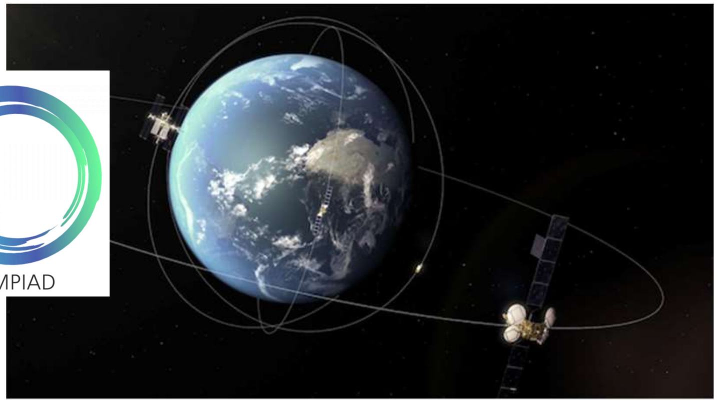
Problem statement:

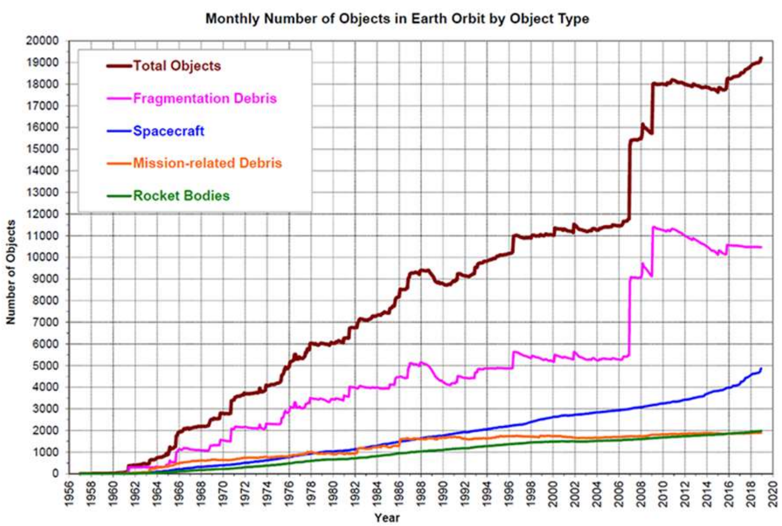


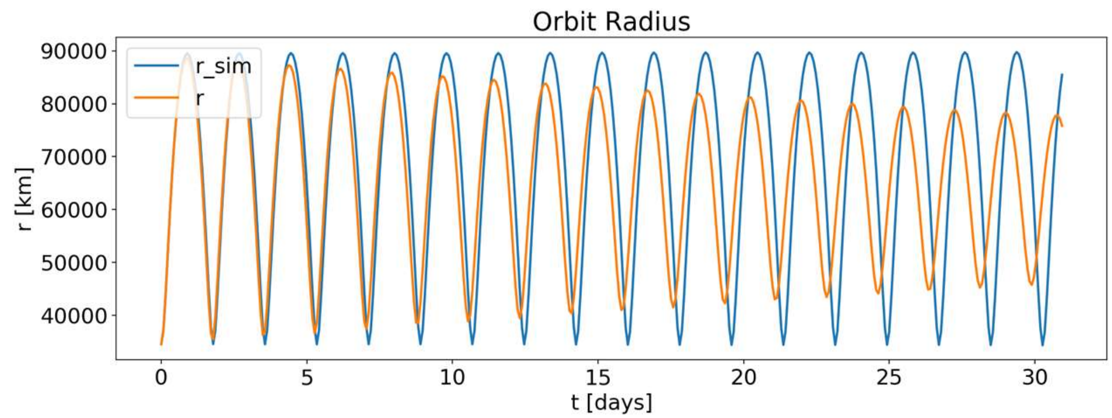
Inaccurate prediction of the Earth satellite orbits can lead to dangerous collisions

