Cairo University **Faculty of Engineering**

Credit hour system **Fall 2023**

MTHS114 **Numerical Analysis**

Assignment 10

Numerical Solution of PDE's

1. **Use finite difference method to set the equations that solve numerically the following partial differential equation: $u_{xx} + u_{yy} = 2u$.**

   with initial and boundary conditions: $u(0, y) = y^2$ $u(x, 1.5) = 4$.

   $u(x, 0) = e^x$ $u_x(1.5, y) = 2y$. **(use backward difference.)**

   where $0 \le x, y \le 1.5$ $(h = \ k = \frac{1}{2})$

   Write the obtained equations in a matrix form.

2. **Find numerically the solution of the partial differential equation: $u_{tt} = 9u_{xx}$ at the points $u(0.1, 0.1)$, $u(0.2, 0.1)$, $u(0.3, 0.1)$ and $u(0.1, 0.2)$, With the following initial and boundary conditions:**

   $u(x, 0) = 3sin5\pi x$

   $u_t(x, 0) = 0$ **(use central difference)**

   $u(0, t) = u(0.4, t) = 0$

   $0 \le x \le 0.4$ **and** $t \ge 0$ **(Let $h = k = 0.1$).**

3. **Use finite difference method to set the equations that solve numerically the following partial differential equation: $u_{xt} = u_{tt} - u_{xx} + e^{-xt}$.**

   with boundary conditions: $u(0, t) = 0$ $u(1, t) = 0$.

   and initial conditions: $u(x, 0) = 1 - x$ $u_t(x, 0) = e^x$. **(use CD)**

   where $0 \le x \le 1$ and $0 \le t$. ($h = \frac{1}{4}$ and $k = \frac{1}{2}$) . Write the equations of *u* for the first two lines (i.e. *t* = 1/2).

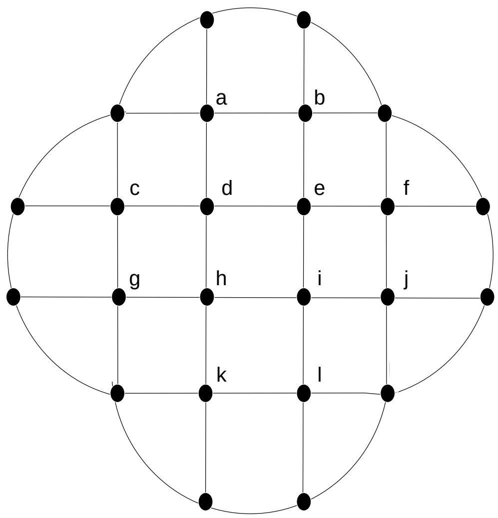
4. **Use finite difference method to find $u(1, 1/3)$ and $u(1, 2/3)$ for the following partial differential equation: $u_{xx} + u_{yy} = u$ for $0 \le x, y \le 1$, with the below initial and boundary conditions:**

   $u(0, y) = 2\cos y$ $u(x, 0) = \ln(x+1)$

   $u_x(1, y) = 2y$ **(B.D.)** $u_y(x, 0) = 2x$ **(C.D.)**

   Let $h = k = 1/3$

5. **Solve the linear system of equations obtained by the finite difference method, for The given partial differential equation: $u_{xx} + u_{yy} - 16u_x = 0$, at the grid points of the section shown below (with $h = k = \frac{1}{4}$). Knowing that $u = 75$ on all boundaries. Use central difference for u_x**



6. **For the figure shown below, use finite difference method to set the equations that solve numerically the following partial differential equation:**

$$u_{xx} + u_{yy} = uxy$$

At the given points: $u_{1,0}$, $u_{1,-1}$, $u_{-2,1}$, $u_{2,1}$, $u_{2,2}$, $u_{3,3}$, $u_{3,2}$. (Do not solve it).
With the following boundary conditions:

$u(x,1) = 1$ $\qquad$ $u(1,y) = 1$.

$u(x,y) = 0$ **on rhombus boundaries.**

$u(x,y) = 9$ **on the line joining the points (0,-1) and (-1,0)**

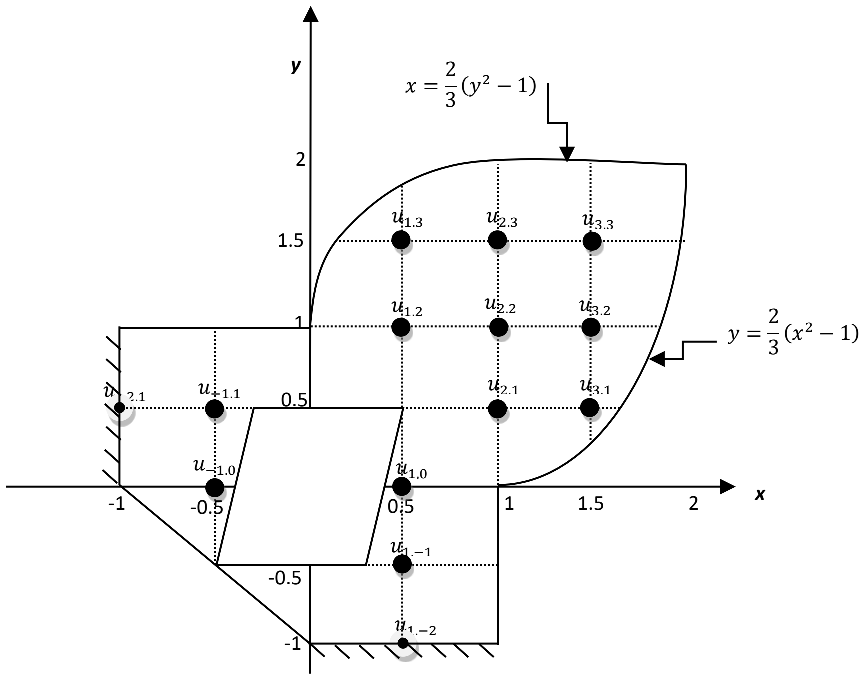
$u(x,y) = 27$ **on the parabolas: $y = \frac{2}{3}(x^2 - 1)$ and $x = \frac{2}{3}(y^2 - 1)$.**

