

MATH 222: Week 4

Sarah Randall

Last updated: May 23, 2017

Contents

1 §14.5 Chain Rule

1

1 §14.5 Chain Rule

The chain rule in 1-dimension is as follows:

For an equation $y = f(x(t))$

$$\frac{dy}{dt} = \frac{df}{dx} \frac{dx}{dt}$$

Example. If $y = (x(t))^2$ and $x(t) = \ln 1 + t$

$$\frac{dy}{dt} = \frac{dy}{dx} \frac{dx}{dt} = 2x \frac{1}{1+t} = \frac{2 \ln 1 + t}{1+t}$$

Example. Suppose $f(x, y) = xy + x^2 + y$

$$x(t) = \ln 1 + t, \quad y(t) = e^{t^2}$$

Turn $f(x, y)$ into a function $g(t)$ with only the time parameter.

$$g(t) = f(x(t), y(t)) = \ln 1 + te^{t^2} + (\ln 1 + t)^2 + e^{t^2}$$

$$\frac{dg}{dt} = \frac{df}{dt} = \frac{e^{t^2}}{1+t} + 2t \ln 1 + te^{t^2} + \frac{2 \ln 1 + t}{1+t} + 2te^{t^2}$$

Wherever we can, replace the values of $x(t)$, $y(t)$ with $x(t)$, $y(t)$

$$\frac{dg}{dt} = \frac{df}{dt} = \frac{y(t)}{1+t} + 2te^{t^2}x(t) + \frac{2}{1+t} + 2te^{t^2}$$