ELCTRO-DYNAMICS

SCALAR AND VECTOR FIELD

- A ***field*** is a spatial distribution of a quantity; in general, it can be either *scalar* or *vector* in nature.

- Region in space, every point of which is characterized by a scalar quantity is known as **scalar field.**
  - An example of a scalar field in electromagnetism is the electric potential. Other examples include temperature field, pressure field , gravitational potential etc.

- Region in space, each point of which is characterized by a vector quantity is known as **vector field.**
  - Examples of vector field are electric field, gravitation field, magnetic field , magnetic potential etc.

DEL OPERATOR

- The operators are mathematical tools or prescriptions. The operators have no direct physical meaning. However, they acquire significance when operated upon another function.
- The del operator is the vector differential operator,
- Represented by "∇"

$$\vec{\nabla} = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z} \right)$$

Note: *DEL operator is not a vector quantity in itself, but it may operate on various scalar or vector fields.*

DEL OPERATIONS

Gradient $\vec{\nabla}\phi$

Divergence $\vec{\nabla}\cdot\vec{\mathrm{A}}$

Curl $\vec{\nabla}\times\vec{\mathrm{A}}$

GRADIENT

Let ϕ (x,y,z) by any scalar field function, then

$$\vec{\nabla}\phi = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)(\phi(x,y,z))$$

$$\vec{\nabla}\phi = \left(\hat{i}\frac{\partial\phi}{\partial x} + \hat{j}\frac{\partial\phi}{\partial y} + \hat{k}\frac{\partial\phi}{\partial z}\right)$$

Physical Significance

The gradient is a fancy word for derivative, or the rate of change of a function. It's a vector that Points in the direction of greatest increase of a function

Hence $\vec{\nabla}\phi$ provides information how ϕ varies in the neighbourhood of any point in scalar field. The gradient of scalar field ϕ represents both magnitude and direction of maximum space rate of increase of scalar field ϕ.

PHYSICAL SIGNIFICANCE

Thus the rate of change of Φ in the direction of a unit vector a is the component of grad Φ in the direction of a (i.e. the projection of grad Φ onto a). The maximum value of the directional derivative occurs when the directional vector a coincides with the direction of grad Φ. Thus the directional derivative achieves its maximum in the direction of the normal to the level surface Φ(x, y, z) = c at P.

https://www.youtube.com/watch?v=fZ231k3zsAA

Then small change in scalar field as we alter all three variables by small amount dx, dy and dz is given by fundamental theorem of partial derivative, i.e.

$$d\phi = \frac{\partial \phi}{\partial x}dx + \frac{\partial \phi}{\partial y}dy + \frac{\partial \phi}{\partial z}dz$$

Let $\vec{r}$ be the position vector of a point, whose co-ordinates are (x,y,z)

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k} \text{ and } d\vec{r} = dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k}$$

Scalar product of $\vec{\nabla}\phi$ with $d\vec{r}$ yields

$$\vec{\nabla}\phi \cdot \overrightarrow{dr} = \left(\hat{i}\frac{\partial \phi}{\partial x} + \hat{j}\frac{\partial \phi}{\partial y} + \hat{k}\frac{\partial \phi}{\partial z}\right) \cdot \left(dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k}\right)$$

$$\vec{\nabla}\phi \cdot \overrightarrow{dr} = \left(\frac{\partial \phi}{\partial x}dx + \frac{\partial \phi}{\partial y}dy + \frac{\partial \phi}{\partial z}dz\right)$$

$$\vec{\nabla}\phi \cdot \overrightarrow{dr} = d\phi$$

$$d\phi = \left|\vec{\nabla}\phi\right| \cdot \left|\overrightarrow{dr}\right| \cos\theta$$

where θ is the angle between $\vec{\nabla}\phi$ and $d\vec{r}$.

From this equation, it can be concluded that the gradient of scalar function is the maximum rate of change of scalar function ϕ with distance 'r 'and directed along the normal to the surface having same value of ϕ i.e. $\theta=0°$.