Task : predict real coordinates from simulated values

$$\begin{bmatrix} V_x\_sim \\ V_y\_sim \\ V_z\_sim \\ x\_sim \\ y\_sim \\ z\_sim \end{bmatrix} \rightarrow \begin{bmatrix} V_x \\ V_y \\ V_z \\ x \\ y \\ z \end{bmatrix}$$

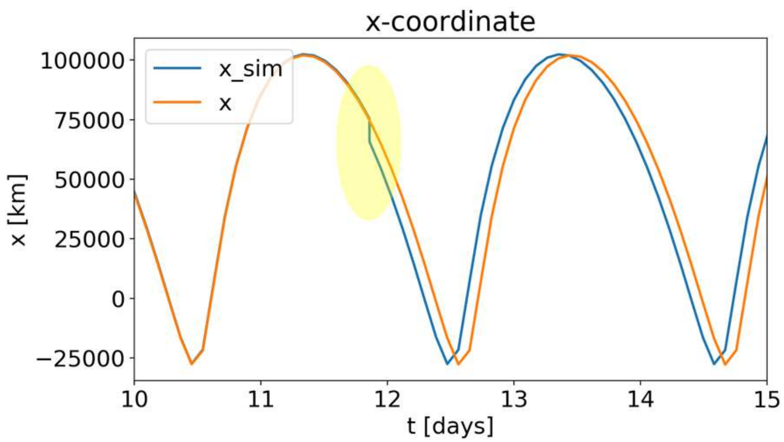
Train data (January 2014):
$[r\_sim]$, $[r\_real]$
600 satellites





Images: isa.int, nasa.gov

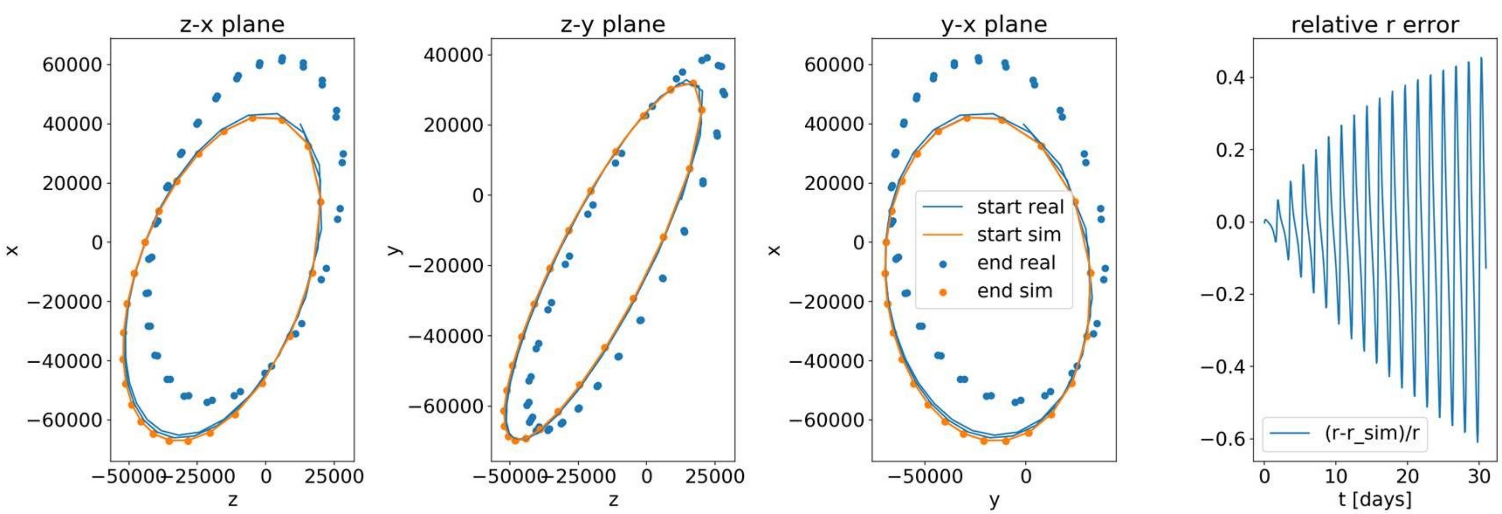
Problem statement:



Remove sudden jumps in simulated signal due to nonuniform time grid

↓

Mapping signal to the nonuniform time grid using interpolation



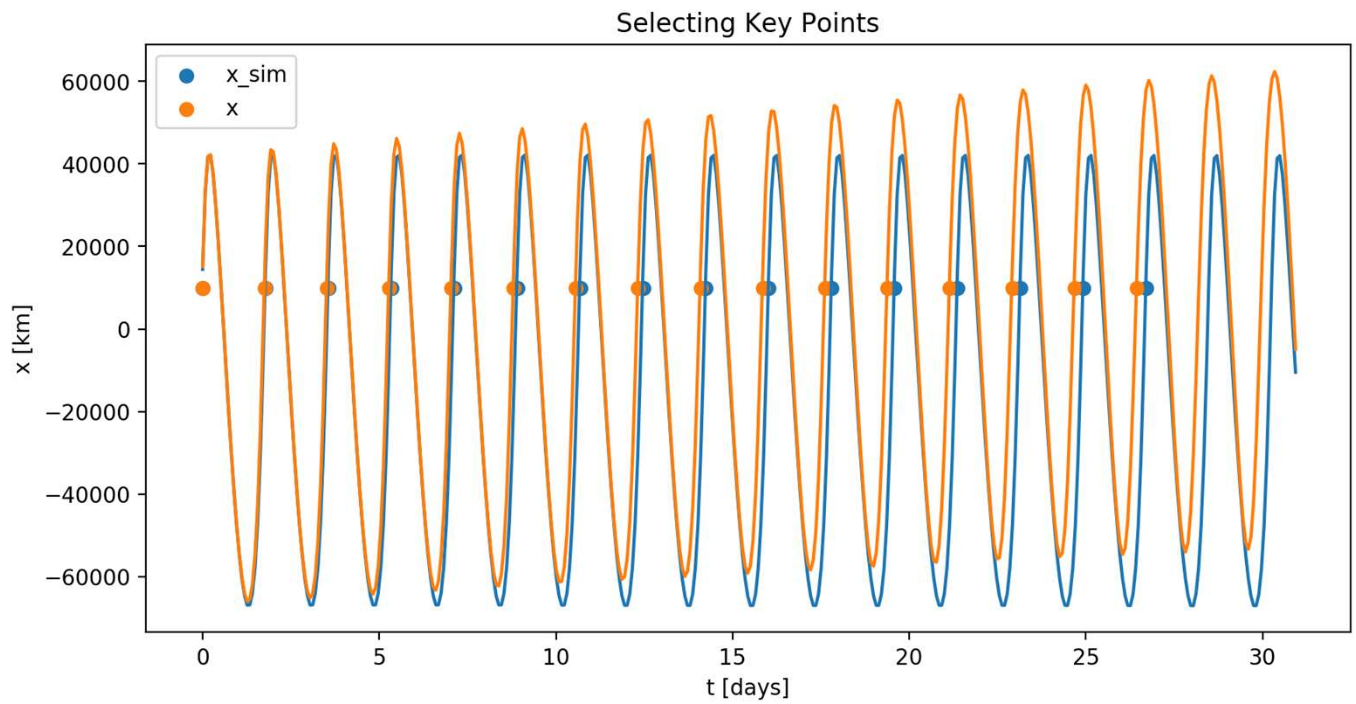
Address gradual dephasing and slow orbit precession

