

Tema 1

Ex 3

$$\bullet \left(\bigcup_{n \in \mathbb{N}} A_n \right)^c = \bigcap_{n \in \mathbb{N}} A_n^c \quad (\Rightarrow)$$

$$\Rightarrow (A_1 \cup A_2 \cup \dots \cup A_n)^c = A_1^c \cap A_2^c \cap \dots \cap A_n^c$$

$$P(2) : (A_1 \cup A_2)^c = A_1^c \cap A_2^c \quad (A) \quad (\text{Legea 1})$$

$$P(n) \Rightarrow P(n+1)$$

$$P(n) : \underbrace{(A_1 \cup A_2 \cup \dots \cup A_n)}_B^c = A_1^c \cap A_2^c \cap \dots \cap A_n^c \quad | \quad \cap A_{n+1}^c$$

$$\Rightarrow B^c \cap A_{n+1}^c = \bigcap_{n \in \mathbb{N}} A_{n+1}^c$$

$$\Rightarrow (B \cup A_{n+1})^c = \bigcap_{n \in \mathbb{N}} A_{n+1}^c$$

$$\Rightarrow (A_1 \cup A_2 \cup \dots \cup A_n \cup A_{n+1})^c = \bigcap_{n \in \mathbb{N}} A_{n+1}^c$$

$$\Rightarrow \left(\bigcup_{n \in \mathbb{N}} A_{n+1} \right)^c = \bigcap_{n \in \mathbb{N}} A_{n+1}^c$$

$$\Rightarrow \text{cf } P \subseteq M. \quad \therefore \left(\bigcup_{n \in \mathbb{N}} A_n \right)^c = \bigcap_{n \in \mathbb{N}} A_n^c$$

$$\bullet \left(\bigcap_{n \in \mathbb{N}} A_n \right)^c = \bigcup_{n \in \mathbb{N}} A_n^c \quad (\Rightarrow)$$

$$(\Rightarrow) (A_1 \cap A_2 \cap \dots \cap A_n)^c = A_1^c \cup A_2^c \cup \dots \cup A_n^c$$

$$P(2) : (A_1 \cap A_2)^c = A_1^c \cup A_2^c \quad (\text{Lemma 2})$$

$$P(n) \Rightarrow P(n+1)$$

$$P(n) : (A_1 \cap A_2 \cap \dots \cap A_n)^c = A_1^c \cup A_2^c \cup \dots \cup A_n^c \quad | \quad \cup A_{n+1}^c$$

$$\Rightarrow \underbrace{A_1^c \cup A_2^c \cup \dots \cup A_n^c}_B \cup A_{n+1}^c = \bigcup_{n \in \mathbb{N}} A_{n+1}^c$$

$$\Rightarrow (B \cap A_{n+1})^c = \bigcup_{n \in \mathbb{N}} A_{n+1}^c$$

$$\Rightarrow (A_1 \cap A_2 \cap \dots \cap A_n \cap A_{n+1})^c = \bigcup_{n \in \mathbb{N}} A_{n+1}^c$$

$$\Rightarrow \left(\bigcap_{n \in \mathbb{N}} A_{n+1} \right)^c = \bigcup_{n \in \mathbb{N}} A_{n+1}^c$$

$$\Rightarrow \text{cf. PIM} \quad \left(\bigcap_{n \in \mathbb{N}} A_n \right)^c = \bigcup_{n \in \mathbb{N}} A_n^c$$

Ex 4

Este mai probabil ca din 4 aruncări să pice cel puțin un 6
decât din 24 de aruncări să pice o dublă de 6.
(cu un zar)
(cu 2 zaruri)

P_1 = prima probabilitate

P_2 = a doua probabilitate

$P_1 = 1 - (P_1)^c$, $(P_1)^c$ = probabilitatea să nu pice niciun 6
în cele 4 aruncări

Pt. o singură aruncare probabilitatea o să fie $\frac{5}{6} \Rightarrow$
pt. 4 aruncări o să fie $\left(\frac{5}{6}\right)^4 \Rightarrow P_1 = 1 - \left(\frac{5}{6}\right)^4$

$P_2 = 1 - (P_2)^c$, $(P_2)^c$ = prob. să nu pice nicio dublă de 6 în
cele 24 de aruncări

Pt. o singură aruncare o să fie $\frac{35}{36} \Rightarrow$

pt. 24 aruncări o să fie $\left(\frac{35}{36}\right)^{24} \Rightarrow P_2 = 1 - \left(\frac{35}{36}\right)^{24}$

$$\begin{array}{cc} P_1 & P_2 \\ \hline 1 - \left(\frac{5}{6}\right)^4 & 1 - \left(\frac{35}{36}\right)^{24} \end{array} \quad (\Rightarrow) \quad \left(\frac{5}{6}\right)^4 > \left(\frac{35}{36}\right)^{24}$$

$$\Rightarrow \left(\frac{5}{6}\right)^4 < \left(\frac{35}{36}\right)^{24}$$

$$(\Rightarrow) \frac{5^4}{36^2} < \frac{55^{24}}{36^{24}}$$

$$(\Rightarrow) 5^4 \cdot 36^{24} < 36^2 \cdot 35^{24}$$

$$(\Rightarrow) 5^4 \cdot 36^{22} < 35^{24}$$

$$(\Rightarrow) \frac{5^4 \cdot 36^{22}}{35^{24}} < 1$$

$$(\Rightarrow) \frac{36^{22}}{35^{20} \cdot 7^4} < 1$$

$$(\Rightarrow) \frac{36^2}{7^4} \cdot \left(\frac{36}{35}\right)^{20} < 1$$

$$(\Rightarrow) \frac{1296}{2401} \cdot 1,756 < 1$$

$$(\Rightarrow) 0,947 < 1(A)$$

Ex 2

$\Delta = \text{suma inițială}$, $2n = nr.$ amcări , $n \geq 1$

$$\Delta \cdot (1+a)^n (1-b)^n < 1 \cdot \Delta \Leftrightarrow$$

$$\Leftrightarrow (1+a)^n (1-b)^n < 1$$

$$(\Rightarrow) (1+a)^n < \frac{1}{(1-b)^n}$$

$$(\Rightarrow) (1+a)^n < \left(\frac{1}{1-b}\right)^n$$

$$(\Rightarrow) 1+a < \frac{1}{1-b}$$

$$(\Rightarrow) a < \frac{1}{1-b} - 1$$

$$(\Rightarrow) a < \frac{1 - 1 + b}{1-b}$$

$$(\Rightarrow) a < \frac{b}{1-b}$$