Divergence is a scalar quantity

$$\vec{\nabla}\cdot\vec{A}=\left(\hat{i}\frac{\partial}{\partial x}+\hat{j}\frac{\partial}{\partial y}+\hat{k}\frac{\partial}{\partial z}\right)\cdot\left(A_x\hat{i}+A_y\hat{j}+A_z\hat{k}\right)$$

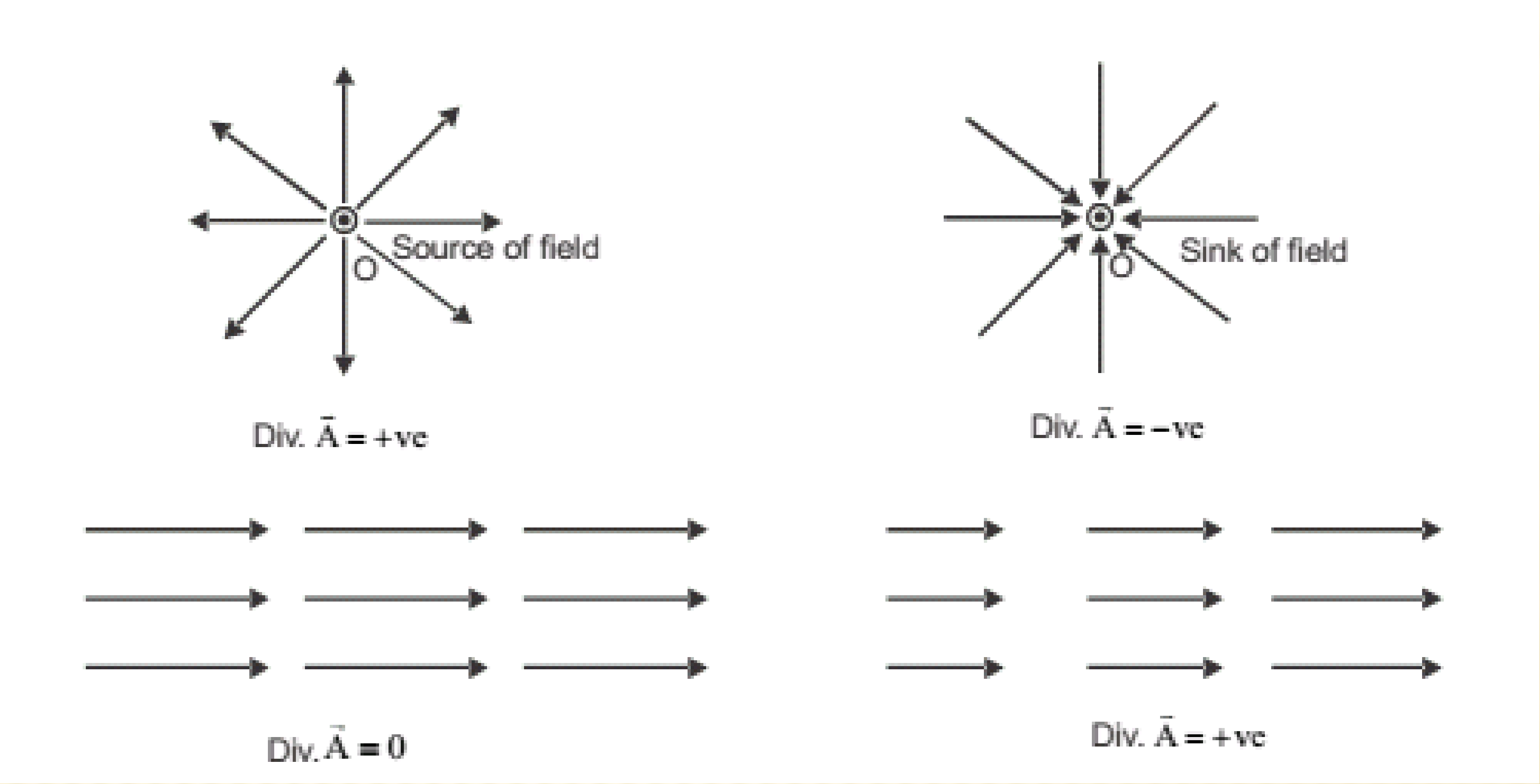
DIVERGENCE

$$\vec{\nabla}\cdot\vec{A}=\left(\frac{\partial A_x}{\partial x}+\frac{\partial A_y}{\partial y}+\frac{\partial A_z}{\partial z}\right)$$

- Physical Significance
  - **Divergence** represents the volume density of the outward flux of a vector field from an infinitesimal volume around a given point

$$\vec{\nabla}\cdot\vec{A} = \underset{V\to 0}{\mathrm{Lt}}\ \frac{1}{V}\oint_s \vec{A}.d\vec{S} = \underset{V\to 0}{\mathrm{Lt}}\ \frac{1}{V}\ \{\text{vector flux}\}$$

  - The divergence of vector field A is defined as the net outward flux per unit volume over a closed surface S.
  - The div. A at a point is measure of how much the vector A spread outs.



If Divergence of vector field is zero , then it is also termed *Solenoidal Field*

CURL

<u>Physical Significance of Curl:</u>
The maximum value of the circulation density evaluated at a point in the vector field is known as curl of vector field

$$\vec{\nabla}\times\vec{A} = \text{Curl }\vec{A} = \underset{S\to 0}{\text{Lt}}\left[\frac{1}{S}\oint \vec{A}\cdot d\vec{l}\right]_{\max}$$

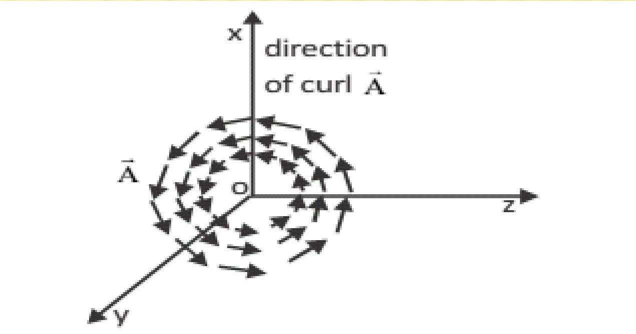
$$\text{Curl }\vec{A} = \vec{\nabla}\times\vec{A} = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)\times\left[A_x\hat{i} + A_y\hat{j} + A_z\hat{k}\right]$$

$$= \hat{i}\left[\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right] + \hat{j}\left[\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right] + \hat{k}\left[\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right]$$