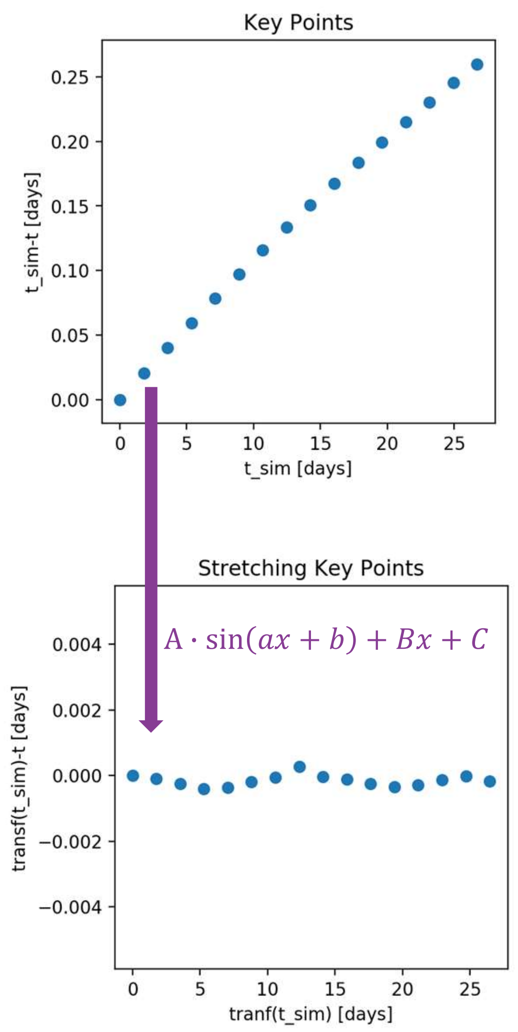
↓

Nonlinear alignment

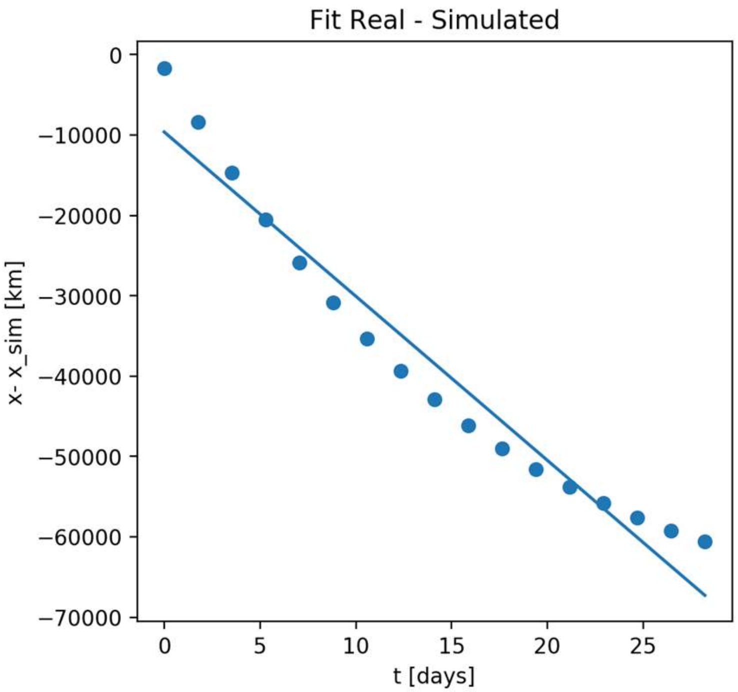
Challenge: finding uniform approach for all satellite

Nonlinear alignment: matching key time points





Nonlinear alignment: fit coordinate difference at key points



Linear fit $X_{real} - X_{sim} = A \cdot t + B$

Full algorithm:

I. Remove time jumps (shift signal using spline interpolation)
II. Perform nonlinear alignment for each generalized coordinate X
   1. Identify families of key time points (by offset, 100 points per period)
   2. For each family of key points:
      a) Transform simulation key points to real key points
      b) Linearly stretch ΔX at key points
   3. Combine stretched coordinates and transformed key points into sparse solution
III. Map the sparse solution onto the initial time grid using 2nd order EOM:

$$a[i] = -\frac{G\ M_E}{r[i]^2}$$

$$v[i+1] = v[i] + a[i] \cdot dt$$

$$r[i+1] = r[i] + (v[i+1] + v[i])dt$$

Nonlinear alignment: combining the result from all key points