Homework 1 (計算方法設計，Design and Analysis of Algorithms)

註: 所有的作業皆以紙本的方式，在截止日以前繳交給助教(台達館 737 或 738 室)，請注意不接受遲交。All homework assignments should be submitted to the TAs (Room 737 or 738 at Delta Building) as hard copy (handwriting or paper printout) by the due date. Please note that late assignment submissions will not be accepted.

Due date: March 18, 2022

1. (25%) Suppose that $f(n) = a_d n^d + a_{d-1} n^{d-1} + \cdots + a_1 n + a_0$ is a polynomial of degree d in n, where $a_i \in R$ for $0 \leq i \leq d$ and $a_d > 0$. Prove that $f(n) = \Theta(n^d)$.

2. (25%) Binary search is a famous algorithm that can efficiently solve the problem of determining whether a given number X is in a list $a_1, a_2, \ldots, a_n$ of n sorted numbers, where $a_1 \leq a_2 \leq \cdots \leq a_n$. Show that the best-case time complexity of the binary search is $O(1)$ (5%), its worst-case time complexity is $O(\log_2 n)$ (5%) and its average-case time complexity is $O(\log_2 n)$ (15%). For simplicity, you may assume that $n = 2^k - 1$, where k is a positvie integer.

3. (25%) Given a sorted list L of n numbers in non-decreasing order and also a number x, the search problem is to determine whether x is in L or not. Use the decision tree method to prove that any comparison-based algorithm needs to perform at least $\lfloor \log_2 n \rfloor$ comparisons for solving the search problem.

4. (25%) Consider a sequence of seven numbers 5,2,6,4,1,3,7. Demostrate how to sort these seven numbers in decreasing order using the heap sort algorithm. You need to show the heap trees in the process of your heap sort.