$$\therefore\ \vec{\nabla}\times\vec{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

The rotation with maximum value is known as curl and is a vector quantity. Thus curl of vector field signifies the whirling nature or circulation of the vector field □A □around any point O.

The direction of the curl is the axis of rotation, as determined by the right-hand rule, and the magnitude of the curl is the magnitude of rotation



CONSERVATIVE FIELDS

- *For a conservative vector field ,*

  CURL IS ZERO

$(\nabla \times \mathbf{F}) \cdot \mathbf{n} \equiv \lim_{A \to 0} \frac{}{A},$

- The curl of a vector field is defined as the vector field having magnitude equal to the maximum "circulation" at each point and to be oriented perpendicularly to this plane of circulation for each point.
- The magnitude of Curl is the limiting value of circulation per unit area.

$$(\nabla \times \mathbf{F}) \cdot \hat{\mathbf{n}} \equiv \lim_{A \to 0} \frac{\oint_C \mathbf{F} \cdot d\,\mathbf{s}}{A},$$

- **Curl** is simply the circulation per unit area, circulation density, or rate of rotation (amount of twisting at a single point).
- To be technical, curl is a **vector**, which means it has a both a magnitude and a direction. The magnitude is simply the amount of twisting force at a point.
- The direction is a little more tricky: it's the orientation of the axis of your paddlewheel in order to get maximum rotation. In other words, it is the direction which will give you the most "free work" from the field. Imagine putting your paddlewheel sideways in the whirlpool - it wouldn't turn at all. If you put it in the proper direction, it begins turning.

