University of Technology Sydney
Department of Mathematical and Physical Sciences

37233 Linear Algebra
Tutorial Problems 1

Question 1.

Let $\mathbf{A} = \begin{pmatrix} 0 & 1 \\ 3 & 2 \\ 4 & 1 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 1 & 2 & -1 \\ 3 & 0 & 1 \end{pmatrix}$. Calculate by hand, where possible: $\mathbf{A} + \mathbf{B}$, $\mathbf{A} + \mathbf{B^T}$, $\mathbf{AB}$, $\mathbf{BA}$, $\mathbf{AB^T}$, and identify any calculations which are **not** possible.

Question 2.

Find all solutions of the following systems. State pivot positions, basic variables and free variables if any. Use:

- Gaussian elimination
- Gauss-Jordan elimination

(a)

$$\begin{aligned} 2x_1 - 4x_2 + x_4 + 7x_5 &= 11 \\ x_1 - 2x_2 - x_3 + x_4 + 9x_5 &= 12 \\ -x_1 + 2x_2 + x_3 + 3x_4 - 5x_5 &= 16 \\ 4x_1 - 8x_2 + x_3 - x_4 + 6x_5 &= -2 \end{aligned}$$

(b)

$$\begin{aligned} 2x_1 - 4x_2 + x_4 + 7x_5 &= 0 \\ x_1 - 2x_2 - x_3 + x_4 + 9x_5 &= 0 \\ -x_1 + 2x_2 + x_3 + 3x_4 - 5x_5 &= 0 \\ 4x_1 - 8x_2 + x_3 - x_4 + 6x_5 &= 0 \end{aligned}$$

(c)

$$\begin{aligned} x_1 - x_2 - 3x_3 + 8x_4 &= -2 \\ 3x_1 - 3x_3 + 9x_4 &= -1 \\ x_1 + x_2 + x_3 - 2x_4 &= 1 \end{aligned}$$

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(d)

$$\begin{aligned} -x_1 + x_2 \quad +x_3 &= 9 \\ 2x_1 + x_2 \quad -x_3 &= -10 \\ 3x_1 \qquad\quad -2x_3 &= -19 \\ -x_1 + 2x_2 - 3x_3 &= -10 \end{aligned}$$

Question 3.

Show that the system does not have a solution. With what number you should replace the right hand side of the third equation in order for this system to have a solution?

$$\begin{aligned} x_1 + \ x_2 + x_3 &= 2 \\ x_1 + 2x_2 + 3x_3 &= 1 \\ x_2 + 2x_3 &= 0 \end{aligned}$$

Question 4.

Consider the system of linear equations, $A\mathbf{x} = \mathbf{b}$ where:

$$\mathbf{A} = \begin{pmatrix} 2 & 4 & 6 \\ 4 & 5 & 5 \\ 3 & 1 & -3 \end{pmatrix} \qquad \mathbf{b} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}.$$

(a) Find $\mathbf{A^{-1}}$ by row reducing the array $(\mathbf{A}|\mathbf{I})$.

(b) Now calculate the matrix of cofactors

$$\mathbf{B} = \begin{pmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{pmatrix}$$

of the matrix A and verify that $\mathbf{A^{-1}} = \dfrac{1}{\det(\mathbf{A})}\mathbf{B}^T$.

(c) Use your inverse matrix to solve the system of equations.

(d) Solve the system by using Gaussian elimination.

Question 5.

Use Mathematica to find solutions of above questions numerically.

Question 6.

You are given the matrices

$$\mathbf{A} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} -1/2 & -\sqrt{3}/2 \\ \sqrt{3}/2 & -1/2 \end{pmatrix}, \quad \mathbf{C} = \begin{pmatrix} -1/2 & \sqrt{3}/2 \\ -\sqrt{3}/2 & -1/2 \end{pmatrix}.$$

Fill in the missing entries of the table overleaf:

Product	Result
AA	
AB	
AC	
BA	**B**
BB	**C**
BC	**A**
CA	
CB	
CC	

(a) Which matrix plays the role of the identity matrix?

(b) Which matrix is the inverse of the matrix C?

(c) Do any of the matrix products commute? Use the table to write down the result of BAC.

Question 7.

Find 2×2 real matrix with the property $\mathbf{A^2} = -\mathbf{I}$, where $\mathbf{I}$ is an identity matrix.