1.1 Module for Stirring Turbulence

Here we summarize the method for driving turbulence used in Federrath et al. (2010, A&A, 512, A81). Please refer to that paper for further details.

1.1.1 Basics of Turbulent Stirring

Turbulence decays in about a crossing time, because the kinetic energy carried by the turbulence dissipates on small scales and turns into heat. In order to study the statistics of turbulence (e.g., the PDF, power spectrum, structure functions, etc.) over a significant time period thus requires continuous stirring (also called driving or forcing) with a turbulent acceleration field, which we call $\vec{f}(\vec{x},t)$ in the following.

The stirring field $\vec{f}$ is often modeled with a spatially static pattern for which the amplitude is adjusted in time. This results in a roughly constant energy input on large scales, but has the disadvantage that the turbulence is not really random, because the large-scale pattern is fixed, which may introduce undesirable systematics. Other studies model $\vec{f}$ such that it can vary in time *and* space to achieve a smoothly varying pattern that resembles the flow of kinetic energy from scales larger than the simulation box scale. The most widely used method to achieve this is the Ornstein-Uhlenbeck (OU) process. The OU process is a well-defined stochastic process with a finite autocorrelation timescale. It can be used to excite turbulent motions in 3D, 2D, or 1D simulations as explained in Eswaran & Pope (1988, Computers & Fluids, 16, 257).

The OU process is a stochastic differential equation describing the evolution of $\widehat{\vec{f}}$ in Fourier space (k-space):

$$\mathrm{d}\widehat{\vec{f}}\,(\vec{k},t) = f_0\,(\vec{k})\;\underline{\mathcal{P}}^{\zeta}(\vec{k})\;\mathrm{d}\vec{\mathcal{W}}(t)\;-\;\widehat{\vec{f}}\,(\vec{k},t)\,\frac{\mathrm{d}t}{T}\;. \tag{1.1}$$

The first term on the right hand side is a diffusion term. This term is modeled by a Wiener process $\vec{\mathcal{W}}(t)$, which adds a Gaussian random increment to the vector field given in the previous time step $\mathrm{d}t$. Wiener processes are random processes, such that

$$\vec{\mathcal{W}}(t) - \vec{\mathcal{W}}(t-\mathrm{d}t) = \vec{\mathcal{N}}(0,\mathrm{d}t)\;, \tag{1.2}$$

where $\vec{\mathcal{N}}(0,\mathrm{d}t)$ denotes the 3D, 2D, or 1D version of a Gaussian distribution with zero mean and standard deviation $\mathrm{d}t$. This is combined with a projection using the projection tensor $\underline{\mathcal{P}}^{\zeta}(\vec{k})$ in Fourier space. In index notation, the projection operator reads

$$\mathcal{P}^{\zeta}_{ij}\,(\vec{k}) = \zeta\,\mathcal{P}^{\perp}_{ij}\,(\vec{k}) + (1-\zeta)\,\mathcal{P}^{\parallel}_{ij}\,(\vec{k}) = \zeta\,\delta_{ij} + (1-2\zeta)\,\frac{k_i k_j}{|k|^2}\;, \tag{1.3}$$

where δ_{ij} is the Kronecker symbol, and $\mathcal{P}^{\perp}_{ij} = \delta_{ij} - k_i k_j/k^2$ and $\mathcal{P}^{\parallel}_{ij} = k_i k_j/k^2$ are the solenoidal (divergence-free) and the compressive (curl-free) projection operators, respectively. The projection operator serves to construct a purely solenoidal stirring field by setting $\zeta = 1$. For $\zeta = 0$, a purely compressive stirring field is obtained. Any combination of solenoidal and compressive modes can be constructed by choosing $\zeta \in [0,1]$. By changing the parameter ζ, we can thus set the power of compressive modes with respect to the total power in the driving field. The analytical ratio of compressive power to total power can be derived from equation (1.3) by evaluating the norm of the compressive component of the projection tensor,