DIVERGENCE THEOREM

- It states that the surface integral of any vector field through a closed surface is equal to volume integral of the divergence of vector field taken over the volume enclosed by the closed surface.
  Mathematically,

$$\oint_s \vec{A} \cdot \overrightarrow{ds} = \int_v \left(\vec{\nabla} . \vec{A} \right) dV$$

where $\vec{A}$ is vector field and V is volume enclosed by surfaces S.

STOKE'S THEOREM

- It states that line integral of the tangential component of a vector field A over a closed path is equal to the surface integral of the normal component of the curl A on the surface enclosed by path.

- Mathematically,

$$\oint_l \vec{A}.d\vec{l} = \int_s \left(\vec{\nabla} \times \vec{A}\right).\overline{ds}$$

GREEN'S THEOREM

- **Green's theorem** gives the relationship between a line integral around a simple closed curve C and a double integral over the plane region D bounded by C.
- Mathematically

Continuity Equation

$$\vec{\nabla}.\vec{J}+\frac{\partial\rho}{\partial t}=0$$

Continuity equation is the mathematical form of law of localized conservation of electric charge. This equation says that the total current flowing out of some volume must be equal to the rate of decrease of the charge with in the volume, (If the charge neither be created nor lost). The current density $\vec{J}$ associated with current I and surface S is

$$I = \oint_s \vec{J}\,.\,\overline{dS} \qquad .....(1.12)$$

Also $\quad I = -\frac{dq}{dt}$

So $\quad \oint_s \vec{J}\,.\,\overline{dS} = -\frac{dq}{dt} = -\frac{d}{dt}\int_v \rho dV$

(Here, –ve sign represents decrease in charge in the selected volume)

$$\therefore \oint_s \vec{J} \cdot \overline{dS} = -\int_v \frac{\partial \rho}{\partial t} dV \quad \text{(using Leibniz integral rule)} \quad ....(1.13)$$

Using Gauss's divergence theorem

$$\oint_s \vec{J} \cdot \overline{dS} = \int_v \left(\vec{\nabla} \cdot \vec{J}\right) dV \qquad ---(1.14)$$

From eqn (1.13) and (1.14)

$$\int_v \left(\vec{\nabla} \cdot \vec{J}\right) dV = -\int_v \frac{\partial \rho}{\partial t} dV$$

$$\int_v \left(\vec{\nabla} \cdot \vec{J} + \frac{\partial \rho}{\partial t}\right) dV = 0 \qquad ---(1.15)$$

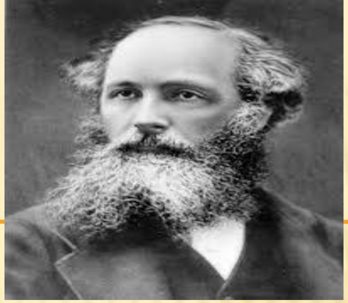
The eqn (1.15) holds true for any arbitrary volume, if

$$\vec{\nabla} \cdot \vec{J} + \frac{\partial \rho}{\partial t} = 0 \qquad ---(1.16)$$

This is known as equation of continuity and represents conservation of charge in a localized volume. In case of stationary current, i.e., its magnitude is not changing with space or times, then charge inside the elementary volume remains constant, $\frac{\partial \rho}{\partial t} = 0$

which implies $\vec{\nabla} \cdot \vec{J} = 0$ ---(1.17)

Maxwell's equations



Maxwell's equations are a set of partial differential equations that, together with the Lorentz force law, form the foundation of classical electrodynamics, classical optics, and electric circuits. These fields in turn underlie modern electrical and communications technologies. Maxwell's equations describe how electric and magnetic fields are generated and altered by each other and by charges and currents. They are named after the physicist and mathematician James Clerk Maxwell, who published an early form of those equations between 1861 and 1862.