$u_x(-1,y) = 0$ **and** $u_y(x,-1) = 0$ **(use central difference)**

Where $-1 \le x, y \le 2$ $\qquad$ $(h = k = \frac{1}{2})$

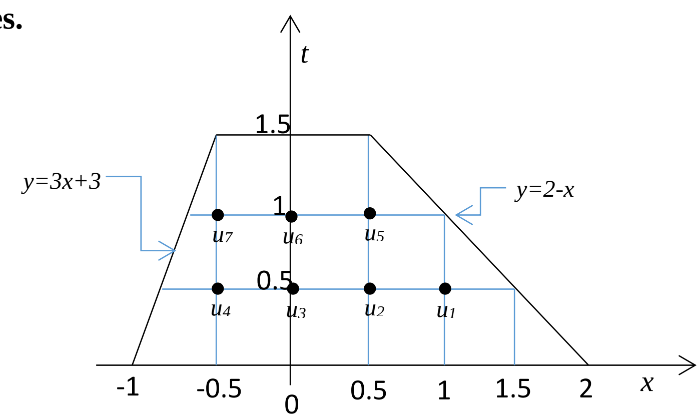
(Hint: The four sides of the rhombus are:

$y = \pm 0.5$, $\qquad$ $y = 4x - 1.5$ $\qquad$ **and** $\qquad$ $y = 4x + 1.5$



7. **Use finite difference method to <u>write</u> the linear system of equations that solves the Poisson equation: $u_{xx} + u_{tt} = 2$, at the shown grid points of the shape in the figure below. $(h = k = 0.5)$**

   The initial and boundary conditions are: $u(x, 0) = 8$, $u(x, 1.5) = 4$, and $u(x, y) = 0$ on the sides.



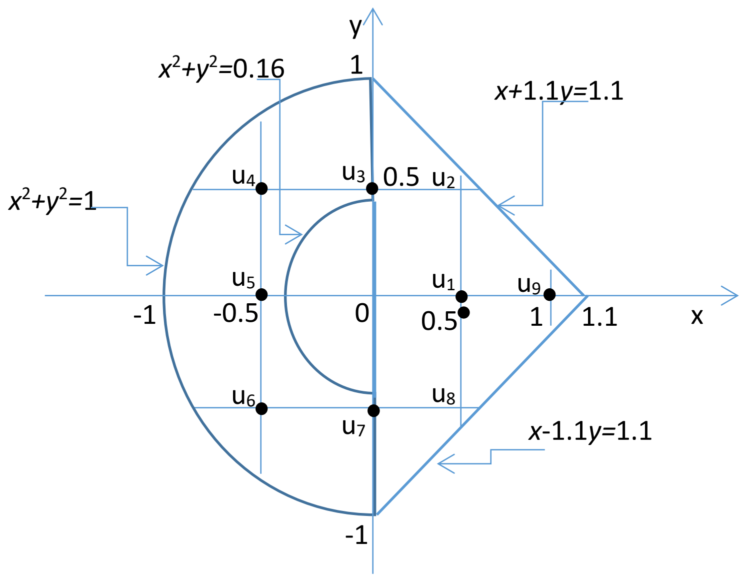
8. **For the figure shown below, use finite difference method to <u>write</u> the equations that solve numerically the following partial differential equation:**

$$u_{xx} + u_{yy} = u + \cos xy$$

for the points: $u_1, u_2, u_3, \ldots u_9$, with the following boundary conditions:

$u(x, y) = 2$ on the large semicircle $\qquad$ $u(x, y) = 1$ on the small semicircle

$u(x, y) = 0$ $\qquad$ on the triangle $\qquad$ $(h = \ k = \frac{1}{2})$ $\qquad$ (Do not solve it)



9. **For the figure shown below, use finite difference method to <u>write</u> the equations that solve numerically the following partial differential equation:**

