

$\frac{1}{\lambda}$: Avg. inter-arrival time = Time between arr. \times proba.

$\frac{1}{\mu}$: Avg. service time = Service time \times proba

Stat acc.

Clock	Q(t)	B(t)	Checkout	Future event list	ND	S
Init	0	0		(A, 1, 0)(E, 20)	0	0
A 0	0	1	(1, 0)	(A, 2, 1.73)(D, 1, 2.90)(E, 20)	0	0
A 1.73	1	1	(1, 0)(2, 1.73)	(A, 3, 3.08)(D, 1, 2.90)(E, 20)	0	0
D 2.90	0	1	(2, 1.73)	(A, 3, 3.08)(D, 2, 4.66)(E, 20)	1	2.90
A 3.08	1	1	(2, 1.73)(3, 3.08)	(A, 4, 3.79)(D, 2, 4.66)(E, 20)	1	2.90
A 3.79	2	1	... (4, 3.79)	(A, 5, 4.41)(D, 2, 4.66)(E, 20)	1	2.90
A 4.41	3	1	... (5, 4.41)	(A, 6, 18.69)(D, 2, 4.66)(E, 20)	1	2.90
D 4.66	2	1	(2, 1.73) ...	(D, 3, 8.05)(A, 6, 18.69)(E, 20)	2	5.83
D 8.05	1	1	(3, 3.08) ...	(D, 4, 12.57)(A, 6, 18.69)(E, 20)	3	10.8
D 12.57	0	1	(4, 3.79) ...	(D, 5, 17.03)(A, 6, 18.69)(E, 20)	4	19.58
D 17.03	0	0	(5, 4.41)	(A, 6, 18.69)(E, 20)(D, 6, 2.39)	5	32.2
A 18.69	0	1	(6, 18.69)	(E, 20)	5	32.2

$N(t) = \text{nb clients}$

$E(t, s) = \text{clients entrées } [t, t+s]$

$D(t, s) = \text{partis } [t, t+s]$

Little : $\bar{N} = \lambda \cdot \bar{T}$ ← temps moy
 \uparrow \uparrow
 $N \text{ moy}$ $\tau \text{ arrivé}$
 clients

$$\bar{N} = \frac{\int_0^{t_{\max}} N(t) dt}{t_{\max}} \quad \bar{T} = \frac{\int_0^t N(t) dt}{n}$$

$$= \frac{\bar{N}}{\bar{T}} = \frac{n}{t_{\max}} = \lambda$$

Intensité du trafic :

Régime stationnaire ? $\rho = \frac{\lambda}{\mu} < 1 \Rightarrow \text{Oui}$
 $\geq 1 \Rightarrow \text{Non}$

Système continu :

Soit $U = [a, b]$

function $t_inter_arr()$:

return $a + (b - a) * (\text{double}) \text{rand}() / \text{RAND_MAX};$

function $t_service()$:

return _____ ;

Système discret :

T_1	T_2	T_3	T_4	T_5
t_{ia1}	t_{ia2}	t_{ia3}	t_{ia4}	t_{ia5}

function $t_inter_arr()$:

← same

$\text{rand_n} = \text{gg} * (\text{double}) \text{rand}() / \text{RAND_MAX};$ ^{for service}

if ($\text{rand_n} < t_{ia1}$)

return T_1

elif ($\text{rand_n} < t_{ia1} + t_{ia2}$)

return T_2

⋮

else

return T_5

boucle de simu. ($t \leq t_{\max}$)

debut

```
si evenement-arrivee ( $t_{\text{arr}} < t_{\text{dep}}$ )
{
  q++;
  delta =  $t_{\text{arr}} - t$ ;
  t-cumul += delta * (long-file + etat);
  t-att-cumul += delta * long-file;
  t-occ += delta * etat;
  t-arr =  $t + \text{generer-}t_{\text{ia}}()$ ;
  t = t-arr;
  si (etat == 0)
  {
    etat = 1;
    t-dep =  $t + \text{generer-}t_{\text{srv}}()$ ;
  }
  // evenement-depart
  // ( $t_{\text{dep}} \leq t_{\text{arr}}$ )
sinon
{
  p++;
  delta =  $t_{\text{dep}} - t$ ;
  t-att-cumul += delta * (long-file + etat);
  t-occ += delta * etat;
  t = t-dep;
  if (long-file > 0)
  {
    long-file--;
    t-dep =  $t + \text{gen-}t_{\text{srv}}()$ ;
  }
  else
  {
    etat = 0;
    t-dep = RAND:MAX;
  }
}
fin
```

```
delta =  $t_{\text{max}} - t$ 
t-att-cumul += delta *
               (long-file + etat)
t-occ += delta * etat
```

```
N-moy =  $t_{\text{cumul}} / t_{\text{max}}$ 
T-moy =  $t_{\text{cumul}} / q$ 
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```
t-att-moy =  $t_{\text{att-cumul}} / q$ 
```

1. M/M/1/1

2. Intensité du trafic :

$$\rho = \frac{\lambda}{\mu}$$

$$\lambda = \frac{1}{E(a)} = \frac{1}{20} = 0,05$$

$$\mu = \frac{1}{E(s)} = \frac{1}{30} = 0,033$$

$$\text{donc } \rho = \frac{\lambda}{\mu} = \frac{0,05}{0,033} < 1 \quad \text{donc}$$

le système est stable et atteint son régime stationnaire

3. On suppose avoir fait le déroulement + graphique.

$$T_{\text{occupation}} = \frac{\int_0^{t_{\text{max}}} B(t) dt}{t_{\text{max}}} = \frac{90}{100} = 0,9 = 90\%$$

$$T_{\text{att. moyen}} = \frac{\int_0^t Q(t) dt}{n} = \frac{80}{5} = 16 \text{ min}$$

$$\begin{aligned} N_{\text{moyen}} &= \frac{\int_0^t N(t) dt}{t_{\text{max}}} = \frac{\int_0^t Q(t) + B(t)}{t_{\text{max}}} \\ &= \frac{80 + 90}{100} \\ &= 1,7 \text{ clients} \end{aligned}$$