James Clerk Maxwell, one of the world's greatest physicists, was Professor of Natural Philosophy at King's from 1860 to 1865. It was during this period that he demonstrated that magnetism, electricity and light were different manifestations of the same fundamental laws, and described all these, as well as radio waves, radar, and radiant heat, through his unique and elegant system of equations. These calculations were crucial to Albert Einstein in his production of the theory of relativity 40 years later, and led Einstein to comment that 'One scientific epoch ended and another began with James Clerk Maxwell'.

MAXWELL'S EQUATIONS

Maxwell's four equations describe the electric and magnetic fields arising from distributions of electric charges and currents, and how those fields change in time.

They were the mathematical distillation of decades of experimental observations of the electric and magnetic effects of charges and currents, plus the profound intuition of Michael Faraday.

Maxwell's own contribution to these equations is just the last term of the last equation -- but the addition of that term had dramatic consequences. It made evident for the first time that varying electric and magnetic fields could feed off each other -- these fields could propagate indefinitely through space, far from the varying charges and currents where they originated.

Previously these fields had been envisioned as tethered to the charges and currents giving rise to them. Maxwell's new term (called the displacement current) freed them to move through space in a self-sustaining fashion, and even predicted their velocity -- it was the velocity of light!

MAXWELL EQUATIONS

Differential form

$$\vec{\nabla} \cdot \vec{\mathbf{E}} = \frac{\rho}{\varepsilon_0} \quad \text{OR} \quad \vec{\nabla} \cdot \vec{\mathbf{D}} = \rho$$

$$\vec{\nabla} \cdot \vec{\mathbf{B}} = 0$$

$$\vec{\nabla} \times \vec{\mathbf{E}} = -\frac{\partial \vec{\mathbf{B}}}{\partial \mathbf{t}}$$

$$\vec{\nabla} \times \vec{\mathbf{B}} = \mu_0 \vec{\mathbf{J}} + \mu_0 \varepsilon_0 \frac{\partial \vec{\mathbf{E}}}{\partial \mathbf{t}} \ \text{OR}\ \vec{\nabla} \times \vec{\mathbf{H}} = \vec{\mathbf{J}} + \frac{\partial \vec{\mathbf{D}}}{\partial \mathbf{t}}$$

$$\text{where,} \vec{\mathbf{D}} = \varepsilon_0 \vec{\mathbf{E}} \quad \& \quad \vec{\mathbf{B}} = \mu_0 \vec{\mathbf{H}}$$

Integral form

$$\oint_S \vec{E} \cdot d\vec{a} = \frac{Q_{enc}}{\varepsilon_0}$$

$$\oint_s \vec{B} \cdot \overrightarrow{dS} = 0$$

$$\oint_C \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \int_S \vec{B} \cdot \overrightarrow{dS}$$

$$\oint_C B \cdot d\vec{l} = \mu_0 \vec{I}_{enc} + \mu_0 \varepsilon_0 \frac{d}{dt} \int_S \vec{E} \cdot \overrightarrow{dS}$$