$$\boldsymbol{u_{xx} + u_{yy} + 2u = e^{x}}$$

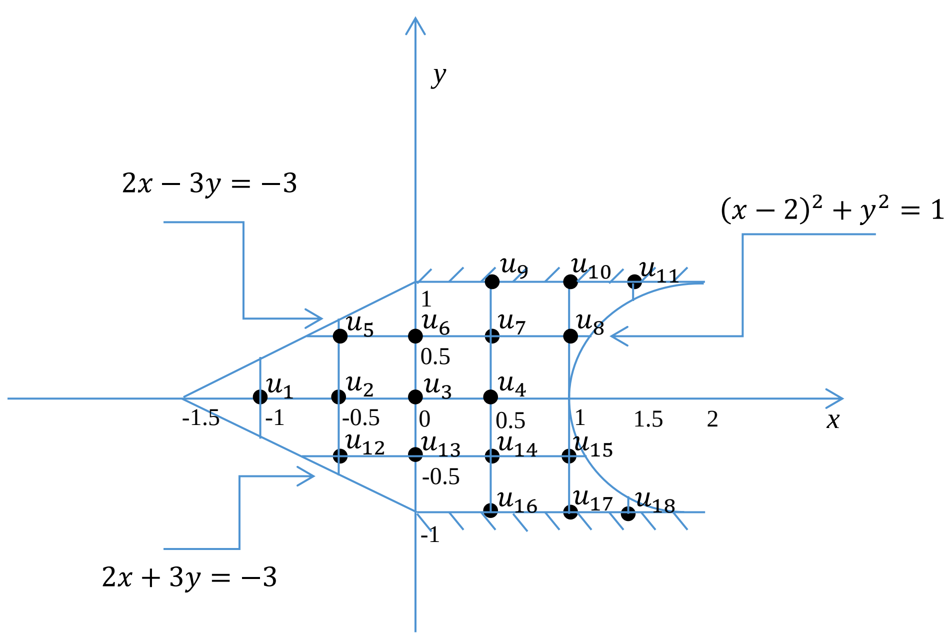
for the points: $u(0, 0.5)$, $u(0.5, 1)$, $u(1, 0.5)$, $u(-0.5, -0.5)$, $u(-1, 0)$, with the following boundary conditions:

$u(x, y) = 2$ on the semicircle

$u(x, y) = 1$ on the two straight lines of the triangle

$u_y = 0$ on the insulated boundaries: $y = \pm 1$ (use Central Difference)

(Do not solve the equations)



10. For the figure shown below, use finite difference method to <u>write</u> the equations that solve numerically the following partial differential equation:

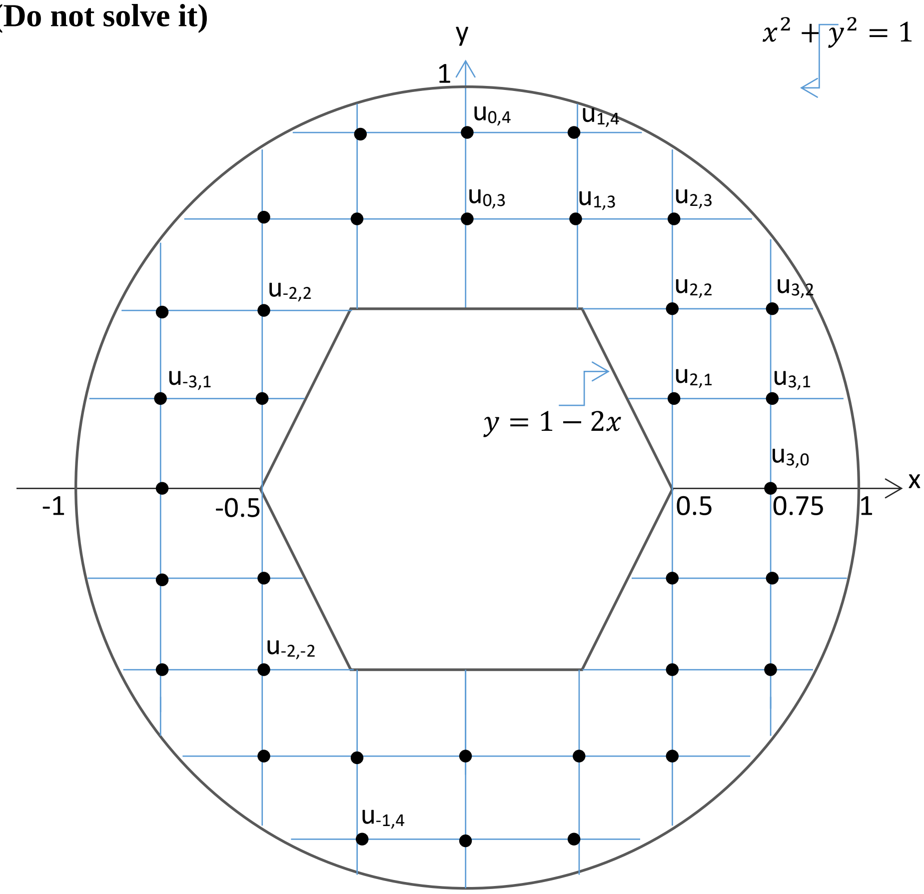
$$u_{xx} + u_{yy} + 2u = 3|xy|$$

for the points: $u_{-2,2}, u_{2,1}, u_{3,2}, u_{0,-4}$ with the following boundary conditions:

$u(x,y) = 5$ on the circle

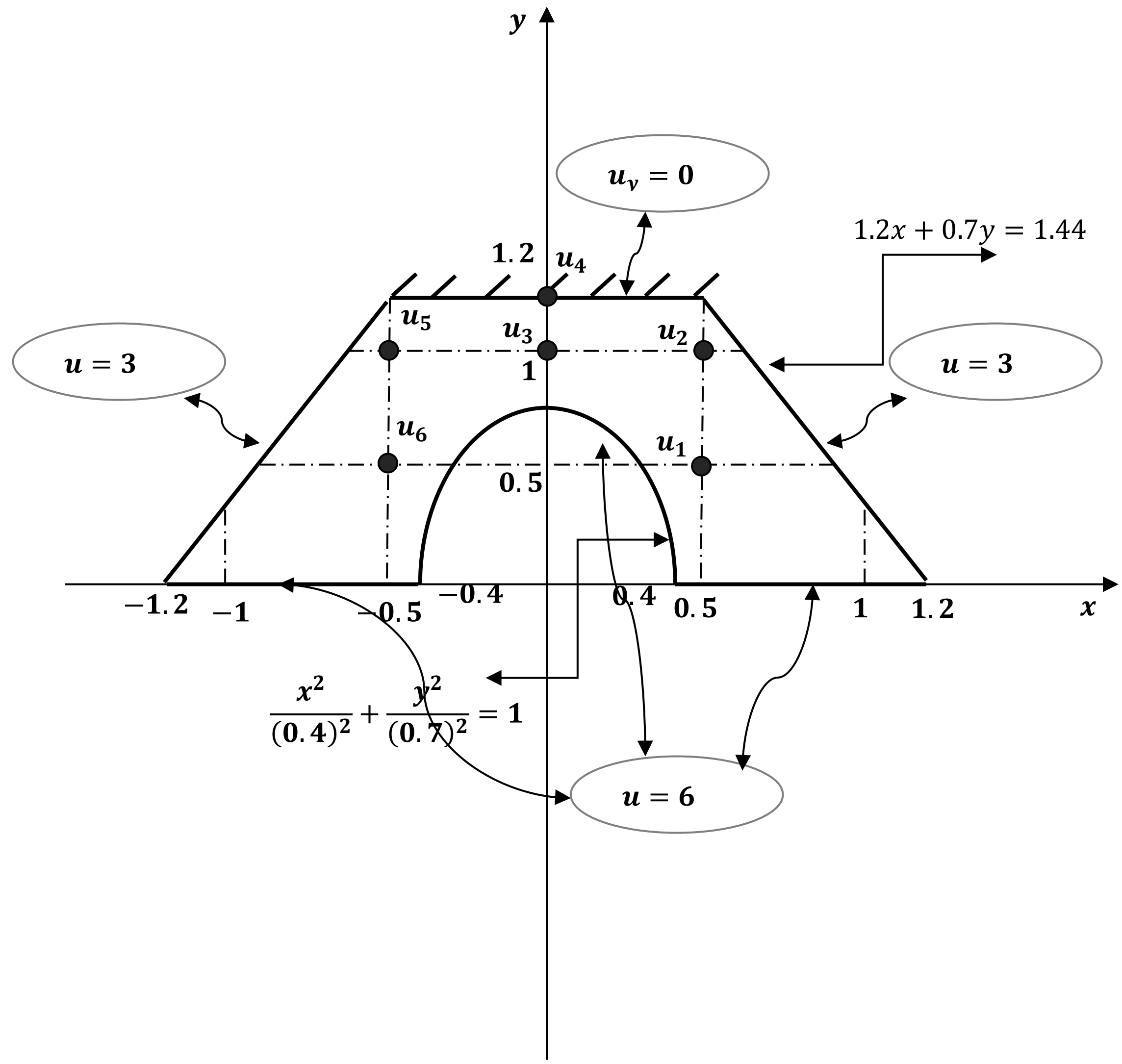
$u(x,y) = 4$ on the hexagon

(Do not solve it)



11. Write the equations required for solving the Partial Differential Equation: $u_{tt} = u_x + u$, for the first two lines, where $0 \le x \le 1$, $t \le 0$ and a mesh with $h = \frac{1}{3}$ and $k = \frac{1}{4}$. The initial and boundary conditions are: $u_t(x,0) = 5x$, $u(1,t) = 1$, $u(x,0) = 4$ and $u(0,t) = 5y$. (Use central difference for u_x, forward difference for u_{tt}, and backward difference for the boundary $u_t(x,0)$)

12. Write the equations that solve the partial differential equation $u_{yy} = 3u_{xx} + u\cos x$ at the six indicated points in the figure below, with the shown initial and boundary conditions. $(h = k = 0.5)$. Use central difference on the insulated boundary.



13. Write the steps required for solving the Partial Differential Equation: $u_t = u_{xx}$, where $0 \le x, t \le 1$ and a square mesh of size 0.1 . The initial and boundary conditions are:

$u(0, t) = u(1, t) = k$ **and** $u(x, 0) = \sin 2\pi x$.

(Use forward difference for the PDE)