$$\left|(1-\zeta)\,\mathcal{P}^{\parallel}_{ij}\right|^2 = (1-\zeta)^2\;, \tag{1.4}$$

and by evaluating the norm of the full projection tensor

$$\left|\mathcal{P}^{\zeta}_{ij}\right|^2 = 1 - 2\zeta + D\zeta^2\;. \tag{1.5}$$

The result of the last equation depends on the dimensionality $D = 1, 2, 3$ of the simulation, because the norm of the Kronecker symbol $|\delta_{ij}| = 1$, 2 or 3 in one, two or three dimensions, respectively. The ratio of equations (1.4) and (1.5) gives the relative power in compressive modes, $F_{\rm long}/F_{\rm tot}$, as a function of ζ:

$$\frac{F_{\rm long}}{F_{\rm tot}} = \frac{(1-\zeta)^2}{1-2\zeta+D\zeta^2}\;. \tag{1.6}$$

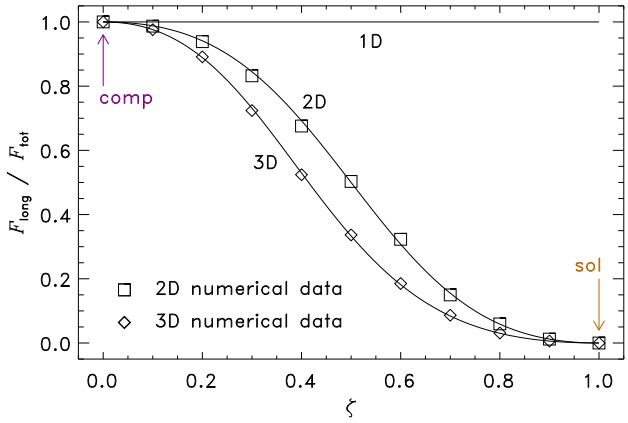

Figure 1.1: Ratio of compressive to total power of the turbulent stirring field, reproduced from Federrath et al. (2010, A&A, 512, A81) with permission by Astronomy & Astrophysics (© ESO). The solid lines labelled with 1D, 2D, and 3D show the analytical expectation for this ratio, equation (1.6), as a function of the stirring parameter ζ for one-, two- and three-dimensional driving, respectively. The diamonds and squares show results of numerical simulations in 3D and 2D with $\zeta = [0, 1]$, separated by $\Delta\zeta = 0.1$. The two limiting cases of purely solenoidal stirring ($\zeta = 1$) and purely compressive stirring ($\zeta = 0$) are indicated as "sol" and "comp", respectively. Note that in any 1D model, all power is in the compressive component, and thus $F\_long/F\_tot = 1$, independent of ζ.

Figure 1.1 provides a graphical representation of this ratio for the 1D, 2D, and 3D case. For comparison, we plot numerical values of the forcing ratio obtained in eleven 3D and 2D hydrodynamical simulations by Federrath et al. (2010, A&A, 512, A81), in which we varied the forcing parameter ζ from purely compressive stirring ($\zeta = 0$) to purely solenoidal stirring ($\zeta = 1$) in the range $\zeta = [0, 1]$, separated by $\Delta\zeta = 0.1$. Note that a natural mixture of stirring modes is obtained for $\zeta = 0.5$, which leads to $F_{\rm long}/F_{\rm tot} = 1/3$ for 3D turbulence, and $F_{\rm long}/F_{\rm tot} = 1/2$ for 2D turbulence. A simple way to understand this natural ratio is to consider longitudinal and transverse waves. In 3D, the longitudinal waves occupy one of the three spatial dimensions, while the transverse waves occupy two of the three on average. Thus, the longitudinal (compressive) part has a relative power of 1/3, while the transverse (solenoidal) part has a relative power of 2/3 in 3D. In 2D, the natural ratio is 1/2, because longitudinal and transverse waves are evenly distributed in two dimensions.