Differential form	Integral Form	Corresponding law
1. $\vec{\nabla}\cdot\vec{D}=\rho$ $\vec{\nabla}\cdot\vec{E}=\frac{\rho}{\varepsilon_o}$	$\oint_s \vec{D}\cdot d\vec{S}=\int_v \rho dV$ $\oint_s \vec{E}\cdot d\vec{S}=\frac{1}{\varepsilon_o}\int_v \rho dV$	Gauss's law in electrostatics
2. $\vec{\nabla}\cdot\vec{B}=0$	$\oint_s \vec{B}\cdot d\vec{S}=0$	Gauss's law in magnetism
3. $\vec{\nabla}\times\vec{E}=\frac{-\partial\vec{B}}{\partial t}$	$\oint_l \vec{E}\cdot d\vec{l}=-\int_s \frac{\partial\vec{B}}{\partial t}.d\vec{S}$	Faraday's law of electromagnetism
4. $\vec{\nabla}\times\vec{B}=\mu_o\left[\vec{J}+\varepsilon_o\frac{\partial\vec{E}}{\partial t}\right]$ $\vec{\nabla}\times\vec{H}=\vec{J}+\frac{\partial\vec{D}}{\partial t}$	$\oint_l \vec{H}\cdot d\vec{l}=\int_s\left[J+\frac{\partial\vec{D}}{\partial t}\right]\cdot d\vec{S}$	Modified form of Ampere's circuital law.

Maxwell 1st Equation: $\vec{\nabla}.\vec{E} = \frac{\rho}{\varepsilon_0}$

Suppose that charge is distributed over a volume V. Let ρ be the volume charge density,
Then total charge.

$$q = \int_v \rho dV$$

Gauss's law can be written as

$$\phi = \oint_s \vec{E}.\vec{d}S = \frac{1}{\varepsilon_0}\int_v \rho dV \qquad \text{---(1.18)}$$

This is known as integral form of Gauss's law.
According to Gauss' divergence therorem,

$$\oint_s \vec{E}\cdot\overline{dS} = \int_v \left(\vec{\nabla}.\vec{E}\right) dV \qquad \text{---(1.19)}$$

From eqn (1.18) and (1.19)

$$\int_v \left(\vec{\nabla}.\vec{E}\right) dV = \frac{1}{\varepsilon_0}\int_v \rho dV \qquad \text{---(1.20)}$$

The above is true for all volume dV if integrands must be equal, i.e.

$$\vec{\nabla}.\vec{E} = \frac{\rho}{\varepsilon_0} \qquad \text{---(1.21)}$$

Also we know that displacement vector $\vec{D} = \varepsilon_o \vec{E}$

$$\therefore \quad \vec{\nabla}.\vec{D} = \rho \qquad \text{---(1.22)}$$

Eqn (1.21) & (1.22) are diffrential form of Gauss's law in electrostatics.

Significance of Maxwell's First Equation

(i) It represents the Gauss's law in electrostatics.

(ii) It is time independent differential equation or steady state equation.

(iii) Maxwell's first equation gives the relationship between electric flux and charge density.

(iv) It also shows that charge acts as source or sink for electric lines of force.

Maxwell 2nd Equation: $\vec{\nabla}\cdot\vec{B}=0$

As we known that magnetic lines of force are continuous and do not appear to have origin or the end. Therefore, the total magnetic flux in closed surface is always zero. That means the number of magnetic lines of force entering the surface is equal to the magnetic lines leaving the surface. Hence, we can write

$$\oint_s \overline{B}\cdot\overline{ds}=0 \qquad \text{---(1.24)}$$

In accordance with Gauss' Divergence theorem,

$$\oint_s \overline{B}\cdot\overline{ds}=\int_V \overline{\nabla}\cdot\overline{B}\,dV=0 \qquad \text{---(1.25)}$$

$$\therefore\ \vec{\nabla}\cdot\vec{B}=0 \qquad \text{---(1.26)}$$

This is analogue of Gauss' law in magnetism and signify that magnetic monopole does not exist.

Significance of Maxwell's second equation

(i) It represents the Gauss's law in magnetism and shows that magnetic monopole does not exist in nature.

(ii) There is no source or sink for the magnetic lines of force.

(iii) It is time independent differential equation.

