Vector Autoregressive Models

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Consider the following two equations:

$$
\begin{aligned}
y_{1,t} &= a y_{1,t-1} + b y_{2,t-1} + v_{1,t} \\
y_{2,t} &= c y_{1,t-1} + d y_{2,t-1} + v_{2,t}
\end{aligned}
$$

These two equations comprise a vector autoregression (VAR). A VAR is the extension of the autoregressive (AR) model to the case in which there is more than one variable under study. The VAR above is a bivariate VAR(1) (lag-one). Generally, a VAR can consist of K variables and have p lags.

In a VAR model each variable is treated as endogenous. There is one equation for each variable in the system, and each equation consists of lags of it's own variable plus each of the others. Technically speaking, this is called a *reduced form* VAR, which we will distinguish from a *structural VAR* in just a bit.

We can write the VAR(p) model more generally as:

$$
\begin{aligned}
y_{1,t} &= \alpha_1 + \sum_{i=1}^{p} a_{1i} y_{1,t-i} + \sum_{i=1}^{p} b_{1i} y_{2,t-i} + \cdots \sum_{i=1}^{p} c_{1i} y_{k,t-i} + v_{1,t} \\
y_{2,t} &= \alpha_2 + \sum_{i=1}^{p} a_{2i} y_{1,t-i} + \sum_{i=1}^{p} b_{2i} y_{2,t-i} + \cdots \sum_{i=1}^{p} c_{2i} y_{k,t-i} + v_{2,t} \\
&\vdots \\
y_{k,t} &= \alpha_k + \sum_{i=1}^{p} a_{ki} y_{1,t-i} + \sum_{i=1}^{p} b_{ki} y_{2,t-i} + \cdots \sum_{i=1}^{p} c_{ki} y_{k,t-i} + v_{k,t}
\end{aligned}
$$