14. **Write the equations that solve the partial differential equation:**

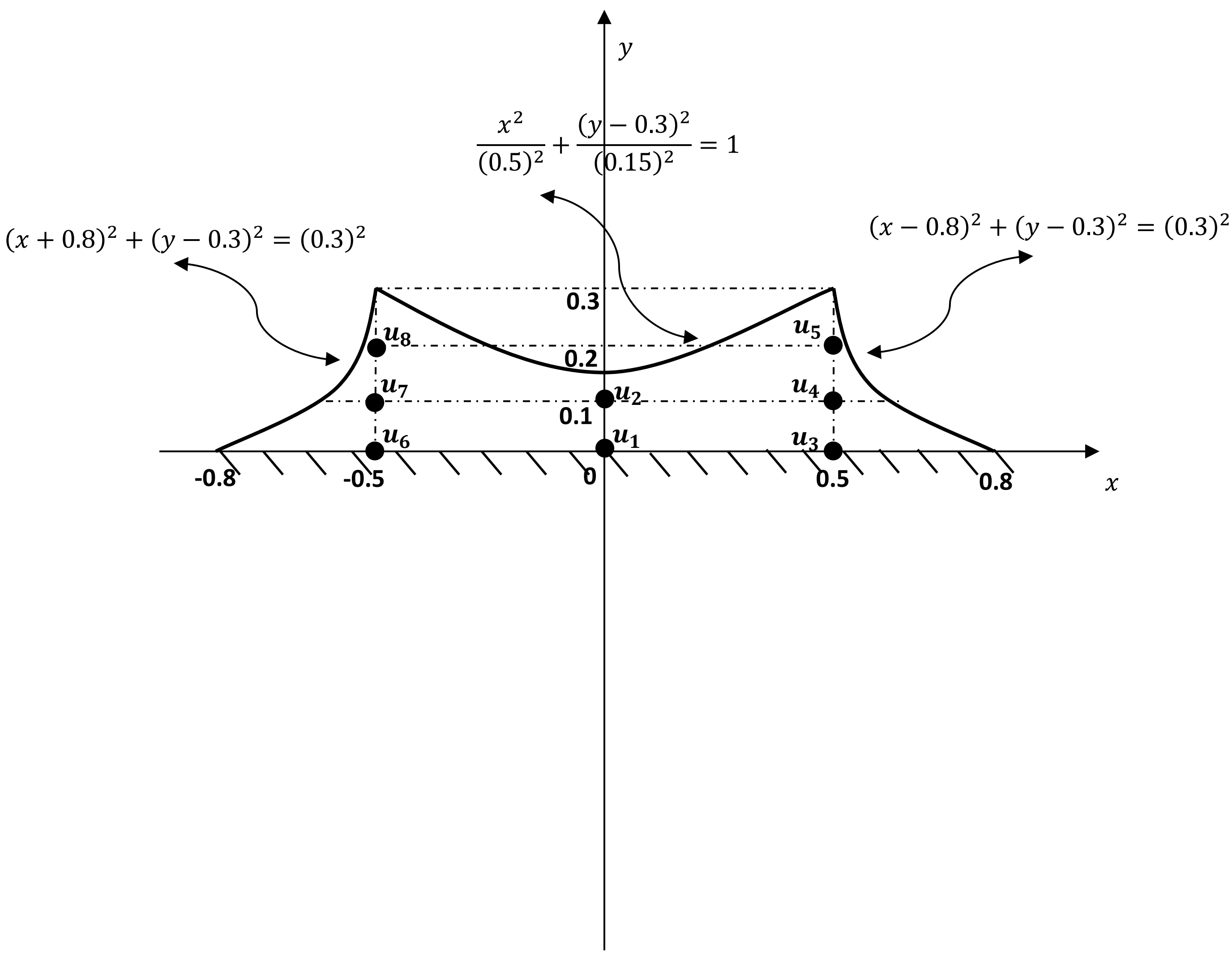
$$2u_{xx} + u_{yy} + e^{x^2}u = y^2 \cos x \ ,$$

at the points indicated in the figure below, with $h = 0.5$ and $k = 0.1$.

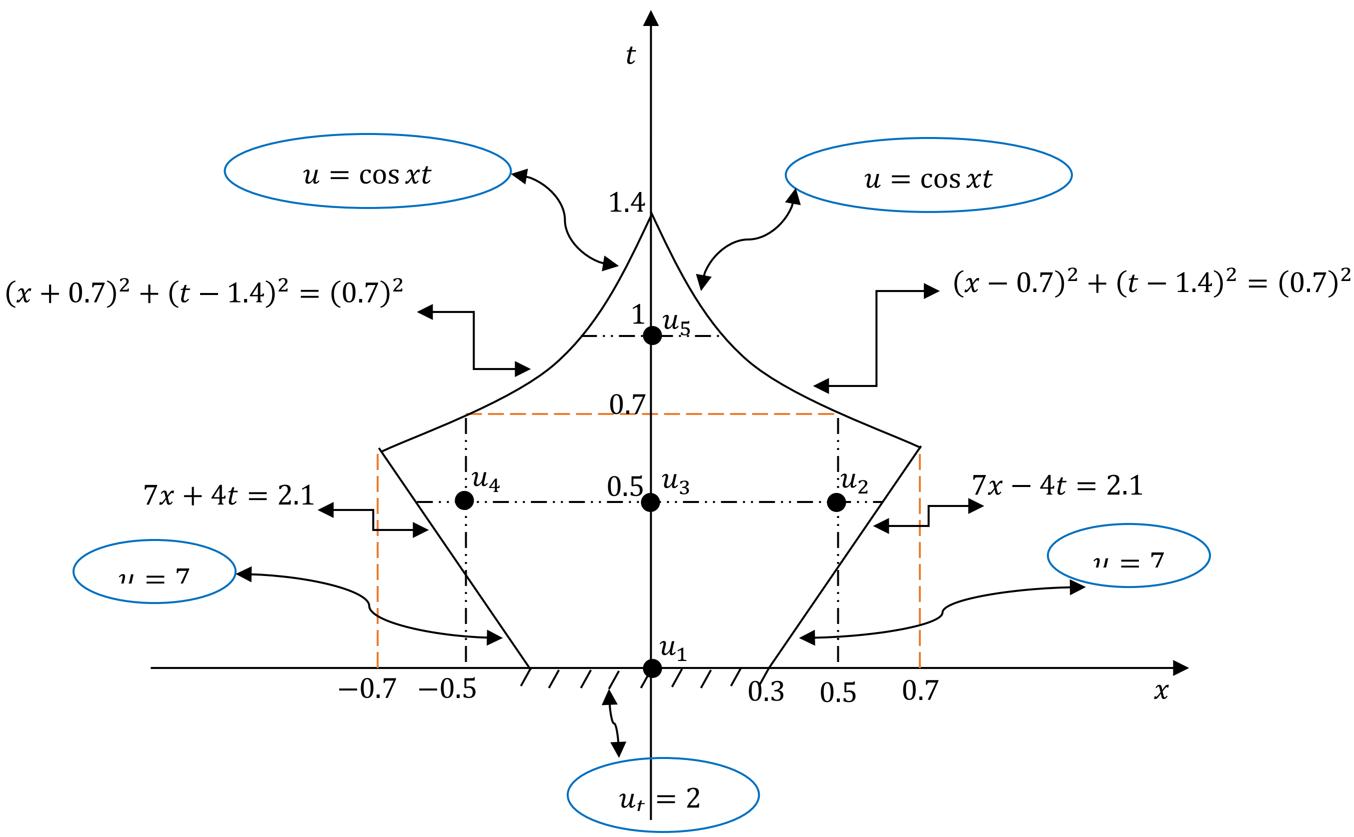
The initial and boundary conditions are:

$u = \sin\left|\frac{x}{y}\right|$ **on the semi-ellipse** $u = x^2y^2$ **on both quarter circles**

$u_y(x,0) = 2,$ **Use backward difference**



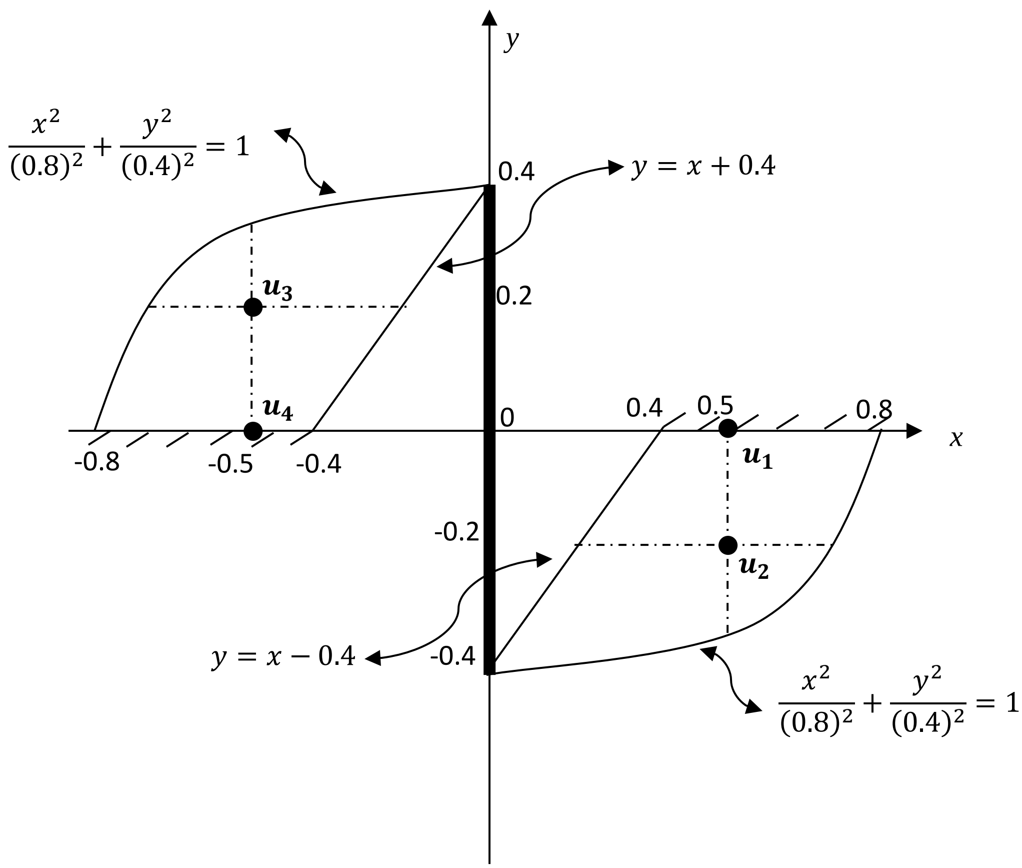
15. Write the equations that solve the partial differential equation $u_{tt} = u_{xx} + u$ at the five points indicated in the figure below, with the shown initial and boundary conditions. $(h = k = 0.5)$. Use central difference on the insulated boundary.



16. A steel rod is attached to two iron sheets of a propeller as shown in the figure. Assuming the thickness of the rod is negligible, write the equations that solve the partial differential equation $3u_{yy} + 2u_{xx} - u = e^{|xy|}$ at the four points indicated, with $h = 0.5$ and $k = 0.2$. The boundary conditions are as follows:

$u = \sin x^2 y^2$, on the quarter ellipse **$u = 8$ on the straight lines**

$u_y(x, 0) = 0$ (use central difference).



