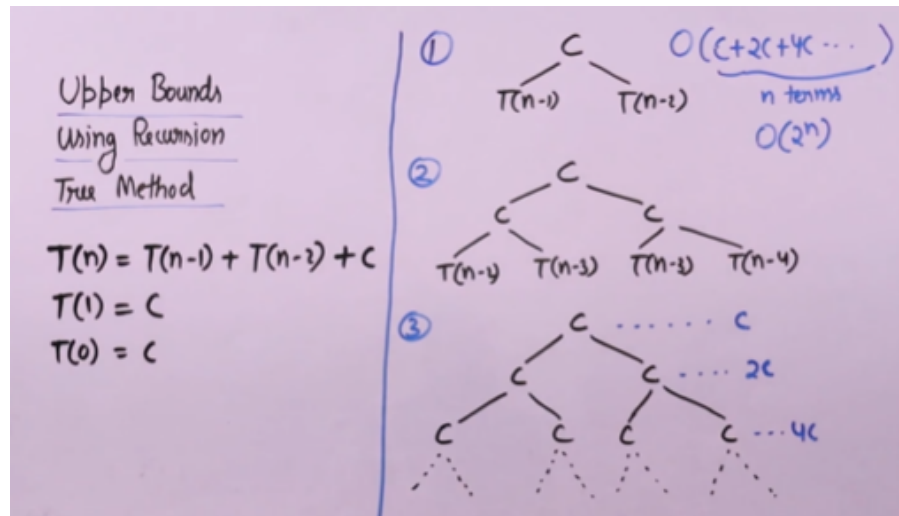
Using Recursion

Tree Method

$$T(n) = T(n/4) + T(n/2) + cn$$

$$T(1) = c$$





Space Complexity –

```
int get_sum1 (int n) {
    return n * (n + 1) / 2;
}
```

$\theta(1)$

```
int get_sum2 (int n) {
    int sum = 0;
    for (int i = 1; i <= n; i++)
        sum = sum + i;
    return sum;
}
```

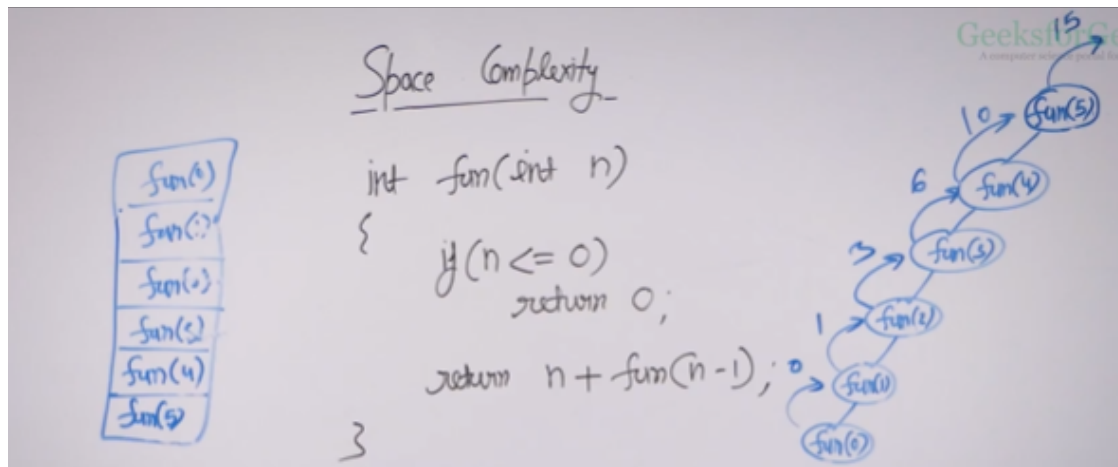
$\theta(1)$

```
int arr_sum (int a [], int n) {
    int sum = 0;
    for (int i = 0; i < n; i++)
        sum += a [i];
    return sum;
}
```

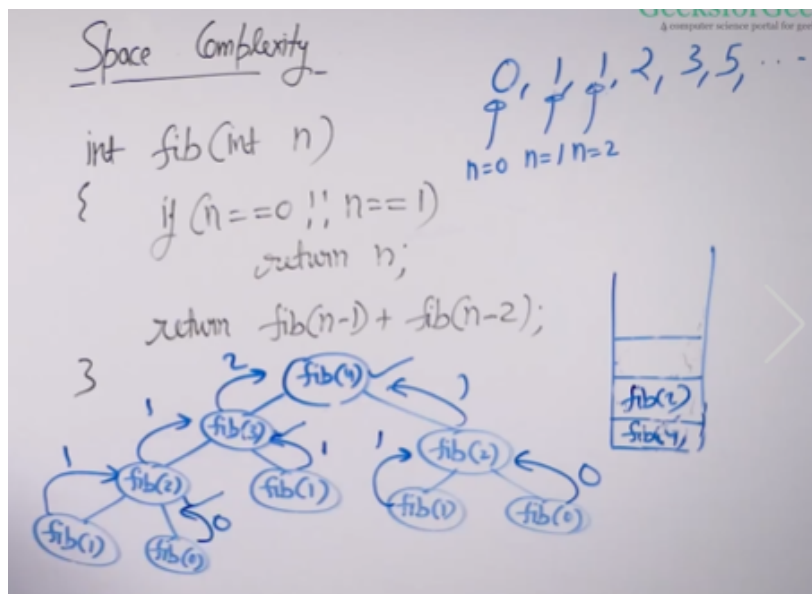
$\theta(n)$

Auxiliary Space – measures the order of growth of extra space

```
int arr_sum (int a [], int n) {  
    int sum = 0;  
    for (int i = 0; i < n; i++)  
        sum += a [i];  
    return sum;  
}  
 $\theta(1)$ 
```



$\theta(n)$



$\theta(n)$