Maxwell 3rd Equation: $\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$

According to Faraday's law,

$$EMF = -\frac{d\Phi}{dt}$$

Φ is the Magnetic Flux within a circuit, and *EMF* is the electro-motive force

$$\Phi(t) = \int_S \mathbf{B}(t)\, d\mathbf{S}$$ [The Magnetic Flux Φ is the Sum (Average) of the $\mathbf{B}$ – field over the area S]

Also,

$$EMF_{total} = \oint_{Circuit} \mathbf{E} \cdot d\mathbf{L}$$ [the total EMF around the circuit is equal to summing up $\mathbf{E}$ field around the length of the circuit]

$$\oint_{Circuit} \mathbf{E} \cdot d\mathbf{L} = \int_S \nabla \times \mathbf{E} \cdot d\mathbf{S} \quad \text{[Stokes' Theorem]}$$

$$\int_S \nabla \times \mathbf{E} \cdot d\mathbf{S} = -\frac{d}{dt} \int_S \mathbf{B}(t) \cdot d\mathbf{S} = \int_S \frac{-d\mathbf{B}(t)}{dt} \cdot d\mathbf{S}$$

$$\Rightarrow \boxed{\nabla \times \mathbf{E} = \frac{-\partial \mathbf{B}(t)}{\partial t}}$$

Significance of Maxwell's third equation

(i) It summarizes the Faraday's law of electromagnetic induction.

(ii) This equation relates the space variation of electric field with time variation of magnetic field

(iii) It is time dependent differential equation.

(iv) It proves that the electric field can begenerated by change in magnetic field

Maxwell 4th Equation: $\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} + \mathbf{J}$

According to Ampere's Law

$$I_{enc} = \oint \mathbf{H} \cdot \mathbf{dL} = \int_S (\nabla \times \mathbf{H}) \cdot d\mathbf{S}$$

$$I_{enc} = \int_S \mathbf{J} \cdot \mathbf{dS}$$

$$\int_S (\nabla \times \mathbf{H}) \cdot d\mathbf{S} = \int_S \mathbf{J} \cdot d\mathbf{S}$$

$$\Rightarrow \quad \nabla \times \mathbf{H} = \mathbf{J}$$

$$\nabla \times \mathbf{H} = \mathbf{J}$$

$$\nabla \cdot (\nabla \times \mathbf{H}) = \nabla \cdot \mathbf{J}$$

$0 = \nabla \cdot \mathbf{J}$ [the divergence of J is always zero?]

However, from the equation of continuity $\vec{\nabla} . \vec{J} = \frac{-\partial \rho}{\partial t}$

Maxwell realized that the definition of the total current density is incomplete and suggested to add another term

$$\vec{\nabla} \times \vec{H} = \vec{J} + \vec{J}\,'$$

Take div. of above eqn

$$\vec{\nabla}(\vec{\nabla}\times\vec{H}) = \vec{\nabla}\cdot(\vec{J}+\vec{J}') = \vec{\nabla}.\vec{J}$$

As $\quad \vec{\nabla}\cdot\left(\vec{\nabla}\times\vec{H}\right)=0$

it implies, $\vec{\nabla}\cdot\ \vec{J}+\vec{\nabla}\cdot\vec{J}'=0$

$$\vec{\nabla}\cdot\vec{J}' = -\vec{\nabla}\cdot J = \frac{\partial\rho}{\partial t}$$

$$\therefore \quad \vec{\nabla}\cdot\vec{J}' = \frac{\partial\rho}{\partial t}$$

According to Gauss's law of electrost

$$\vec{\nabla}\cdot\vec{D} = \rho$$

$$\therefore \quad \vec{\nabla}\cdot J' = \frac{\partial}{\partial t}(\vec{\nabla}\cdot\vec{D}) = \vec{\nabla}\cdot\frac{\partial\vec{D}}{\partial t}$$

Therefore,

$$J' = \frac{\partial\vec{D}}{\partial t} = \varepsilon_o\frac{\partial\vec{E}}{\partial t}$$

modified form of Ampere's circutal law is

$$\vec{\nabla}\times\vec{H} = \vec{J} + \frac{\partial\vec{D}}{\partial t} = \vec{J} + \varepsilon_o\frac{\partial\vec{E}}{\partial t}$$

Significance of Maxwell's fourth equation

(i) It summarizes the modified form of Ampere's ciruital law.