17. Write the equations that solve the partial differential equation:

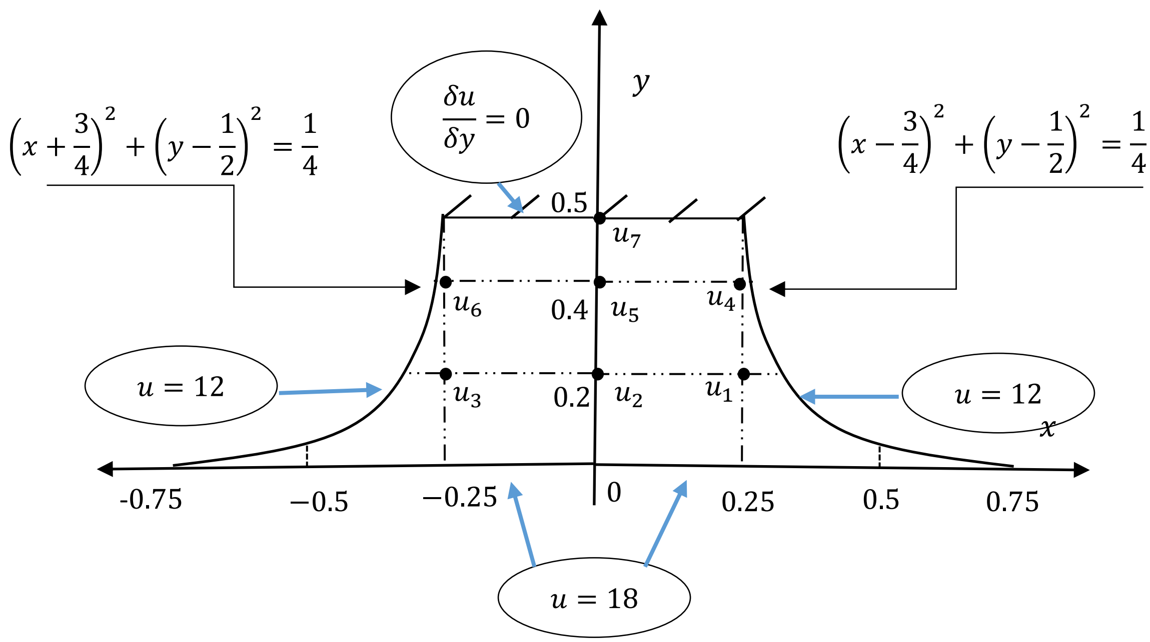
$\boldsymbol{2u_{yy} + 3u_{xx} - u_x = \ln|xy|}$, at the indicated points on the figure below, where the initial and boundary conditions are as follows:

$\boldsymbol{u_y = 0}$ on the line $\boldsymbol{y = 0.5}$

$\boldsymbol{u = 12}$ on the two quarter circles.

$\boldsymbol{u = 18}$ on the x-axis.

Use central difference for all partial derivatives ($\boldsymbol{h = 0.25 \text{ and } k = 0.2}$) [8]



18. **Find the solution of the partial differential equation: $\boldsymbol{u_{xx} - 2u_x = u_{tt}}$ at the point (0.4,-0.3), where $\boldsymbol{0 \le x \le 0.8}$ and $\boldsymbol{t \ge -0.9}$. The initial and boundary conditions are:**
$\boldsymbol{u(0,t) = u(0.8,t) = t^2}$, $\boldsymbol{u(x,-0.9) = 1}$ and $\boldsymbol{u_t(x,-0.9) = x}$ (use forward difference).
The mesh size is : $\boldsymbol{h = 0.2}$ and $\boldsymbol{k = 0.3}$

19. <u>Write</u> the equations that solve the partial differential equation:

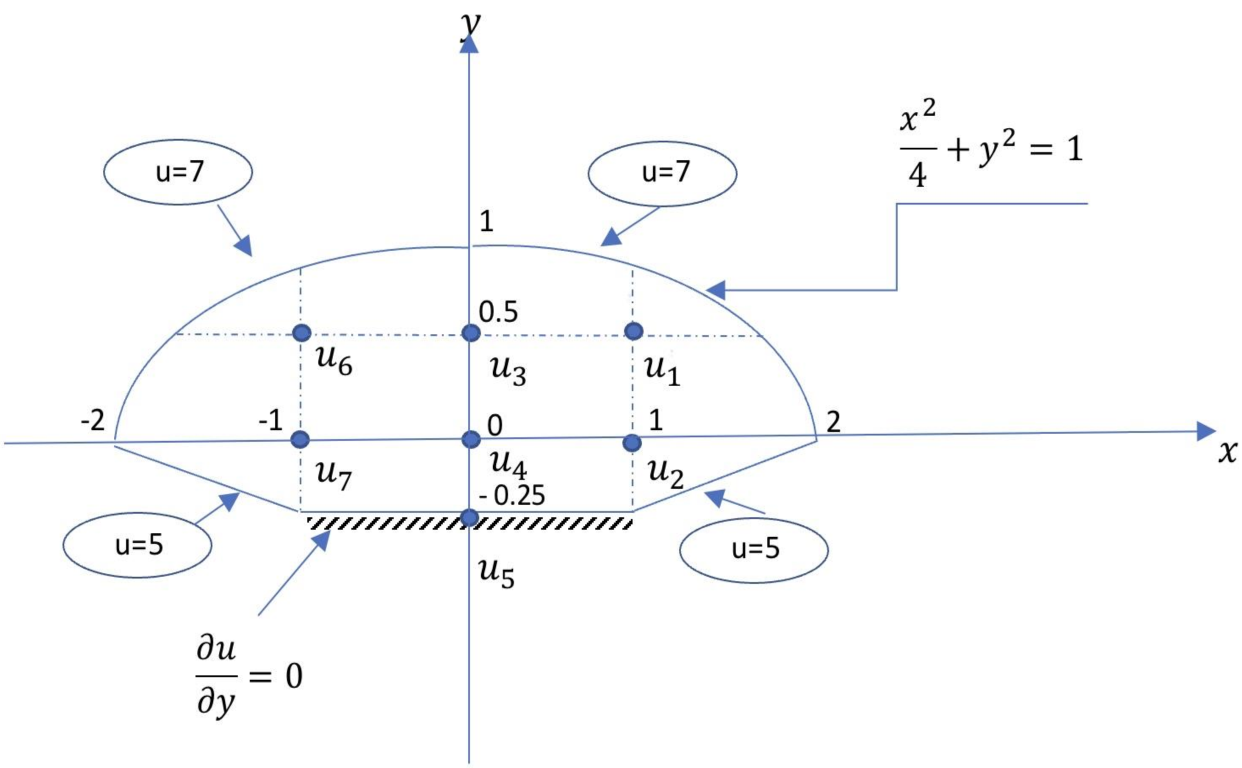
$x^2u_{xx} + yu_{yy} - u = 0$, at all points on the figure below, where the initial and boundary conditions are as follows:

$u_y = 0$ on the line $y = -0.25$

$u = 7$ on the ellipse whose equation is: $\frac{x^2}{4} + y^2 = 1$

$u = 5$ on the straight lines.

Use central difference for all partial derivatives ($h = 1$ and $k = 0.5$)



20. Write the equations that solve the partial differential equation: $3\,u_{xx} + 2u_{tt} = 5$ at the points: (0,0) , (0.4 , 0) and (-0.4 , -0.25) in the region bounded by the straight lines

$x = \pm 0.4$ and the functions $y = |x| + 0.25$ and $y = -|x| - 0.25$.

The initial and boundary conditions are:

$u_x(\pm 0.4, y) = 1$ (use Central difference) and $u = 0$ elsewhere. ($h = 0.4$ and $k = 0.25$)

21. Write the equations that solve the partial differential equation:

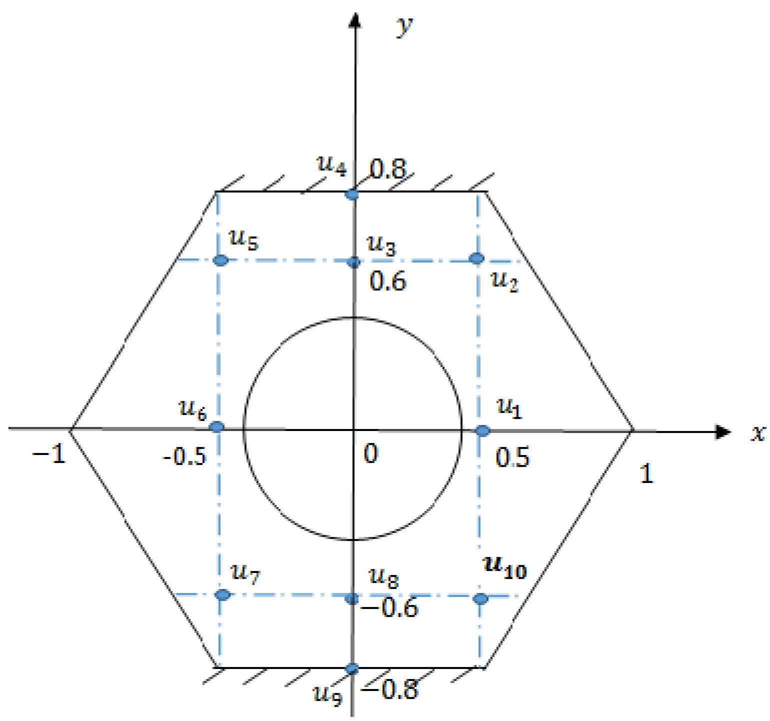
$\boldsymbol{u_{xx} + y u_{yy} - 4u = 0}$, at all points on the figure below, where the initial and boundary conditions are given by:

$\boldsymbol{u_y = 0}$ on the line $\boldsymbol{y = \pm 0.8}$.

$\boldsymbol{u = 4}$ on the circle whose equation is: $\boldsymbol{x^2 + y^2 = 0.16}$

$\boldsymbol{u = 5}$ on the straight lines.

Use central difference for all partial derivatives ($\boldsymbol{h = 0.5}$ **and** $\boldsymbol{k = 0.6}$) [8]



22. Find the solution of the partial differential equation: $\boldsymbol{2u_{xx} - u_{tt} + u_x + 20 = 0}$ **at the points: (0.25,0.1), (0.25, 0.2) and (0.5, 0.1) in the region bounded by the straight lines** $\boldsymbol{x \geq 0}$ **and** $\boldsymbol{0 \leq t \leq 0.5}$ **, given the following initial and boundary conditions:** $\boldsymbol{u_x(0,t) = 0}$ **(B.D),** $\boldsymbol{u(0,t) = 4t}$ **and** $\boldsymbol{u(x,0) = u(x,0.5) = 2}$**. (**$\boldsymbol{h = 0.25}$ **and** $\boldsymbol{k = 0.1}$**)**