The second term on the right-hand side of equation (1.1) is a drift term describing the exponential decay of the autocorrelation of $\vec{f}$. The usual procedure is to set the autocorrelation timescale equal to the turbulent crossing time, $T = L_{\rm peak}/V$, on the scale of energy injection, $L_{\rm peak}$. This type of stirring models the kinetic energy input from large-scale turbulent fluctuations breaking up into smaller and smaller structures.

1.1.2 Using the StirFromFile Unit

1.1.2.1 Runtime Parameters

Table 1.1: Runtime parameters for the stirring module.

Variable	Type	Default	Description
`useStir`	boolean	.true.	switch stirring on/off
`st_computeDt`	boolean	.false.	restrict timestep based on stirring
`st_infilename`	string	"forcingfile.dat"	file containing the stirring time sequence

Table 1.1 lists the runtime parameters for the StirFromFile unit. This includes a switch for turning the stirring module on/off and a switch to restrict the timestep based on the acceleration field used for stirring (`st_computeDt` is switched off by default, because it is normally sufficient to restrict the timestep based on the gas velocity). Finally, `st_infilename` is the name of the input file containing the time and mode sequence used for stirring. This file must be prepared in advance with a separate Fortran program located in `SimulationMain/StirFromFile/forcing_generator/`. The reason for this structural splitting is to predetermine what the code is going to do. For instance, by preparing the time sequence of the stirring in advance, one can always reproduce exactly the full evolution of all driving patterns applied during a simulation. It also has the advantage that exactly identical stirring patterns can be applied in completely different codes, because they read the time and mode sequence from the same stirring file (Price & Federrath, 2010, MNRAS, 406, 1659).

The stirring module is compatible with any hydro or MHD solver and any grid implementation (uniform or AMR). Upon inclusion in a FLASH setup or module, the `StirFromFile` module defines three additional grid scalar fields, `accx`, `accy`, and `accz`, holding the three vector components of the stirring field $\vec{f}$.

1.1.2.2 Preparing the Stirring Sequence (`st_infilename`)

Table 1.2: Parameters in `forcing_generator.F90` to prepare a stirring sequence.

Variable	Type	Default	Description
`ndim`	integer	3	The dimensionality of the simulation (1, 2, or 3)
`xmin, xmax`	real	-0.5, 0.5	Domain boundary coordinates in x direction
`ymin, ymax`	real	-0.5, 0.5	Domain boundary coordinates in y direction
`zmin, zmax`	real	-0.5, 0.5	Domain boundary coordinates in z direction
`st_spectform`	integer	1	Spectral shape (0: band, 1: paraboloid)
`st_decay`	real	0.5	Autocorrelation time of the OU process, $T = L_{\rm peak}/V$
`st_energy`	real	2e-3	Determines the driving amplitude
`st_stirmin`	real	6.283	Minimum wavenumber stirred (e.g., $k_{\rm min} \lesssim 2\pi/L_{\rm box}$)
`st_stirmax`	real	18.95	Maximum wavenumber stirred (e.g., $k_{\rm max} \gtrsim 6\pi/L_{\rm box}$)
`st_solweight`	real	1.0	Mode mixture $\zeta = [0, 1]$ in Eq. (1.6). Typical values are 1.0: solenoidal; 0.0: compressive; 0.5: natural mixture.
`st_seed`	integer	140281	Random seed for stirring sequence
`end_time`	real	5.0	Final time in stirring sequence
`nsteps`	integer	100	Number of realizations between $t = 0$ and `end_time`
`outfilename`	string	"forcingfile.dat"	Output name (input file `st_infilename` for FLASH)

The code requires a time sequence of stirring modes at runtime, which have to be prepared with the standalone Fortran program `forcing_generator.F90` in `SimulationMain/StirFromFile/forcing_generator/`. A Makefile is provided in the same directory. This program prepares the time sequence of Fourier modes, which is then read by FLASH during runtime, to construct the physical acceleration fields used for stirring. It controls the spatial structure and the temporal correlation of the driving, its amplitude, the mode mixture, and the time separation between successive driving patterns. The user has to modify `forcing_generator.F90` to construct a requested driving sequence and to tailor it to the desired physical situation to be modeled.