(ii) It is time dependent differential equation.

(iii) Maxwell's fourth equation relates the space variation of magnetic field with time variation of electric field

(iv) It also proves that magnetic field can be generated by changing electric field

PROPAGATION OF ELECTROMAGNETIC WAVE IN FREE SPACE

The Maxwell's equation for free space ($\rho=0$ and $J=0$) can be written as

$$\vec{\nabla} \cdot \vec{E} = 0 \qquad \text{---}(1.39)$$

$$\vec{\nabla} \cdot \vec{H} = 0 \qquad \text{---}(1.40)$$

$$\vec{\nabla} \times \vec{E} = -\mu_0 \frac{\partial \vec{H}}{\partial t} \qquad \text{---}(1.41)$$

$$\boxed{\vec{B} = \mu_o \vec{H}}$$

$$\vec{\nabla} \times \vec{H} = \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \qquad \text{---}(1.42)$$

Take curl of eqn (1.41) we have

$$\vec{\nabla} \times \vec{\nabla} \times \vec{E} = -\mu_0 \frac{\partial}{\partial t} (\vec{\nabla} \times \vec{H}) \qquad \text{---}(1.43)$$

According to vector calculus,

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = \vec{\nabla}(\vec{\nabla}.\vec{E}) - \nabla^2 \vec{E} \quad \left[\because \vec{A} \times \left(\vec{B} \times \vec{C}\right) = \vec{B}\left(\vec{A}.\vec{C}\right) - \left(\vec{A}.\vec{B}\right)\vec{C}\right]$$

From eqn (1.39) , $\vec{\nabla}.\vec{E} = 0$

$$\therefore \vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = -\nabla^2 \vec{E} \qquad \text{---(1.44)}$$

Using eqn (1.42) and eqn (1.44), eqn (1.43) can be written as,

$$\nabla^2 \vec{E} - \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} = 0$$

One can write the above eqn as a wave equation for $\vec{E}$

$$\nabla^2 \vec{E} - \frac{1}{v^2} \frac{\partial^2 \vec{E}}{\partial t^2} = 0 \qquad \text{---(1.45)}$$

since the dimension of $1/\mu_0\varepsilon_0$ is same as that of velocity.

Similarly the curl of Eqn (1.42) gives, wave eqn for $\vec{H}$ as

$$\nabla^2 \vec{H} - \mu_0 \varepsilon_0 \frac{\partial^2 \vec{H}}{\partial t^2} = 0$$

$$\nabla^2 \vec{H} - \frac{1}{v^2} \frac{\partial^2 \vec{H}}{\partial t^2} = 0 \qquad \text{---(1.46)}$$

$$\therefore \quad \frac{\omega}{k} = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3\times10^{8}\text{m/sec} = c = \text{speed of light}$$

where, $\varepsilon_o = 8.854\times10^{-12}\, F/m$; $\mu_o = 4\pi\times10^{-7}\, H/m$

Hence the phase velocity $v_p = \dfrac{\omega}{k}$ of light is equal to the speed of light (c) in free space or vacuum.

WHAT , WHY & HOW??????

- Define : Curl, Divergence & Gradient
- Explain the physical significance: Curl, Divergence & Gradient
- Write the expression for Del-operator
- Write continuity equation and its physical significance
- Derive an equation which express conservation of charge in a localized volume.
- Write Maxwell equations in both differential and integral form
- Derive differential Maxwell equations. Also write their physical significances.
- Derive the equations for electromagnetic wave propagation in free space using Maxwell equations, and hence calculate the value of c (velocity of light).
- Write Stoke’s and Divergence’ theorems.
- What are the conditions for irrotational, solenoidal and conservative fields, resp.?