Table 1.2 lists all the parameters that can be adjusted in the main routine of `forcing_generator.F90`. Most of them are straightforward to set (`ndim`, `xmin, xmax, ymin, ...`[1]), but others may require some explanation. For example, `st_spectform` determines the shape of the driving amplitude in Fourier space. Many colleagues drive a band (`st_spectform`=0), i.e., equal power injected between wavenumber modes $k_{\rm min}$ = `st_stirmin` and $k_{\rm max}$ = `st_stirmax`. This produces a sharp transition between stirred modes and modes that are not stirred. Here we set the default to a smooth function, a paraboloid (`st_spectform`=1),

[1]Note that we typically assume a cubic box with side length $L\_{\rm box}$ = `xmax` − `xmin` = `ymax` − `ymin` = `zmax` − `zmin`

such that most power is injected on wavenumber $k_{\rm peak} = (k_{\rm min} + k_{\rm max})/2$ and falls off quadratically towards both wavenumber ends, normalized such that the injected power at $k_{\rm min}$ and $k_{\rm max}$ vanishes. This has the advantage of defining a characteristic peak injection scale $k_{\rm peak}$ and achieves a smooth transition between stirred and non-stirred wavenumbers.

`st_decay` and `st_energy` determine the autocorrelation time of the OU process and the total injected energy, which is simply a measure for the normalization of the acceleration field. These parameters must be adjusted according to the physical setup. For instance, for a given target velocity dispersion V on the injection scale $L_{\rm peak} = 2\pi/k_{\rm peak}$, the autocorrelation time should be set equal to the turbulent crossing time, $T = L_{\rm peak}/V$. In contrast, setting `st_decay` to a very small or a very large number results in white noise driving or in a static driving pattern, respectively.

The parameter `st_solweight` determines whether the acceleration field will be solenoidal (divergence-free) or compressive (curl-free) or any mixture, according to Equation (1.6). Incompressible gases should naturally be driven with a purely solenoidal field ($\zeta = 1$), while compressible turbulence in the interstellar medium may be driven primarily by a mixture of solenoidal and compressive modes. A detailed study of the influence of ζ is presented in Federrath et al. (2010, A&A, 512, A81).

`st_seed` is the random seed for the OU sequence and determines the pseudo random number sequence for the integrated Box-Muller random number generator.

Finally, `end_time` and `nsteps` determine the final physical time for stirring and the number of driving patterns to be prepared within the time period from $t = 0$ to $t =$ `end_time`. This sets the number of equally-spaced times at which FLASH is going to read a new stirring pattern from the file. This allows the user to control how frequently a new driving pattern is constructed. A useful time separation of successive driving patterns is about 10% of a crossing time (or autocorrelation time), i.e., setting `nsteps` $= 10 \times$ `end_time`/`st_decay`. This will sample the smooth changes in the OU driving sequence sufficiently well for most applications.

1.1.3 Stirring Unit Test

An example setup using the `StirFromFile` unit is located in `SimulationMain/StirFromFile/`. The unit test can be invoked by

`./setup StirFromFile -auto -3d -nxb=16 -nyb=16 -nzb=16 +ug -with-unit=physics/Hydro`.

The FLASH executable must be copied into the run directory together with the standard `flash.par` for this setup, and together with the default forcing file (to be constructed using the standard parameters; see section 1.1.2.2). During runtime the code writes a file with the time evolution of spatially integrated quantities, amongst others, the rms Mach number and vorticity, which can used as basic code checks.